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Unit of account, sovereign debt, and optimal currency area

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ABSTRACT

This paper considers how the choice of a *unit of account* affects the formation of an optimal currency area (OCA). First, we show that forming a currency union internalizes the exchange rate risk and leads to smoothing of consumption levels. However, changing the *unit of account* of the inherited sovereign debt to a common currency may increase a country's debt burden if a debtor country is more likely to face a trade deficit within the union. Therefore, the OCA is determined by this trade-off and the debtor country may be better off choosing not to enter the currency union when it faces a high inherited sovereign debt.

1. Introduction

Following the 2011 sovereign debt crisis in the eurozone, there has been considerable renewed interest in optimal currency area (OCA) theory, with academics and policy makers debating whether the eurozone must be an OCA to survive.

Since Mundell (1961) pioneered the theory of the OCA, some consensus has been reached about the preconditions for a smoothly functioning monetary union, including a convergence of economic conditions, labor mobility, and fiscal integration. In accordance with these criteria, the Maastricht Treaty was signed in 1992 and the euro was established in 1999.

Although the introduction of the euro has enhanced the economic activities of the eurozone as a whole, several indicators have suggested that the economic disparity between the member countries has widened. Fig. 1 shows the trade balance of eight key euro countries within the euro area during 2000–2010. The figure shows that (1) although the northern countries, such as Germany and the Netherlands, have enjoyed trade surpluses with the euro area, the southern countries, such as Portugal, Greece, and Spain, have faced trade deficits, and (2) the trade account imbalances have widened gradually among the member countries since the introduction of the euro.[2]

This trade imbalance within the eurozone changes the intraregional movements of currency in the union, which in turn may affect the burden of sovereign debt in each member country. This is because, when countries enter the currency union, they lose monetary sovereignty over the currency in which their debt is denominated so that if deficit (surplus) countries attract less (more) currency, their burden of debt payment becomes more (less) severe. Fig. 2 shows the net debt interest payment to gross domestic product (GDP) ratio for the eurozone countries. As it indicates, while the burden of debt payment has become less severe in northern countries, it has become more severe in southern countries since the middle of the 2000s. As Eichengreen (2014) notes, the 2011 sovereign debt crisis revealed that a heavy inherited debt burden in southern European countries caused a severe budget deficit

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[2] As Lane (2012) notes, the current account imbalance was a precrisis risk factor in the European sovereign debt crisis.

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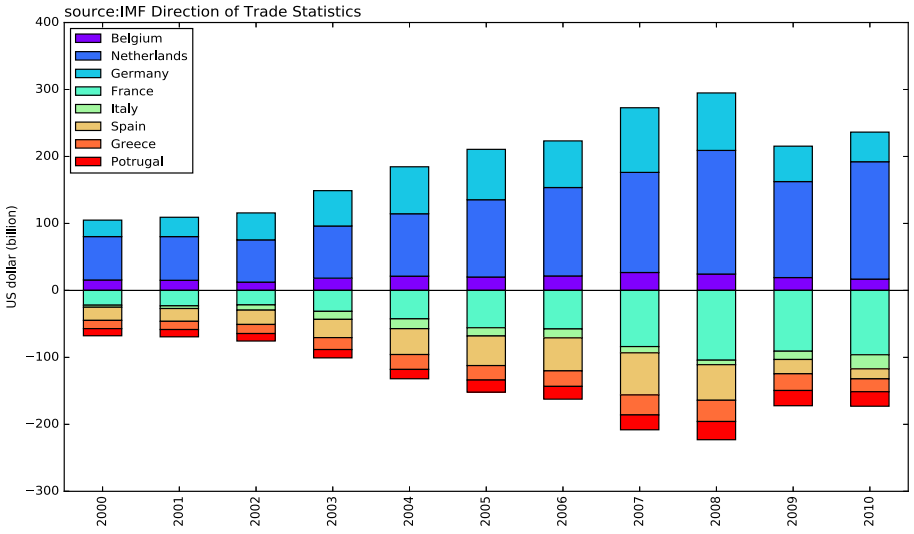

Fig. 1. Trade imbalance within the euro area.

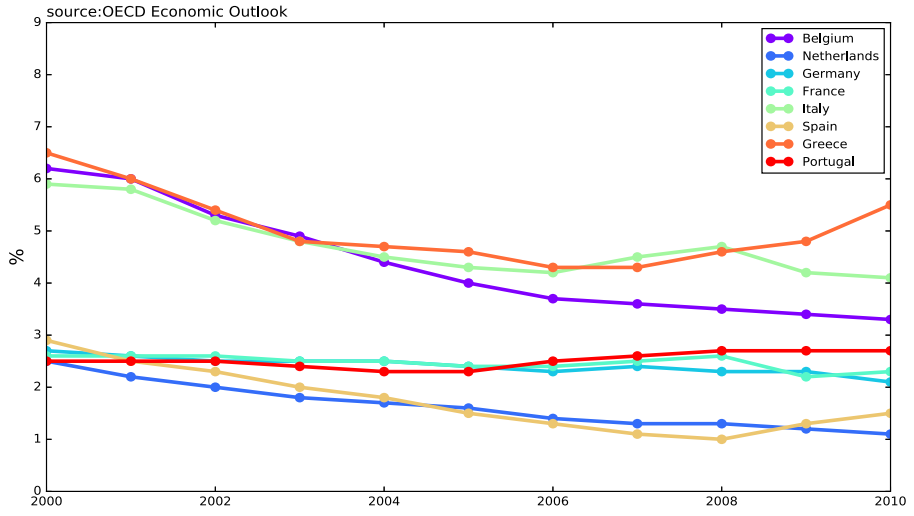

Fig. 2. Net debt interest payment/GDP ratio.

and a rise in debt-to-GDP ratios.[3] These facts reveal that the classical OCA theory overlooked not only the burden of inherited debt overhang, but also how the economic disparity in the union affects the burden of sovereign debt in each of the member countries.

To incorporate these issues into the analytical framework of OCA theory, the role of a common currency as a *unit of account* should be highlighted. This is because, when a country enters a currency union, a common currency is widely used as a *unit of account* in intraregional trade and the inherited debt contracts denominated in the national currency are newly denominated in the common currency. Although the existing OCA literature has highlighted various functions of a common currency, its role as a *unit of account* has received little attention.[4] Therefore, the main objective of this paper is to explain how the choice of the *unit of account* affects the formation of the OCA when a country faces an inherited sovereign debt and economic disparity exists in the union.

We outline the model as follows. There are two island countries, with identical households and banks. A household, which is endowed with one unit of a country-specific good, deposits the endowment with a bank in the same country and, in return, receives a banknote. This banknote represents the national currency of the island. In the next period, households exchange their banknotes to obtain the goods they want. Here, we introduce a preference shock that is common to all households. We assume that the financial

[3] Kalemli-Özcan et al. (2019) find that firms in eurozone countries reduced their investments during the sovereign debt crisis, and this depression of investment was stronger when firms were linked to weak banks with large exposures to sovereign risk.

[4] Mongelli (2002) provides a detailed survey on the evolution of OCA theory.

market is incomplete in that it is not possible to arrange mutual insurance among households before the realization of the preference shock.

Under these settings, if a good in a country is preferred more (less), the currency of the country is demanded more (less). Therefore, fluctuations in the value of the currencies induce fluctuations in the value of the households' assets, so that consumption levels also fluctuate. However, if the central bank can introduce a common currency, it will be chosen as an optimal *unit of account* in a deposit contract because the common currency enables households to share the risk to their consumption. This risk-sharing effect in relation to consumption levels is the benefit of the common currency.

However, when a country owes debt to another country in the union, the result alters. Changing the *unit of account* of the inherited debt from a national currency to a common currency may increase a country's debt burden if the good of that country is likely to be less preferred. This variation of the debt burden is the cost of the common currency. Therefore, we can derive the condition in which the benefit of common currency is equal to its cost and thus can depict the OCA. Moreover, we investigate whether the creditor country has an incentive to bail out the debtor country and show that, in some cases, bailout is effective not only for a debtor country but for a creditor country to maintain the common currency regime.

This trade-off between the risk-sharing effect in relation to consumption levels and the variation of the debt burden illustrates the importance of the choice of the *unit of account* and its effect on the formation of the OCA. Moreover, our paper shows the effectiveness of bailout in maintaining a currency union. Therefore, we believe that our results provide a new perspective on the benefits and costs of a common currency and contribute to developing a new framework for OCA theory.

Related literature: The OCA is determined by the trade-off between the benefits and the costs of adopting a common currency. Classical OCA theory supposes that various transaction costs and nominal rigidities in prices exist. Under these circumstances, the benefits of adopting a common currency are derived from decreasing the transaction costs, whereas the costs are derived from a lack of monetary sovereignty to stabilize the economy. Therefore, as Mundell (1961) suggests, if there were no nominal rigidities, the benefit would always outweigh the costs so that a unique common currency would exist for the entire world.

However, as Goodhart (1998) and Eichengreen (2014) note, the classical OCA theory overlooks the crucial political economy factors, such as sovereign debt problems. After the 2011 sovereign debt crisis, several papers considered the relationship between the sovereign debt problem and monetary policy in a currency union. Corsetti and Dedola (2016) consider the role of the central bank as a backstop for government funding and show that monetary authorities can rule out a self-fulfilling sovereign debt crisis by adopting an unconventional monetary policy. Aguiar et al. (2015) show that a high-debt country facing a potential rollover crisis may be better off if it belongs to a currency union with an intermediate mix of high- and low-debt members. Bolton and Huang (2018) consider the value of monetary sovereignty and find that the OCA is determined by the trade-off between monetary flexibility and the costs of strategic monetizations. These papers show that the burden of sovereign debt is affected by the centralized monetary policy in a currency union. In contrast, our paper shows that the burden of sovereign debt is affected by the country's choice of a *unit of account* and the economic disparity within the union.

Several empirical papers examine the relationship between the lack of monetary sovereignty and the 2011 sovereign debt crisis.[5] De Grauwe and Ji (2013) and Ghosh et al. (2013) show that when the crisis erupted, sovereign bond yield spreads rose more sharply for eurozone member countries than for other countries with monetary sovereignty. Dell'Erba et al. (2013) also find that the positive relationship between debt levels and sovereign bond yield spreads is amplified by the presence of large net foreign liabilities and that this amplifying effect becomes larger in eurozone countries than in other advanced countries with monetary sovereignty. These empirical findings suggest that the lack of monetary sovereignty affects the fiscal condition of a debtor country in the currency union. In contrast to these papers, we show that the lack of monetary sovereignty affects the burden of inherited sovereign debt and thus affects the formation of the OCA.

Moreover, following the contribution of Reinhart and Rogoff (2010), a significant strand of literature has investigated whether countries with large public debt overhangs tend to experience persistent stagnation.[6] Although Reinhart and Rogoff's (2010) research provoked controversy (e.g., Herndon et al., 2014), the 2011 sovereign debt crisis stimulated the debate about the relationship between public debt overhang and growth in eurozone countries (e.g., Checherita-Westphal and Rother, 2012; Baum et al., 2013; Gómez-Puig and Sosvilla-Rivero, 2015; De Vita et al., 2018). In contrast to these papers, we focus on the problem of how the burden of public debt overhang is affected by the trade imbalance within the union.

Our research is related to several other strands of literature. First, Asdrubali et al. (1996) and Sørensen and Yosha (1998) consider mechanisms for achieving income insurance and consumption smoothing in a currency union. They show how risk sharing in a union is achieved via financial markets and government transfers. In contrast to these papers, we show that a common currency *per se* provides the function of consumer insurance when it is accepted as a *unit of account* in financial contracts.

A growing body of research has investigated the special role of the dollar in international trade contracts. Ize and Yeyati (2003), Bocola and Lorenzoni (2020) and Gopinath and Stein (2021) consider the role of the *unit of account* in international trade contracts, such as trade invoices or debt contracts, and explain financial dollarization as the result of the choice of currency denomination of the contracts. In contrast to these works, our paper derives the endogenous need for common currency as an optimal *unit of account* in contracts.

Finally, although the benefits and costs of a common currency as a *medium of exchange* are often considered in an environment in which search-matching frictions in transactions exist (e.g., Matsuyama et al., 1993; Trejos and Wright, 1996; Ravikumar and

[5] As Corsetti (2010) and De Grauwe (2011) note, because the members of the currency union do not have monetary sovereignty, the countries that join a currency union face problems similar to those of emerging market countries attempting to borrow in a foreign country.

[6] Panizza and Presbitero (2013) provide a comprehensive survey of the literature on the relationship between public debt and growth.

Wallace, 2002; Kiyotaki and Moore, 2003), the benefits and costs of a common currency as a *unit of account* have been considered only rarely. Freeman and Tabellini (1998) find that even when privately issued IOUs can be circulated in an economy, a common currency is chosen not only as a *medium of exchange* but also as a *unit of account*. Doepke and Schneider (2017) show that countries choose a common currency as a *unit of account* if the intensity of cross-border trade increases and the value of a national currency is too volatile. In contrast to these papers, we show that the inherited debt burden affects the choice of a *unit of account* and the formation of an OCA.

The remainder of this paper is organized as follows. Section 2 provides the model. Section 3 considers the benchmark case and derives the socially optimal allocation and the competitive equilibrium allocation without a common currency. Section 4 shows that the introduction of a common currency issued by the central bank improves welfare. Section 5 shows that, under the existence of inherited sovereign debt, choosing a national currency may be optimal for a debtor country with a less preferred tradable good, indicating the required conditions for the OCA. We also show that bailout may be effective in maintaining the currency union. Section 6 concludes.

2. The model

Time consists of two periods, 0 and 1. There are two island countries, indexed by $i \in \{1,2\}$. In each country, there are two agents, households and banks. In the subsequent analysis, a household and a bank located in country i are denoted as household i and bank i, respectively.

In country i, there is a continuum of households, the population of which is normalized to one. At the beginning of period 0, a representative household i is endowed with one unit of country-specific good i, e_i. Households do not engage in any production activity in this economy. In period 1, a representative household i receives utility by consuming both e_1 and e_2. The value of utility from consumption for a representative household i in period 1, U_i, is denoted by:

$$U_i(\omega_s) = \alpha(\omega_s)\ln C_i^1(\omega_s) + (1-\alpha(\omega_s))\ln C_i^2(\omega_s),$$

where $C_i^j(\omega_s)$ is consumption of e_j ($j \in \{1,2\}$) by a representative household i when a state ω_s ($s \in \{a,b\}$) occurs. In terms of α and ω_s, we assume that all households face the same preference shock at the beginning of period 1, such that ω_a (ω_b) occurs with probability q $(1-q)$ and:

$$\alpha(\omega_s) = \begin{cases} \alpha & \text{when } \omega_s = \omega_a, \\ 1-\alpha & \text{when } \omega_s = \omega_b, \end{cases}$$

where $0 < \alpha < 1$.[7] For example, suppose that $\alpha = 2/3$. Then, all households in both countries put twice (half) as much weight on consuming e_1 as on consuming e_2 when ω_a (ω_b) is realized.

Here, we assume that the financial market is incomplete so that households cannot arrange mutual insurance against the preference shock in period 0. This is a key assumption for the following arguments and it is appropriate for the eurozone because, as Sørensen and Yosha (1998) show, financial markets in the eurozone were not integrated before the introduction of the euro, as neither factor income flows nor cross-border flows of physical goods contributed significantly to international risk sharing.[8]

The good with which the representative household i is endowed is stored until period 1. Here, we assume that households do not have a storage technology, so they rely on another agent, the "bank", which has a large "freezer" in which the goods can be perfectly stored until period 1. In each country, there is a continuum of banks, the population of which is normalized to one. Banks in country i are perfectly competitive and, due to spatial separation, a representative household i deposits its endowment only with bank i.[9] A representative bank i is endowed with one unit of an asset, which is common to banks 1 and 2, at the beginning of period 0 and consumes the asset in period 1. Banks are risk neutral and the value of utility from consumption for a representative bank i in period 1, V_i, is simply given by:

$$V_i(\omega_s) = A_i(\omega_s),$$

where A_i is bank i's consumption of a bank asset when ω_s is realized.

Goods are traded in perfectly competitive markets at $t = 1$. In these markets, there is no friction in goods transactions so that the equilibrium price of goods is determined competitively.

3. Benchmark allocation

First, we derive the socially optimal allocation. Next, we derive the allocation of a competitive equilibrium without a common currency.

[7] Although we adopt this type of shock to simplify the exposition, our results generalize to a more general preference shock under which both households receive independent and identically distributed preference shocks.

[8] Note that to focus on the role of the *unit of account* in transactions, we consider the simple endowment economy. However, even when each household engages in a specific production sector, the following arguments can be applied if the household cannot work in multiple sectors because they possess sector-specific skills.

[9] This assumption can be interpreted in another way. That is, the "freezer" is good-specific equipment such that bank i can store only good i.

3.1. Socially optimal allocation

First, we consider the socially optimal allocation. Given the level of (α, q), the social planner solves the following maximization problem:

$$\max_{C_i^j(\omega_s), A_i(\omega_s)} \sum_i \Big(E\big[U_i(\omega_s)\big] + E[V_i(\omega_s)] \Big)$$

$$\text{s.t.} \quad \sum_i C_i^j(\omega_s) = 1 \tag{1}$$

$$\sum_i A_i(\omega_s) = 2 \tag{2}$$

$$C_1^j(\omega_s) = C_2^j(\omega_s), \quad A_1(\omega_s) = A_2(\omega_s) \tag{3}$$

for $j \in \{1,2\}$ and $s \in \{a,b\}$. (1) and (2) denote the resource constraint for good j and the asset, respectively. (3) means that, at the socially optimal resource allocation, the level of consumption in period 1 does not depend on the place in which a household and a bank are located. Solving this maximization problem, we can derive the following proposition.

Proposition 1 (*Socially optimal allocation*). *The socially optimal allocation is* $C_i^j(\omega_s) = \frac{1}{2}, A_i(\omega_s) = 1 \ \ \forall i, j \in \{1,2\}, s = \{a,b\}$.

Proposition 1 states that because a preference shock is common across all households, it is socially optimal to allocate the same amount of endowments to both households, irrespective of the state that occurs in period 1. In addition, Proposition 1 indicates that any bank cannot obtain excess returns in period 1 so that $A_i(\omega_s) = 1$ is optimal.

3.2. Equilibrium without a common currency

Next, we consider the case in which a common currency does not exist in the economy. In period 0, a representative household i deposits its endowment, e_i, in a representative bank i and receives bank i's banknote, which represents the national currency of an island i. Banknote i is a bearer note so that a holder of the banknote is certain to receive a unit of e_i in period 1. In this sense, the *unit of account* of this deposit contract is the amount of e_i. In period 1, after the realization of a preference shock, all households exchange their banknotes to obtain the goods they want. Because a banknote is supposed to be divisible, banknote i can be used as a *medium of exchange* in these transactions.

First, the maximization problem of a representative household i is defined as:

$$\begin{aligned} \max_{C_i^j(\omega_s)} \quad & U_i(\omega_s) \\ \text{s.t.} \quad & \sum_j P_j(\omega_s) C_i^j(\omega_s) = P_i(\omega_s), \end{aligned} \tag{4}$$

where $P_i(\omega_s)$ is the goods-market price of good i and it also denotes the price of banknote i in period 1 when ω_s is realized. Therefore, the constraint (4) represents the budget constraint for a representative household i.

Next, the maximization problem of a representative bank i is defined as:

$$\begin{aligned} \max_{A_i(\omega_s)} \quad & V_i(\omega_s) \\ \text{s.t.} \quad & P^a(\omega_s) A_i(\omega_s) = P^a(\omega_s), \end{aligned} \tag{5}$$

where $P^a(\omega_s)$ denotes the asset-market price of an asset in period 1 when ω_s is realized. Therefore, the constraint (5) represents the budget constraint for a representative bank i. The market-clearing conditions in goods and an asset are identical to the resource constraints given by (1) and (2), respectively.

Then, given the set of parameters, (α, q), an equilibrium in this economy consists of a vector of parameters $(C_i^j(\omega_s), A_i(\omega_s), P_i(\omega_s), P^a(\omega_s))$, such that: (1) a representative household i solves (4) for given levels of α and $P_i(\omega_s)$; (2) a representative bank i, facing perfect competition among banks in the same country, solves (5) under given levels of α and $P^a(\omega_s)$; (3) the exchange rate of banknotes is determined competitively; and (4) markets in goods and an asset clear.

The following proposition characterizes the optimal allocation in the economy without a common currency.

Proposition 2 (*The economy without a common currency*). *For all* $j \in \{1,2\}$, $C_1^j(\omega_s) = \alpha(\omega_s), C_2^j(\omega_s) = 1 - \alpha(\omega_s), P_1(\omega_s)/P_2(\omega_s) = \alpha(\omega_s)/(1-\alpha(\omega_s))$, *and* $A_i(\omega_s) = 1, P^a(\omega_s) = 1$.

Proof. See Appendix A.

Proposition 2 shows that the consumption level of a representative household i varies depending on the realization of the states: because the value of banknote i equals the price of e_i, the value of banknote i becomes high (low) when e_i is preferred more (less) in period 1. Therefore, a preference shock induces fluctuations in the relative price of banknotes, so that the consumption levels of households also fluctuate. On the other hand, Proposition 2 shows that a representative bank i consumes its own assets in both states so that a preference shock does not affect the behavior of banks in this case.[10] Therefore, when a common currency does not

[10] As a result, a market for bank assets is not established in this case.

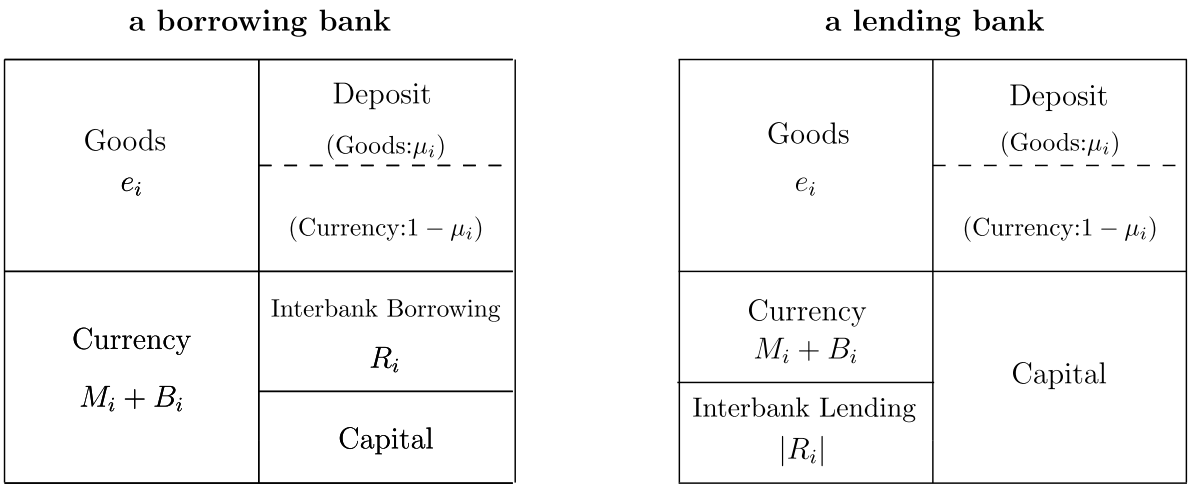

Fig. 3. Banks' balance sheets at the beginning of period 1.

exist, the risk induced by a preference shock directly damages the payoff of risk-averse households. This inefficiency occurs because households cannot diversify the relative price shock *ex ante* owing to the incomplete financial market.

4. Introduction of a common currency

Next, we introduce the third agent, called "the central bank", which is located between the two island countries. The central bank has the ability to supply its tickets to both banks in exchange for a bank asset if it is considered an eligible asset at the end of period 0. We call these tickets "currency". In this sense, the supply of currency can be interpreted as liquidity provision by the central bank through purchasing banks' assets.[11]

In this environment, if a bank chooses to exchange its asset for currency, the *unit of account* in a deposit contract can be denominated not only in goods but also in currency. That is, in a deposit contract, a representative household i can demand a deposit contract in which a fraction γ_i is repaid in the form of e_i and a fraction $1-\gamma_i$ is repaid in the form of currency, m_i, in period 1.

On the other hand, a representative bank i offers a deposit contract in which a fraction μ_i is repaid in the form of e_i and a fraction $1-\mu_i$ is repaid in the form of currency. Then, to comply with the contract, a representative bank i should collect at least $(1-\mu_i)m_i$ units of currency from the central bank and the interbank market at the beginning of $t=1$. Let M_i be the amount of currency that a bank i is supplied by the central bank in exchange for its asset, let B_i be the amount of currency that it borrows from other banks in period 0, and let R_i be the amount of currency that it repays to other banks in period 1. Note that when a bank i borrows from (lends to) other banks, B_i and R_i become positive (negative). Then, the banks' balance sheets at the beginning of period 1 can be depicted as in Fig. 3.

Under this modification, the maximization problem of a representative household i in period 1 is given by:

$$\begin{aligned} \max_{C_i^j(\omega_s)} \quad & U_i(\omega_s) \\ \text{s.t.} \quad & \sum_j P_j(\omega_s) C_i^j(\omega_s) = \gamma_i P_i(\omega_s) + (1-\gamma_i) m_i. \end{aligned} \tag{6}$$

The budget constraint of (6) differs from that of (4) because a fraction $1-\gamma_i$ of a deposit is repaid in the form of currency. Then, in period 0, a representative household i chooses γ_i to maximize its expected utility.

Next, the maximization problem of a representative bank i in period 1 is given by:

$$\begin{aligned} \max_{A_i(\omega_s)} \quad & V_i(\omega_s) \\ \text{s.t.} \quad & P^a(\omega_s) A_i(\omega_s) = \Delta m_i - R_i + (1-\mu_i) P_i(\omega_s), \\ & \Delta m_i \equiv M_i + B_i - (1-\mu_i) m_i, \end{aligned} \tag{7}$$

[11] This type of monetary policy operation is considered by Kiyotaki and Moore (2019), who show that the central bank can improve resource allocation by replacing real assets with money, if real assets are subject to a resaleability constraint. In contrast to their work, this paper presents an environment in which agents need currency issued by the central bank endogenously.

where Δm_i denotes the balance of the amount of currency that it holds. Consider the third term on the right-hand side of the constraint. According to the deposit contract, a fraction $1-\mu_i$ of the deposit is repaid in currency so that a representative bank i can sell $1-\mu_i$ units of e_i at the competitive price $P_i(\omega_s)$.

The maximization problem of a representative bank i in period 0 is given by:

$$\begin{aligned} \max_{\mu_i} \quad & E[V_i(\omega_s)] \\ \text{s.t.} \quad & M_i \le E[P^a(\omega_s)] \\ & \Delta m_i \ge 0. \end{aligned} \tag{8}$$

The first constraint implies that the central bank supplies currency only up to the expected value of a bank asset.[12] The second constraint implies that the amount of currency that a representative bank i can commit to paying a depositor (i.e., $(1-\mu_i)m_i$) is an amount up to the sum of the currency supplied by the central bank and the currency borrowed from other banks.

Given the set of parameters, (α, q), an equilibrium is characterized as in the previous case, except that we add the following three equations concerning the demand for and supply of currency:

$$\gamma_i = \mu_i, \tag{9}$$

$$\sum_i m_i = \sum_i M_i \equiv M, \tag{10}$$

$$\sum_i B_i = \sum_i R_i = 0. \tag{11}$$

(9) shows that in a deposit contract, the demand for currency by a representative household i equals the supply of currency by a representative bank i. (10) shows that the total demand for currency by banks equals the total supply of currency generated by the central bank. (11) shows the condition for the interbank market to be cleared.

Solving this equilibrium, we can derive the following proposition.

Proposition 3 (*Introduction of a common currency*). *When a common currency is introduced, it is optimal for both a representative household i and bank i to choose $\mu_i = \gamma_i = 0 \quad \forall i \in \{1,2\}$ so that a common currency becomes the unique unit of account in the deposit contract. Under these circumstances, $C_1^j(\omega_s) = \rho, C_2^j(\omega_s) = 1-\rho, A_1(\omega_s) = 1-2\rho + P_1(\omega_s)/P^a(\omega_s), A_2(\omega_s) = 2\rho - 1 + P_2(\omega_s)/P^a(\omega_s) \quad \forall j \in \{1,2\}$ and $s \in \{a,b\}$, where $\rho \equiv q\alpha + (1-q)(1-\alpha)$. In addition, $P_1(\omega_a) = P_2(\omega_b) = \alpha M$, $P_1(\omega_b) = P_2(\omega_a) = (1-\alpha)M$, $P^a(\omega_s) = M/2$.*

Proof. See Appendix B.

Proposition 3 indicates that in an equilibrium with a common currency, all households choose a common currency as an optimal *unit of account* in the deposit contract (i.e., $\gamma_i = \mu_i = 0$). This is because households can share the risk induced by the preference shock *among* countries by committing to using a common currency as a *unit of account* in a deposit contract. In this sense, a common currency emerges endogenously as a result of the households' need to share relative price risk among countries. In addition, $E[V_i] = 1$ holds for $i \in \{1,2\}$ so that the relative price shock faced by the household sector is fully absorbed by the risk-neutral banks.[13]

In equilibrium, the currency is circulated in this economy as follows. In the first transaction, the currency is provided to the banking sector by the central bank in exchange for a bank asset at the end of period 0. In the second transaction, because the currency is considered as the optimal *unit of account* in the deposit contract, the currency is paid to households at the beginning of period 1. In the third transaction, banks receive currency from households as the price of the goods because banks sell the deposited goods in the goods market. In the final transaction, the central bank receives the currency from banks as the price of assets.[14] As a result of these transactions, all the currency is finally returned to the central bank and it then vanishes from this economy at the end of period 1. These transactions show that the common currency, which is an intrinsically valueless ticket, becomes a valuable instrument for both households and banks when the common currency satisfies two functions. The first function is that it is chosen as an optimal *unit of account* in deposit contracts by households, as otherwise the currency would not be supplied into this economy. The second function is that it is accepted by the central bank as the means of payment for bank assets in the final transaction. Because of these functions of the common currency, it is also widely accepted as a *medium of exchange* in the third transaction in this economy.

To achieve the allocation derived in Proposition 3, the interbank market plays an important role. That is, when e_i is more (less) likely to be preferred in period 1, bank i should promise household i to provide more (less) currency in period 1 in return for receiving a unit of e_i in period 0.[15] Because the amount of currency supplied by the central bank (i.e., M_i) is the same for all banks, a bank that demands more (less) currency – i.e., m_i is high (low) – borrows from (lends to) other banks in the interbank market.

[12] In the Eurosystem, assets are subject to specific valuation haircuts, the rates of which differ depending on the quality, risk, and the issuer of the assets. In this paper, for simplicity, we assume that a bank asset is subject to a haircut of 0%.

[13] The result in Proposition 3 is robust because we can obtain the same result even if we consider different environments, such as varying the population of the islands or introducing a home bias in consuming goods.

[14] In this transaction, the bank asset is supplied only by the central bank. Therefore, the provision and withdrawal of currency in this model can be interpreted as an open market operation, as the central bank exchanges currency for assets at the rate it sets.

[15] For example, e_1 is more likely to be preferred in period 1 when 1) q is high and α is high or 2) q is low and α is low.

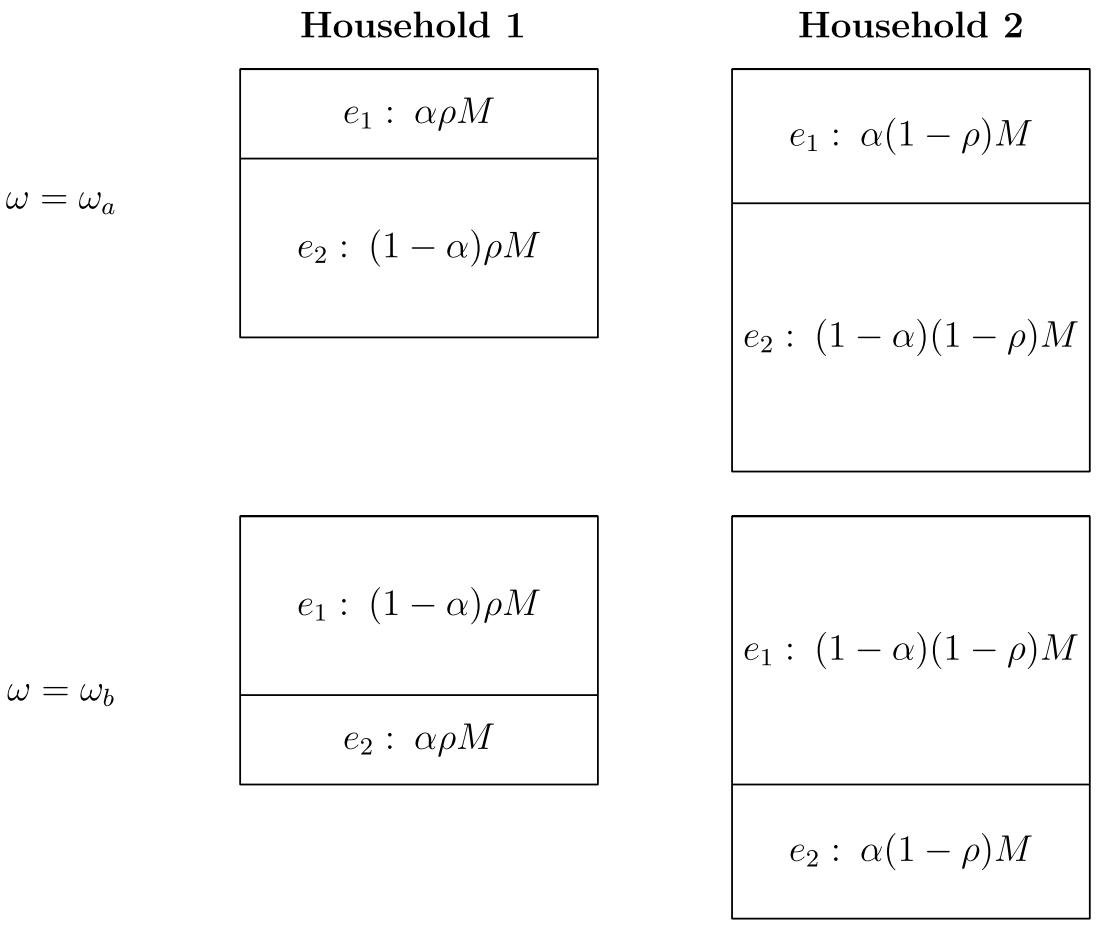

Fig. 4. Expenditure of both households when $\alpha < 1/2$ and $q < 1/2$.

For a borrowing bank, the deposited good is more likely to be sold at a high price so that it can collect enough currency to satisfy the repayment to other banks at the end of period 1. Therefore, all banks can satisfy the liquidity needs in period 1 because of the existence of the interbank market.

In equilibrium, a representative household 1 and 2 receives ρM and $(1-\rho)M$ units of currency, respectively. Fig. 4 shows the expenditure of representative households 1 and 2. When ω_a is realized, both households allocate fractions α and $1-\alpha$ of their income to e_1 and e_2, respectively. Then, country 1 receives $\alpha(1-\rho)M$ units of currency from country 2. On the other hand, when ω_b is realized, country 1 receives $(1-\alpha)(1-\rho)M$ units of currency from country 2. Then, note that $\rho \equiv q\alpha + (1-q)(1-\alpha)$, the expected amount of imports of country 1, can be written as:

$$q\alpha(1-\rho)M + (1-q)(1-\alpha)(1-\rho)M = \rho(1-\rho)M.$$

The expected amount of exports of country 1 can be written as:

$$q(1-\alpha)\rho M + (1-q)\alpha\rho M = \rho(1-\rho)M.$$

Therefore, in equilibrium, the *ex ante* trade account is balanced.[16]

Although the equilibrium derived in Proposition 3 can achieve a more efficient allocation than can an economy without a common currency, it cannot achieve the first-best allocation when $q \neq 1/2$. This is because when $q \neq 1/2$, the expected value of banknotes varies and, thus, m_i varies between households 1 and 2, which leads to their consumption levels varying.

5. Sovereign debt and the optimal currency area

Next, we introduce inherited sovereign debt into the model and consider how it affects the choice of the *unit of account* and the formation of the OCA.

[16] Note that, for example, if $q < 1/2$ and $\alpha < 1/2$ hold, then country 1's trade with country 2 becomes a deficit (surplus) when $\omega_a(\omega_b)$ is realized.

Table 1
Euro conversion rates for participating currencies.

Currency	Units of national currency for 1 euro
Belgian franc	40.3399
German mark	1.95583
Spanish peseta	166.386
French franc	6.55957
Irish pound	0.787564
Italian lira	1936.27
Luxembourg franc	40.3399
Dutch guilder	2.20371
Austrian schilling	13.7603
Portuguese escudo	200.482
Finnish markka	5.94573

From January 1, 1999, the euro began to be substituted for the national currencies of the participating member countries of the European Union (EU). This substitution required the adoption of irrevocable conversion rates, which were determined according to the principle described in a joint communiqué issued on May 2, 1998. In compliance with the legal framework for the use of the euro, the irrevocable conversion rate for each participating currency became the only rate used for conversion (Table 1). Using this rate, the inherited sovereign debt denominated in each national currency was transformed into the common currency, the euro.

As De Grauwe (2011) notes, when a country enters a currency union, it ceases to have control over the currency in which its debt is denominated. Therefore, we consider how the change of the *unit of account* affects the burden of inherited sovereign debt and the formation of the OCA.

5.1. Optimal currency area

To consider how the choice of the *unit of account* affects the debt burden, we assume that country 1 owes country 2 d $(0 < d < 1)$ units of debt denominated by e_1, which is an external debt denominated in domestic currency that must be repaid by the end of period 1. First, we consider the case in which the representative household 1 chooses not to enter the currency union. In this case, because the debt is denominated by e_1, the budget constraint of household 1 becomes:

$$\sum_j P_j(\omega_s)C_1^j(\omega_s) = (1-d)P_1(\omega_s). \tag{12}$$

On the other hand, if the household chooses to enter the currency union, the debt is newly denominated in the common currency according to the predetermined irrevocable conversion rate, r. Thus, when $\gamma_i = 0$, its budget constraint becomes:

$$\sum_j P_j(\omega_s)C_1^j(\omega_s) = m_1 - r \cdot d. \tag{13}$$

Next, we focus on the two maximization problems for representative household 1.[17] One is (4), except that the budget constraint is (12), and the other is (6) with $\gamma_i = 0$, except that the budget constraint is (13). As the equilibrium cannot be solved analytically, we derive the expected utility of both cases and consider numerical examples when $r \in \{0.7, 1.2\}$ and $d \in \{0.2, 0.7\}$. Proposition 4 shows the results.

Proposition 4 (*Optimal currency area*)**.** *Suppose that representative household 1 owes d units of debt denominated by e_1 to country 2. Then, there arises a parameter space (α, q) in which representative household 1 prefers to choose a national currency as a unit of account. In addition, the region expands as d becomes larger and r becomes higher.*

Proof. See Appendix C.

In Fig. 5, representative household 1 chooses a common currency when (α, q) exists in the white region, whereas it chooses a national currency in the blue region. Panel (1) of Fig. 5 shows that representative household 1 prefers a national currency when q is high and α is low, or q is low and α is high. The former (latter) case means that $\omega_a(\omega_b)$ is more likely to be realized and in that state, e_1 is less preferred by all households. Therefore, in both cases, the expected value of banknote 1 becomes low. Thus, representative household 1 faces a trade-off. That is, to achieve the risk-sharing effect on its consumption level, it would prefer to choose a common currency. However, this may increase its debt burden because, as (13) shows, m_1 becomes smaller due to less demand for e_1, but the amount of debt repayment is constant even if the undesirable state for country 1 is more likely to occur.[18]

[17] Note that from Proposition 3 households in country 2 always have an incentive to choose a common currency as a *unit of account*. In addition, without loss of generality, the amount of currency, M, is set to be 2 in the subsequent analysis.

[18] In contrast, if e_1 is more likely to become more preferred, the burden of outstanding debt becomes less severe for country 1 when it enters the currency union.

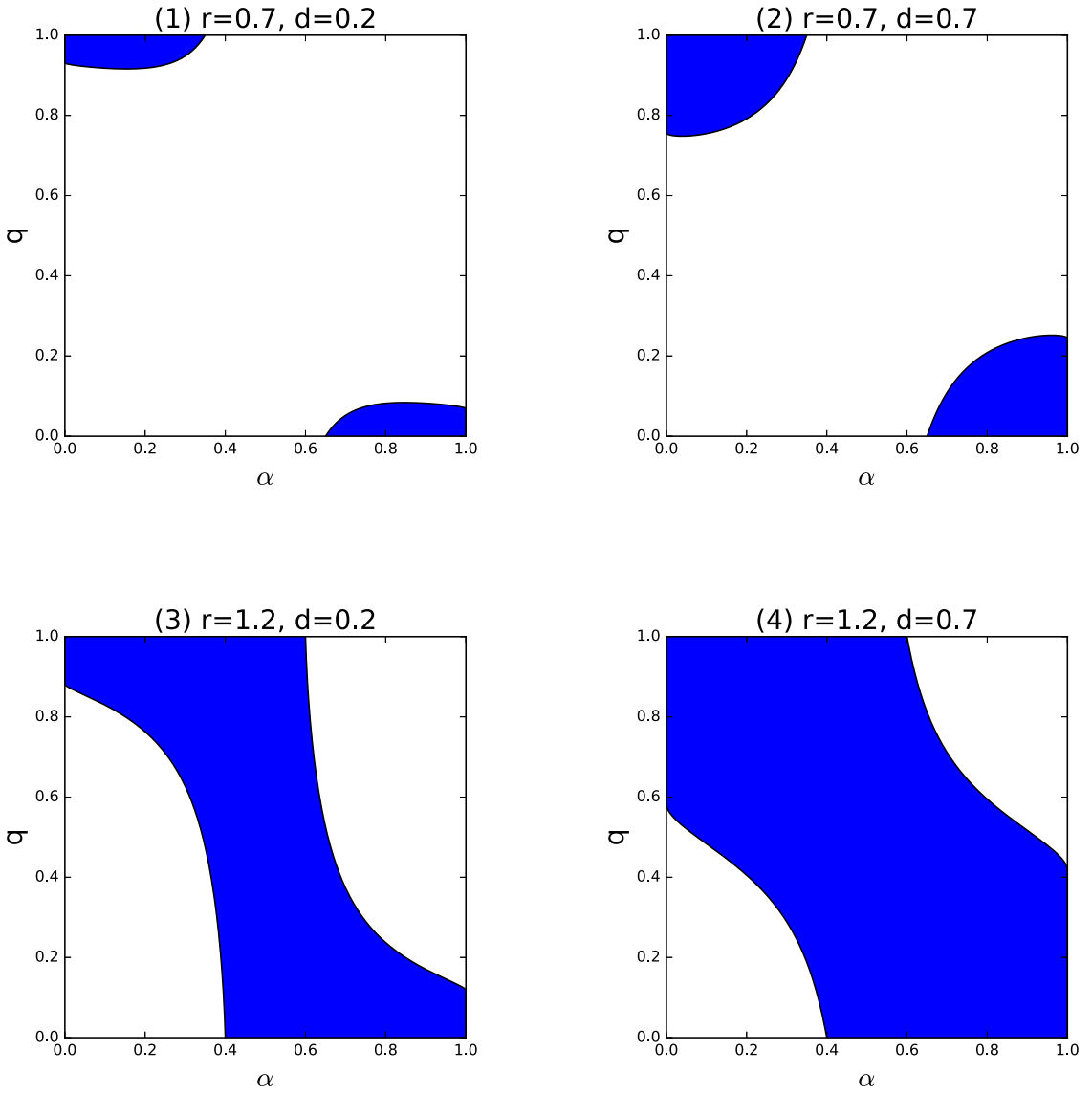

Fig. 5. Optimal currency area.

On the other hand, when it chooses a national currency, as (12) shows, the debt-to-income ratio is constant (i.e., d) irrespective of the state realized. Therefore, if e_1 is more likely to become less preferred in the union, the debt burden of country 1 becomes more severe and, thus, representative household 1 may be better off choosing a national currency as a *unit of account*. Moreover, the other panels in Fig. 5 show that representative household 1 prefers to choose a national currency when d and r become larger.

Proposition 4 highlights the cost of a common currency. That is, adopting a common currency as a *unit of account* generates variations in the debt burden. Therefore, a debtor country with less preferred tradable goods may be better off choosing a national currency. Moreover, as the blue region expands from the upper-left or lower-right region to the center of the figure, representative household 1 prefers to choose a national currency when the values of α and q are located around $1/2$. This means that, in these regions, the risk-sharing effect of a common currency is less attractive so that the benefit of a common currency will be less than its cost.

Recent empirical studies on the eurozone crisis find that a trade imbalance within the union affects a country's burden of sovereign debt. Gros (2013) considers the link between external imbalance and sovereign risk within the union and finds that when a member country faces a trade deficit, its debt burden becomes more severe as its external debt grows. Berger and Nitsch (2014) find that, after the introduction of the euro, bilateral trade imbalances among eurozone countries persistently widened and that countries with low trade surpluses exhibit fiscal deficits. Arghyrou and Kontonikas (2012) find that the real effective exchange rates of the southern countries persistently appreciated after the introduction of the euro, and that the appreciation positively affected their

sovereign bond yield spreads relative to those of Germany during the 2008 financial crisis period. Focusing on the case of Portugal, Silva (2020) finds that after it entered the eurozone, Portugal's external debt-to-GDP ratio became higher and it developed a deficit in its net balance of interest payments. Therefore, the result of Proposition 4 provides a theoretical foundation for these empirical findings about the relation between trade imbalances within the union and the burden of sovereign debt. In addition, as we show in Fig. 5, this relation generates the cost of common currency and thus affects the formation of the OCA.

5.2. Optimal currency area with bailout

Next, we consider whether a creditor country has an incentive to bail out a debtor country in the union. In the eurozone crisis, many debtor countries faced a limited fiscal space before the bailout schemes were designed.[19] Paniagua et al. (2017) find that many debtor countries reacted less actively to the rise of the debt-to-GDP ratio after they entered the eurozone. Okano and Eguchi (2020) show that when the policy authorities of a debtor country do not react to the default risk, introducing a strict fiscal policy generates significant welfare costs if the interest spread is high. Moreover, Kriwoluzky et al. (2019) show that these behaviors of a debtor country may increase the expectation that the country will exit from the union and, thus, raise the rollover risk of the existing sovereign debt. As a result of these behaviors, the crisis of the debtor country is undesirably reinforced.

To counter these difficulties, the euro area authorities designed several financial support programs.[20] First, during 2010–2011, the European Financial Stabilization Facility (EFSF) and the European Financial Stability Mechanism (EFSM) were designed as temporary measures; then, in 2012, the European Stability Mechanism (ESM) was set up as a permanent facility to help euro area countries in severe financial distress.[21] Through these programs, debtor countries received financial support from the EU institutions.[22]

To consider the effect of bailout in the union, we modify the model such that country 2 has an option to reduce the amount of country 1's debt to $\acute{d}$ ($0 \leq \acute{d} < d$), with an additional operating cost of s per unit of renounced debt.[23] This cost includes various operating costs such as those involved with negotiating bailout agreements or renewing a financial contract.

Under this modification, if country 2 accepts the bailout scheme under a common currency regime, the budget constraint for a representative household 2 becomes:

$$\sum_j P_j(\omega_s)C_2^j(\omega_s) = m_2 + r \cdot \acute{d} - s(d - \acute{d}). \tag{14}$$

On the other hand, when country 2 does not accept the bailout scheme, country 1 does not enter the union, so the debt remains denominated by e_1. Therefore, a budget constraint for a representative household 2 becomes:

$$\sum_j P_j(\omega_s)C_2^j(\omega_s) = P_2(\omega_s) + dP_1(\omega_s). \tag{15}$$

Then, we compare the outcomes of the two maximization problems for representative household 2 when (α, q) is located in a blue region in Fig. 5. One is (4), except that the budget constraint is (15), and the other is (6) with $\gamma_i = 0$, except that the budget constraint is (14). Proposition 5 shows the result.

Proposition 5 (*Optimal currency area with bailout*)**.** *Suppose that country 2 can reduce the amount of debt to $\acute{d}$. Then, $\acute{d} = \max\left\{2\left(\rho - \alpha^q(1-\alpha)^{(1-q)}(1-d)\right)/r, 0\right\}$ and there arises a parameter space (α, q) in which country 1 remains in the currency union. The region expands as s becomes smaller.*

Proof. See Appendix D.

Fig. 6 shows the numerical results when $r \in \{0.7, 1.2\}$, $d \in \{0.2, 0.7\}$, and $s = 0.2$. This figure shows that when (α, q) exists in the light blue region, country 2 has an incentive to bail out country 1 because, in that region, a creditor country also becomes better off if a debtor country stays in the common currency regime. This incentive becomes stronger as r and d become larger. Moreover, the region expands as s becomes smaller.[24] However, as the blue region remains in Fig. 6, if more debt must be renounced for country 1 to stay in the currency union, the common currency regime will collapse because country 2 cannot afford to accept this level of bailout.

Recent studies on the eurozone crisis also suggest that bailout is beneficial, not only for debtor countries, but also for creditor countries in the union.[25] Gourinchas et al. (2020) argue that if a member country has a high level of debt, it is beneficial for a

[19] Ghosh et al. (2013) denotes that fiscal space is a notion of fiscal fatigue under which there are limits to the government's ability to raise the primary surplus in response to higher debt.

[20] Article 125 of the Treaty on the Functioning of the European Union, referred to as the "no bailout clause", prohibits member countries taking on the debts of other member countries. However, the European Treaty does not rule out the possibility of financial assistance to distressed member countries through lending, loan restructuring, and so on.

[21] For details on the development of the eurozone crisis resolution frameworks, see Corsetti et al. (2017) and Gourinchas et al. (2020).

[22] The European Central Bank (ECB) also introduced the Securities Markets Programme in 2010 to purchase the sovereign bonds of distressed member countries on the secondary markets. Krishnamurthy et al. (2018) investigate the effect of ECB policies involving government bond purchases.

[23] This setting can also be interpreted as country 2 implicitly transferring $r(d - d')$ units of currency to country 1 in period 1.

[24] In a hypothetical situation in which there is no cost in renouncing a debt (i.e., $s = 0$), country 1 always chooses a common currency regime if (α, q) exists in the blue region in Fig. 5.

[25] Reinhart and Trebesch (2016) show that, from a long-term perspective, the economic performance of debtor countries improves significantly after the introduction of hard forms of debt relief such as debt write-offs.

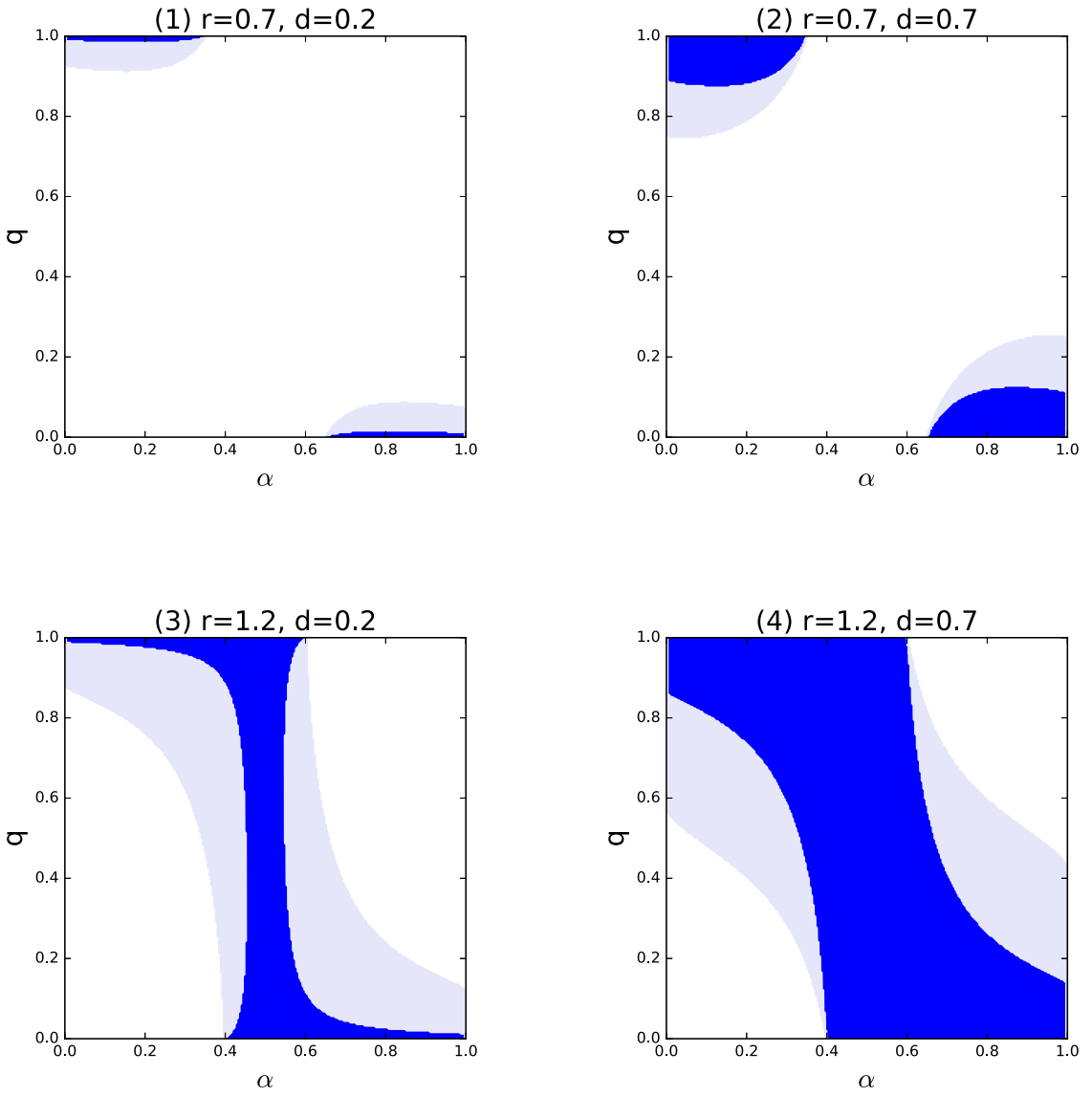

Fig. 6. Optimal currency area with bailout.

creditor member country to bail it out to prevent its immediate insolvency. Indeed, they provide estimates of the implicit transfers during the eurozone sovereign debt crisis periods and find that the EU provides implicit transfers to Ireland and Spain of around 0.4% of their 2011 outputs, to Portugal of 3% of its 2011 output, and to Greece of 43.7% of its 2011 output. In addition, they find that these implicit transfers were not observed for non-eurozone program countries, such as Hungary, Latvia, and Romania, so that the size of the bailout is tied to membership in the currency union, not to membership of the EU. In addition, Pancrazi et al. (2020) determine that, for a debtor country, the welfare gains of bailouts are hump-shaped regarding the size of bailout, and show that although the EFSF/ESM financial assistance provided to Portugal was commensurate with a level that would maximize welfare gains, the assistance provided to Greece may have been too large.

These arguments suggest that bailout of a high-debt member country may be beneficial to maintaining the currency union and, as Fig. 6 shows, the size of the bailout affects the formation of the OCA. These results highlight the importance of designing an *ex post* crisis resolution framework to enhance the sustainability of the currency union. Conversely, to reduce the blue region in Fig. 6, it is necessary for a debtor country to reduce its outstanding debt *ex ante* before it enters a currency union. This gives a theoretical rationale for the Maastricht Treaty, which lays down the preconditions for the level of sovereign debt.

6. Conclusion

This paper provides a theoretical model in which the choice of a *unit of account* may affect not only the terms of trade but the burden of debt repayment, which therefore influences the formation of an OCA. In particular, a debtor country with less preferred tradable goods may be better off when it chooses a national currency as a *unit of account* and, thus, does not enter a currency union. These results indicate the importance of the *unit of account* because it affects the total surplus accrued from various transactions in the country. We believe that our results will contribute to the development of a new theoretical framework for currency unions, which will be particularly relevant to the euro and the eurozone.

Finally, to focus in a simple way on how a burden of sovereign debt affects the formation of an OCA, we assume that the level of an outstanding sovereign debt is given exogenously. As Paniagua et al. (2017) suggests, bailout of a debtor member country may generate moral hazard behavior. If we extend our model to endogenize the level of debt in future research, we will derive richer implications for the literature on OCAs.

CRediT authorship contribution statement

Kenta Toyofuku: Concept, design, analysis of the content, Writing, or revision of the manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

By solving the maximization problem of a representative household i, (4), $C_i^j(\omega_s)(j \in \{1,2\})$ can be written as follows:

$$\begin{cases} C_i^1(\omega_s) &= \dfrac{\alpha(\omega_s)P_i(\omega_s)}{P_1(\omega_s)} \\ C_i^2(\omega_s) &= \dfrac{(1-\alpha(\omega_s))P_i(\omega_s)}{P_2(\omega_s)}. \end{cases}$$

By inserting these equations into (1), we can derive $C_1^j(\omega_s) = \alpha(\omega_s), C_2^j(\omega_s) = 1-\alpha(\omega_s)$ and $P_1(\omega_s)/P_2(\omega_s) = \alpha(\omega_s)/(1-\alpha(\omega_s))$.

In addition, as a representative bank i does not deal with other banks in this case, it is obvious that $A_i(\omega_s) = 1$. (q.e.d.)

Appendix B

First, consider the maximization problem of a representative household i in period 1, (6). As in Appendix A, $C_i^j(\omega_s)$ is written as follows:

$$\begin{cases} C_i^1(\omega_s) = \dfrac{\alpha(\omega_s)(\gamma_i P_i(\omega_s) + (1-\gamma_i)m_i)}{P_1(\omega_s)} \\ C_i^2(\omega_s) = \dfrac{(1-\alpha(\omega_s))(\gamma_i P_i(\omega_s) + (1-\gamma_i)m_i)}{P_2(\omega_s)}. \end{cases} \tag{B.1}$$

Then, inserting (B.1) into (1), we can derive the equilibrium price levels as:

$$\begin{cases} P_1(\omega_s) = \dfrac{\alpha(\omega_s)\bar{m}}{1-\alpha(\omega_s)\gamma_1 - (1-\alpha(\omega_s))\gamma_2}, \\ P_2(\omega_s) = \dfrac{(1-\alpha(\omega_s))\bar{m}}{1-\alpha(\omega_s)\gamma_1 - (1-\alpha(\omega_s))\gamma_2}, \end{cases} \tag{B.2}$$

where $\bar{m} \equiv (1-\gamma_1)m_1 + (1-\gamma_2)m_2$.

Next, consider the maximization problem of a representative household i in period 0. Given $(\alpha, q, P_i(\omega_s))$, it chooses γ_i to satisfy the following first-order condition:

$$\frac{\partial E[U_i]}{\partial \gamma_i} = E\left[\sum_j \frac{\alpha_j(\omega_a)}{C_i^j(\omega_a)} \frac{\partial C_i^j(\omega_a)}{\partial \gamma_i}\right] = 0. \tag{B.3}$$

From (B.1), we have:

$$\frac{\partial C_i^j(\omega_s)}{\partial \gamma_i} = \frac{\alpha_j(\omega_s)(P_j(\omega_s) - m_i)}{P_j(\omega_s)}. \tag{B.4}$$

From the maximization problem of a representative bank i, (7) and (8), we have:

$$m_i = E[P_i(\omega_s)]. \tag{B.5}$$

Then, inserting (B.2), (B.4), and (B.5) into (B.3), we can derive:

$$\begin{aligned}\frac{\partial E[U_i]}{\partial \gamma_i} &= \sum_s \left(\frac{P_i(\omega_s) - m_i}{\gamma_i P_i(\omega_s) + (1-\gamma_i)m_i} \right) \\ &= q(1-q)(P_i(\omega_a) - P_i(\omega_b)) \left(\frac{1}{\gamma_i P_i(\omega_a) + (1-\gamma_i)m_i} - \frac{1}{\gamma_i P_i(\omega_b) + (1-\gamma_i)m_i} \right).\end{aligned}$$

The last term in parentheses in the second equation becomes zero only when $\gamma_i = 0$. Therefore, $\gamma_i = 0$ is optimal for all households.

Inserting $\gamma_i = 0$ and (10) into (B.1) and (B.2), we can derive:

$$\begin{cases} C_1^j(\omega_s) = q\alpha + (1-q)(1-\alpha) \equiv \rho \\ C_2^j(\omega_s) = q(1-\alpha) + (1-q)\alpha = 1-\rho \end{cases}$$

$$\begin{cases} P_1(\omega_a) = P_2(\omega_b) = \alpha M \\ P_1(\omega_b) = P_2(\omega_a) = (1-\alpha)M. \end{cases}$$

Inserting (9), (B.5), and $\gamma_i = 0$ into the second constraint of (8) implies that:

$$E[P_i(\omega_s)] - E[P^a(\omega_s)] \leq B_i.$$

Therefore, when $E[P_i(\omega_s)] - E[P^a(\omega_s)] > 0$, B_i becomes positive so that a representative bank i borrows from other banks.

Now, we focus on the case in which ω_a is realized in period 1. Then, the budget constraint of a representative bank i becomes:

$$P^a(\omega_a)A_i(\omega_a) = -R_i + P_i(\omega_a).$$

As (2) and (11), we can derive $P^a(\omega_a) = M/2$. In addition, as $R_i = -B_i$ and (B.5), we can derive:

$$\begin{cases} A_1(\omega_a) = -2\rho + 1 + 2\alpha \\ A_2(\omega_a) = -1 + 2\rho + 2(1-\alpha). \end{cases}$$

Using the same procedure, we can derive $P^a(\omega_b) = M/2$ and:

$$\begin{cases} A_1(\omega_b) = -2\rho + 1 + 2(1-\alpha) \\ A_2(\omega_b) = -1 + 2\rho + 2\alpha. \end{cases}$$

(q.e.d.)

Appendix C

First, consider a maximization problem (4) subject to the budget constraint (12). As in Appendix A, the equilibrium exchange rate becomes:

$$\frac{P_1(\omega_s)}{P_2(\omega_s)} = \frac{\alpha(\omega_s)}{1-\alpha(\omega_s)}. \tag{C.1}$$

Using (C.1), $C_1^j(\omega_s)$ becomes:

$$C_1^j(\omega_s) = \alpha(\omega_s)(1-d). \tag{C.2}$$

Let $E[U_1^N]$ be the expected payoff of representative household 1 when it chooses a national currency. Then, from (C.2), $E[U_1^N]$ is given by:

$$E[U_1^N] = \ln \alpha^q (1-\alpha)^{1-q}(1-d).$$

Next, consider a maximization problem (6) subject to the budget constraint (13) and $\gamma_i = 0$. As in the previous case, $C_1^j(\omega_s)$ is given by:

$$C_1^j(\omega_s) = \frac{m_1 - r \cdot d}{m_1 + m_2}.$$

Here, from (10) and the assumption that $M = 2$, (B.5) can be rewritten as:

$$m_1 = \rho\bar{m}, \quad m_2 = (1-\rho)\bar{m}.$$

Then, letting $E[U_1^C]$ be the expected utility when a common currency is chosen, we can write:

$$E[U_1^C] = \ln \frac{2\rho - r \cdot d}{2}.$$

Therefore, representative household 1 prefers a national currency if

$$\alpha^q (1-\alpha)^{1-q}(1-d) > \frac{2\rho - r \cdot d}{2} \tag{C.3}$$

is satisfied. (q.e.d.)

Appendix D

First, from (C.3), representative household 1 chooses a common currency if d is reduced to $\acute{d}$, which satisfies:

$$\acute{d} \geq \max\left\{ \frac{2}{r}\left(\rho - \alpha^q (1-\alpha)^{(1-q)}(1-d)\right), 0 \right\}.$$

Let $E[U_2^N]$ and $E[U_2^D]$ be the expected payoff of representative household 2 when it chooses a national currency and a common currency with debt relief, respectively. Then, we can derive:

$$E[U_2^N] = \ln\left(\alpha d + (1-\alpha)\right)^q \left(\alpha + (1-\alpha)d\right)^{(1-q)}$$

$$E[U_2^D] = \ln \frac{2 - 2\rho + r \cdot \acute{d} - s(d - \acute{d})}{2}.$$

Therefore, representative household 2 accepts the debt relief if and only if

$$\frac{2 - 2\rho + r \cdot \acute{d} - s(d - \acute{d})}{2} > \left(\alpha d + (1-\alpha)\right)^q \left(\alpha + (1-\alpha)d\right)^{(1-q)}$$

is satisfied. (q.e.d.)

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