



# Risk evaluation and retail electricity pricing using downside risk constraints method



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## ABSTRACT

Electricity in the retail market has a different value for different types of consumers. Therefore, different retail prices are usually determined for various consumers in the retail market. However, imposed risks from uncertain parameters are a big challenge in the real-time retail market pricing process. This paper proposed a real-time pricing (RTP) framework for various users including residential, commercial, and industrial consumers by the electricity retailer. In addition, uncertainties of various input parameters such as output power of renewable energy resources, electricity demand, and pool market price are modeled using scenario-based stochastic approach while downside risk constraints method is proposed to model risk associated with uncertainties. By implementing this method, electricity retailer will be able to select various risk-based strategies. Furthermore, numerical results illustrate the various risks versus various profits by the occurring of each scenario which helps the retailer for decisions-making in different scenarios. According to obtained results, retailer by choosing of zero risk strategy can reduce its risk by 100% while expected profit is reduced by 2.07%. In addition, offered RTP by the retailer is higher for industrial, commercial, and residential customers, respectively. Finally, risk-averse and risk-neutral strategies of electricity retailer are determined in the power procurement problem.

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## 1. Introduction

Pricing problem of electricity retailer is a new challenge in following of deregulating in the electricity market. In order to find the optimal retail prices, the objective function of these pricing problems is varied in the last years. In some researches, by defining a market efficiency evaluation index, the market social welfare is maximized to obtain the optimal price of the retail market [1]. In Ref. [2], in order to determine the optimal selling price of electricity retailer, a stochastic energy procurement problem of electricity retailer in the presence of various energy resources is developed which the objective function is maximizing the retailer profit. In addition, by the implementation of the smart electricity grid, the needed infrastructures for implementing dynamic pricing in the retail market will be provided [3]. Dynamic retail market pricing

allows consumers to response and adjusts their electricity demand according to received real-time retail market prices from the smart grid infrastructures [4]. The economic impacts of load shifting by electricity customers in response to the RTP without demand-side management is studied in Ref. [5].

In Ref. [6], the electricity retailer is participated in the demand response exchange market to procure a part of its required demand in which uncertainty of pool market price has been considered using robust optimization approach. In Ref. [7] the ESSs are used by electricity retailer improve its flexibility in the energy management strategy. In Ref. [8], an overview of technology and markets of home energy storage systems have been provided. In addition, in order not to experience any loss of their customers, commercial customers may be accepted to pay higher prices for electricity during peak hours [9]. Also, residential customers may have more elasticity to the offered price and reduce their consumption by a few increases in the offered retail price to them [9]. In Ref. [10] an approach is introduced to integrate wholesale market and retail market. In Ref. [11], considering the pool market price uncertainty, real-time pricing of electricity retailers is compared with TOU and

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Nomenclature			
<b>Index</b>		$\eta$	Efficiency related to Discharging state of ESS [%]
$b$	Used index for bilateral contract	$\lambda_{b,t}$	Price related to bth bilateral contract [\$/kWh]
$h$	Used index for DG units generation block	$\lambda_{t,\omega}$	Price related to pool market at time t and scenario w [\$/kWh]
$i$	Used index for min OFF-time and min ON-time limits	$\alpha$	Risk control parameter in the proposed risk approach
$j$	Used index for number of DG unit	<b>Variables</b>	
$\omega$	Used index for Scenario	$A(c, z, t)$	A binary variable for the selected selling price step in the price-quote curve [0,1]
$t$	Used index for Time	$C_B$	The cost of procured energy from the forward contracts [\$]
$z$	Used index for segments of the price-quota curve	$C_P$	The cost of procured energy from the pool market [\$]
$c$	Used index for client	$C_{DG}$	The cost of procured energy from the DG units [\$]
<b>Parameters</b>		$D(c, t, s)$	The demand of client c at time t in scenario w[kW]
$Dn_{j,i}$	Used Auxiliary variable in the MDT constraint modeling	$EDR$	Expected downside risk [\$]
$D^{offer}(c, z, t, \omega)$	Selected power of client in price-quota curve [kW]	$P_{b,t}$	Amounts of procured power from the forward contracts [kW]
$G_{t,\omega}^a$	Solar irradiation [W/m2]	$P_t^{BC}$	Total procured power from the forward contracts [kW]
$G_{a0}$	Predetermined standard Solar irradiation [W/m2]	$P_{t,\omega}^{charge}$	Amounts of Charging power in ESS [kW]
$NOTC$	Normal operating cell temperature [°C]	$P_{t,\omega}^{disc}$	Amounts of Discharging power in ESS[kW]
$\rho_\omega$	Probability of each scenario	$P_{t,\omega}^P$	Procured power from the pool market [kW]
$P_b^{max}$	Max amount of forward contracts [kW]	$P_{j,h,t,\omega}^{DG}$	Amounts of purchased power from the DG units [kW]
$P_b^{min}$	Min amount of forward contracts [kW]	$Profit_\omega^{No Risk}$	Profit of retailer in each scenario without considering downside risk constraints method
$P_{j,h}^{MAX}$	Max power of hth block of DG unit's piecewise linear cost function [kW]	$Profit_\omega$	Profit of retailer in each scenario [\$]
$P_{t,\omega}^{PV}$	Produced power in PV array [kW]	$Risk_\omega$	Risk-in-profit of retailer in each scenario [\$]
$P_{Max,0}^M$	Predetermined standard condition Max power of PV array [kW]	$R_R(c, t)$	The revenue of client group c in time t [\$]
$P_{t,\omega}^{wind}$	Produced power by wind-turbine [kW]	$s_b$	Binary variable related to selected forward contracts [0,1]
$P_r$	Wind-turbine rated power [kW]	$SP(c, t)$	Selling price for the client c [\$/kWh]
$P_{charge}^{max}$	Maximum charging limit of battery [kW]	$SP(c, z, t)$	Price-quota curve interval price for the client l [\$/kWh]
$P_{disc}^{max}$	Maximum discharging limit of battery [kW]	$SP^{RTP}(c, t)$	The real-time retail price offered by the electricity retailer to the client c in time t [\$/kWh]
$R_j^{up}$	Amount of DG unit's Ramp up rate limit [kW/h]	$Target_{profit}$	Target profit of retailer without considering downside risk constraints method
$R_j^{down}$	Amount of DG unit's Ramp down rate limit [kW/h]	$U_\omega$	Binary variable which is equal to 1 in the situation that profit is less than target profit
$S_{j,h}^{DG}$	The slop of hth block in DG unit's piecewise linear cost function [\$/kWh]	$U_{t,\omega}^{charge}$	Binary variable related to charging state of ESS [0,1]
$SP^{offer}(c, z, t)$	Offered retail price to each client in the price-quota curve [\$/kWh]	$U_{t,\omega}^{disc}$	Binary variable related to the discharging state of ESS [0,1]
$T_{t,\omega}^a$	Temperature [°C]	$U_{j,t}^{DG}$	Binary variable related to on/off modes in DG units [0,1]
$T_{M,0}$	Predetermined standard condition temperature[°C]	$X_{t,\omega}^b$	Amounts on total stored energy in the ESS [kWh]
$Up_{j,i}$	Introduced auxiliary variable to MUT modeling	$M_1, M_2$	Big positive constant
$V_{t,\omega}^w$	Wind speed [m/s]		
$V_r, V_{ci}, V_{co}$	Amounts of cut-in and cut-out rates of wind turbines [m/s]		
$X_b^{max}$	Max amounts of stored energy in the ESS [kW]		
$X_b^{min}$	Min amounts of stored energy in ESS [kW]		
$\chi$	Efficiency related to Charging state of ESS[%]		

fixed prices (FP). The imposed risks from uncertain parameters impose financial risk for electricity retailers which are addressed in recent researches. Besides, in Ref. [12] considering the consumers constraints, a bi-level programming problem is proposed for electricity pricing in the retail market in which retailer has access to various resources such as bilateral contracts (BC), distributed generators (DGs), and demand response program. Also, a short-term

decision-making problem and robust-based power procurement problem of electricity retailer considering the uncertainties has been solved in Ref. [13]. In Ref. [14], the effects of time-of-use and time-of-export tariffs on residential consumers have been evaluated under various penetrations of energy storage. Robust electricity pricing of the retailer in the smart grid environment has been considered in ref [15], in which various pricing tariffs such as

TOU pricing, RTP, and FP have been determined and compared with each other. Also, ref [16] using the drawdown-based method, develops the problem of decision making under uncertainty for electricity retailers whose objective function is maximizing the total expected rate of return. Hydrogen energy storage in Ref. [17] and plug-in electric vehicles in Ref. [18] are used by electricity retailers to improve their flexibility in the energy management strategy. In order to model the faced risks by an electricity retailer considering other retailers' strategy, in Ref. [19] a mid-term decision-making problem is considered for electricity retailers in the retail electricity market. In Ref. [20], robust decisions of electricity retailers in the presence of the various energy procurement options is presented in which the effects of demand response program is evaluated on the total cost of retailers. In Ref. [21], a single-period model is used to optimal sizing of energy storage along with the photovoltaic system in an apartment house. An electricity pricing way is planning optimization problems considering consumer's response to offered prices by DisCo's in which real-time retail price is determined with the aim of DisCo's profit maximization [22]. In Ref. [23], pool market and bilateral contracts are used as energy resources in energy procurement problem of electricity retailer in which demand response program has been implemented as a demand-side management option. Stochastic based time of use (TOU) pricing framework from the regulators perspective has been introduced in Ref. [24]. Evaluation of imposed financial risks from pool market price uncertainty using the expected downside risk methodology has been considered in Ref. [25]. Another method to determine the hourly retail price in the literature is determining by retailer, utility companies, or distribution company (DisCo) in which total electricity supply cost at any time should be minimized [26]. In Ref. [27], the retail price is determined in the optimization problem in which retailers or DISCOs as energy providers by optimal determination of the offered retail price, tend to maximize its profit using smart metering devices. In Ref. [28], TOU retail price is determined for electricity consumers in which stochastic optimization framework has been used to handle the uncertainties. The uncertainty of price and load are the main uncertain parameters in the decision-making problems of retailer, which these uncertainties in the energy management problem of electricity retailers have been considered in Ref. [29]. In order to supply their required demands, industrial customers maybe pay more price per procured kWh [30]. The offered retail price by electricity retailers to consumers considering risk assessment has been determined in Ref. [31]. In Ref. [32], a stochastic framework is proposed to risk assessment in pricing and contract determining problem of electricity retailer. In addition, the effects of demand response programs on the retailers' decision-making problems have been analyzed in the proposed stochastic framework. When the dynamic pricing is implemented in the retail market, operation planning of the smart distribution grid is fundamentally different from the other conventional grids, which is evaluated in Ref. [33].

### 1.1. Novelty and contribution

In this work, a new risk-constrained stochastic framework is used to RTP electricity pricing in the retail market by electricity retailer for three different consumers, including residential, industrial, and commercial consumers. Then, the response of mentioned consumers to obtained real-time prices is investigated, and new retail price according to consumers' responses is determined. According to the abovementioned contexts, electricity retailers can benefit from various options such as distributed generators (DGs), RER, and ESS in power procurement process. Therefore in uncertain environments, risk evaluation of electricity retailer in the decision-making process is necessary for electricity

retailers. In addition, risk measurement tool should be clearly shown the impacts of uncertain parameters on the retailer total profit and cost. Also, direct measurement of risk can be given a good idea on how to choose optimal scenarios for electricity retailers. Therefore, the main contribution of this paper in a new stochastic risk-evaluation method to investigating the imposed risk from various uncertain sources such as RER, pool market price uncertainty, and load uncertainty. This risk-evaluation method called the downside risk-constraint (DRC) method, which analyzes the results of stochastic optimization and introduces a zero risk strategy for electricity retailers in the real-time pricing and energy procurement process. Also, exact amounts of risk can be calculated in each scenario using DRC, which gives a good idea to electricity retailers in optimal scenario selection. Therefore, the contribution of this paper can be summarized as follows:

- Real-time pricing by electricity retailers in the risk environment is determined.
- Uncertainties of input parameters are modeled via stochastic programming.
- Downside risk constraint method is proposed to model risk related to uncertainties.
- Risk-averse and risk-neutral strategies for electricity retailers are introduced.
- Exact amounts of retailer risk is directly measured and reduced in each scenario.

### 1.2. Paper organization

The remainder of this paper is structured as follows. In Section 2, real-time pricing problem for three customers are modeled. Section 3 is demonstrated the used risk evaluation framework, which called the downside risk constraints method. Input data, numerical results, and risk-based scheduling of electricity retailers are expressed in Section 4. In Section 5, a discussion is provided about the proposed framework. Finally, Section 6 is devoted to the conclusion.

## 2. Problem formulation

As mentioned in previous sections, the objective can differ from retailer's cost function, profit function, etc in the retailer pricing problem. In the proposed problem, the objective is to maximize the retailer profit in the power procurement process. The retailer profit function is the revenue from sold power minus power procurement cost. Purchasing power by electricity retailer can be accomplished from various options such as wholesale market, forward contracts, and DG units, and RERs. Also, the electricity retailer in the power procurement process can use from ESS to increase its flexibility, which is considered in the proposed objective function. By considering that operation cost of RER and ESS are neglectable in comparison with the other options, hence the operation cost of RER and ESS in the objective function are ignored. Therefore, according to the mentioned contexts, retailer profit function can be written as follows:

$$\begin{aligned} \text{Max} \quad & \sum_{s=1}^Q p_s \times \left\{ \sum_{t=1}^T \sum_{l=1}^L SP(c, t) D(c, t, \omega) - \sum_{t=1}^T \lambda_{t, \omega} P_{t, \omega}^p - \sum_{t=1}^T \right. \\ & \left. \times \sum_{j=1}^J \sum_{h=1}^H S_{j, h}^{DG} P_{j, h, t, \omega}^{DG} \right\} - \sum_b \sum_{t=1}^T \lambda_{b, t} P_{b, t} \end{aligned} \quad (1)$$

In the mentioned objective function (1), purchasing power from the wholesale market is considered in the first term, and purchasing power from all DG units and contracted forward contracts are considered in second and third terms, respectively. In the proposed objective function,  $SP(c, t)$  is real-time retail price offered by the electricity retailer to the clients [\$/kWh],  $D(c, t, \omega)$  is the demand of each client group. The variables  $P_{b,t}$ ,  $P_{j,h,t,\omega}^{DG}$  and  $P_{t,\omega}^P$  are respectively the purchasing power from the market, DG units, and bilateral contracts. In addition, parameters  $\lambda_{t,\omega}$ ,  $S_{j,h}^{DG}$  and  $\lambda_{b,t}$  denote the related prices to the abovementioned energy provision options, respectively.

In addition, the power balance equation in the proposed retailer power procurement problem can be written as follows:

$$\sum_{b=1}^B P_{b,t} + \sum_{j=1}^J \sum_{h=1}^H P_{j,h,t,\omega}^{DG} + P_{t,\omega}^P + P_{t,\omega}^{wind} + P_{t,\omega}^{PV} + P_{t,\omega}^{disc} = \sum_{l=1}^L D(c, t, \omega) + P_{t,\omega}^{charge} \quad \forall t, \omega \quad (2)$$

According to the stated power balance equation in Eq. (2), the total demand of electricity retailer plus charged power in ESS should be equal to procured power from various options at time  $t$  of scenario  $\omega$ .

In the following section, the formula for each part of the power balance equation will be presented.

### 2.1. Forward contract model

Electricity retailers can procure a part of the required energy using the forward contracts for future periods. Using Eq. (3), electricity retailer can calculate total imposed costs from contracted forward contracts [34].

$$C_B = \sum_b \sum_{t=1}^T \lambda_{b,t} P_{b,t} \quad (3)$$

Decision variables related to the forward contract are first-stage variables and amounts of these variables do not depend on each realization of the stochastic process.

Other constraints related to forward contracts are presented as follows:

$$P_b^{\min} s_b < P_{b,t} < P_b^{\max} s_b \quad \forall b, t \quad (4)$$

$$P_t^{BC} = \sum_{b=1}^B P_{b,t}; \quad \forall t \quad (5)$$

Eq. (4) defines the upper and lower bound for each block of forward contracts. In addition, the total bought power from forward contracts can be calculated using Eq. (5).

### 2.2. Pool market model

Mathematical modeling of the considered electricity market in the power procurement problem of electricity retailers is formulated in the following of this sub-section. The cost of total procured power from the electricity market by retailer can be modeled using Eq. (6).

$$C_P = \sum_{\omega=1}^Q \rho_{\omega} \times \sum_{t=1}^T \lambda_{t,\omega} P_{t,\omega}^P \quad (6)$$

As is evident in Eq. (6), pool market price is an uncertain parameter that is modeled using the stochastic framework. In addition, it should be mentioned that variables related to electricity market are second-stage variables that depend on scenario realization.

### 2.3. Distributed units (DGs) model

In the literature, a linear-piecewise model is used to model the DG unit's cost function. According to the model represented in Ref. [35], DG unit's cost function can be modeled as shown in Eq. (7).

$$C_{DG} = \sum_{\omega=1}^Q \rho_{\omega} \times \sum_{t=1}^T \sum_{j=1}^J \sum_{h=1}^H S_{j,h}^{DG} P_{j,h,t,\omega}^{DG} \quad (7)$$

Other technical constraints related to DG units are represented by Eqs. (8)–(15).

$$0 \leq P_{j,h,t,\omega}^{DG} \leq P_{j,h}^{MAX} - P_{j,h-1}^{MAX} \quad \forall j, t, \omega, h = 2, \dots, N \quad (8)$$

$$0 \leq P_{j,1,t,\omega}^{DG} \leq P_{j,1}^{MAX} \quad \forall j, t, \omega \quad (9)$$

$$\sum_{h=1}^H P_{j,h,t,\omega}^{DG} - \sum_{h=1}^H P_{j,h,t-1,\omega}^{DG} \leq R_j^{up} \times U_{j,t}^{DG}; \quad \forall j, t, \omega \quad (10)$$

$$\sum_{h=1}^H P_{j,h,t-1,\omega}^{DG} - \sum_{h=1}^H P_{j,h,t,\omega}^{DG} \leq R_j^{down} \times U_{j,t-1}^{DG}; \quad \forall j, t, \omega \quad (11)$$

$$U_{j,t}^{DG} - U_{j,t-1}^{DG} \leq U_{j,t+Up_{ji}}^{DG}; \quad \forall j, t, i \quad (12)$$

$$U_{j,t-1}^{DG} - U_{j,t}^{DG} \leq 1 - U_{j,t+Dn_{ji}}^{DG}; \quad \forall j, t, i \quad (13)$$

$$Up_{j,i} = \begin{cases} i & i \leq MUT_j \\ 0 & i > MUT_j \end{cases} \quad (14)$$

$$Dn_{j,i} = \begin{cases} i & i \leq MDT_j \\ 0 & i > MDT_j \end{cases} \quad (15)$$

According to the model introduced in Ref. [35], constraints (8) and (9) demonstrates that each piece in the DG linear-piecewise cost function is limited between min and max amounts. Also, according to constraints (10) and (11), the ramp-up rate and ramp-down rate of DG units are less than a predetermined amount. Finally, DG units usually have a time limit for up and down states called minimum up/down time limits, which are defined in constraints (12) and (13). In addition, in order to linear modeling of the DG unit's minimum up/downtime constraints, two additional auxiliary variables are needed, which are defined by constraints (14) and (15).

### 2.4. Renewable energy resources models

The output power of the wind turbine at any time can be defined using a speed-dependent function. Also, Weibull distribution, due to the adequately fitting on the historical wind data, can be used to generate the wind speed scenarios. Generated power per any wind speed can be calculated using Eq. (16).

$$p_{t,\omega}^{wind} = \begin{cases} 0 & V_{t,\omega}^w < V_{ci} \\ p_r \times \left( \frac{V_{t,\omega}^w - V_{ci}}{V_r - V_{ci}} \right)^3 & V_{ci} < V_{t,\omega}^w < V_{cr} \\ p_r & V_r < V_{t,\omega}^w < V_{c0} \\ 0 & V_{t,\omega}^w > V_{c0} \end{cases} \quad (16)$$

Furthermore, the output power of PV units depends on solar irradiation and temperature at any time. Therefore, generated power amounts at any time can be calculated by Eq. (17). Hence, in this problem, the normal distribution is used to generate scenario for solar irradiation and temperature in the proposed stochastic programming.

$$p_{t,\omega}^{PV} = \frac{G_{t,\omega}^a}{G_{a0}} \times \left\{ P_{Max,0}^M + \mu_{Pmax} \times \left( T_{t,\omega}^a + G_{t,\omega}^a \times \frac{NOCT - 20 [^\circ C]}{800 [W/m^2]} - T_{M,0} \right) \right\} \quad (17)$$

## 2.5. Energy storage model

ESS can be used by the electricity retailer to meet the required flexibility and decrease operation costs. Based on the model derived from Ref. [36], constraints (18)–(23) are demonstrated the technical constraints of ESS.

$$X_{t_0}^b = X_0^b \quad (18)$$

$$P_{t,\omega}^{charge} \leq P_{charge}^{max} \times U_{t,\omega}^{charge}; \forall t, \omega \quad (19)$$

$$P_{t,\omega}^{disc} \leq P_{disc}^{max} \times U_{t,\omega}^{disc}; \forall t, \omega \quad (20)$$

$$X_b^{\min} \leq X_{t,\omega}^b \leq X_b^{\max}; \forall t, \omega \quad (21)$$

$$U_{t,\omega}^{charge} + U_{t,\omega}^{disc} \leq 1; \forall t, \omega \quad (22)$$

$$X_{t,\omega}^b = X_{t-1,\omega}^b + \chi \times P_{t,\omega}^{charge} - \frac{P_{t,\omega}^{disc}}{\eta}; \forall t, \omega \quad (23)$$

The initial stored energy in ESS is considered by Eq. (18). Eqs. (19) and (20) defines an upper bound for power in charge/discharge operation modes. Stored energy in the energy storage system is bounded, which constraint (21) defines lower and upper bounds for stored energy in ESS. The constraint (22) ensures that energy storage can not be operated in charge or discharge mode simultaneously. In addition, stored energy in ESS at any time can be calculated using Eq. (23). Also, variables related to ESS are second-stage variables or scenario dependence variables.

## 2.6. Supplied demand by the electricity retailer

After determining the offered real-time price ( $SP(c, t)$ ) by electricity retailer, clients can respond to offered price by retailer and adjust their demand based on a price-quota curve [37]. Using price-quota curve, consumers can be adjusted their demand in response to the offered retail price by electricity retailers. In other words,

using customers response historical data, each retailer can estimates a price-quota curve to determine its client response ( $D(c, t, \omega)$ ) to the offered retail price ( $SP(c, t)$ ).

Therefore, the price quota curve can be formulated mathematically for each client or client group at each scenario at period  $t$ . The mathematical model of the supplied demand by a retailer can be written as Eqs. (24)–(27):

$$D(c, t, \omega) = \sum_{z=1}^Z D^{offer}(c, z, t, \omega) A(c, z, t) \quad ; \forall c, t, \omega \quad (24)$$

$$SP(c, t) = \sum_{z=1}^Z SP(c, z, t) \quad ; \forall c, t \quad (25)$$

$$SP^{offer}(c, z-1, t) A(c, z, t) \leq SP(c, z, t) \leq SP^{offer}(c, z) A(c, z, t); \forall c, z, t \quad (26)$$

$$\sum_{z=1}^Z A(c, z, t) = 1; \forall c, t \quad (27)$$

From Eqs. (24)–(27), it can be shown that the total demand of clients supplied by the retailer at any time is a function of offered price by the electricity retailer. According to the introduced equations above, the supplied demand by an electricity retailer is equal to energy level of indicated step in the price quota-curve which indicated by the binary variable  $A(c, z, t)$ .

The revenue of electricity retailer from selling energy to client  $c$  at period  $t$  can be stochastically formulated as following:

$$R_R(c, t) = \sum_{\omega=1}^Q \rho_{\omega} \times SP(c, t) D(c, t, \omega) \quad (28)$$

## 2.7. Real-time pricing (RTP) model

In the proposed real-time pricing framework, the objective function (1) is maximized subject to constraints (2)–(27). In the proposed framework, the real-time retail price determined by electricity retailers is varying at any time. Therefore, this retail price is similar to the determination of the real-time prices. It should be noted that the real-time retail price offered by the retailer is determined based on the constraint (29) in the proposed framework. In Fig. 1, the price-quota curve based RTP pricing algorithm is illustrated for residential, industrial, and commercial customers.

$$SP(c, t) = SP^{RTP}(c, t) \quad (29)$$

## 3. Downside risk constraints (DRC) model

Unlike other risk measures in the literature, downside risk constraints approach falls into the non-equilibrium approaches category. In this section, a set of constraints are introduced, which links the risk-in-profit and electricity retailer's target profit. In the stochastic programming, more profitable scenarios than expected profit are favorable scenarios for the retailer. The difference between target profits and scenario's with less profit than expected profits are defined as the downside risk. Hence, the downside risk for each scenario can be introduced as Eq. (30).

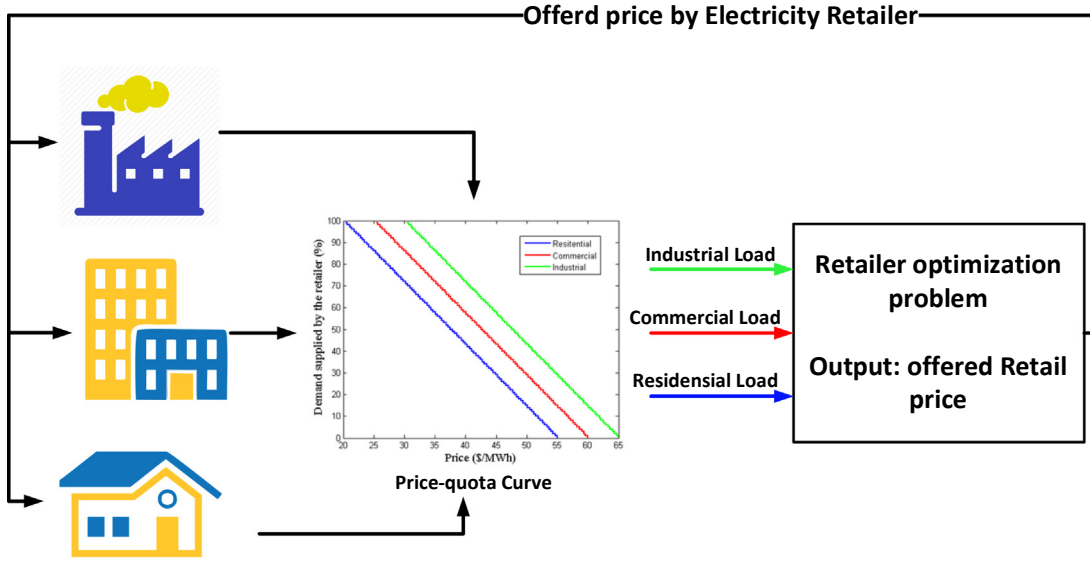


Fig. 1. Price-quota based RTP algorithm.

$$\begin{aligned} \text{If } \text{profit}_\omega < \text{Target}_{\text{profit}} \text{ then } \text{Risk}_\omega \\ = \text{Target}_{\text{profit}} - \text{profit}_\omega \text{ otherwise } \text{Risk}_\omega = 0 \end{aligned} \quad (30)$$

Mathematically form of Eq. (30) can be rewritten as follow:

$$0 \leq \text{Risk}_\omega - (\text{Target}_{\text{profit}} - \text{profit}_\omega) \leq M_1 \cdot (1 - U_\omega) \quad (31)$$

$$0 \leq \text{Risk}_\omega \leq M_2 \cdot U_\omega \quad (32)$$

In Eqs. (31) and (32),  $M_1$ ,  $M_2$  are big positive constant and  $U_\omega$  is a binary variable which is equal to 1 in the situation that profit is less than target profit ( $\text{profit}_\omega < \text{Target}_{\text{profit}}$ ).

Finally, in the proposed decision-making problem of electricity retailer, the expected downside risk (EDR) for introduced retailer's profit function can be modeled as follows:

$$\begin{aligned} \sum_{\omega=1}^{Ns} \rho_\omega \cdot \text{Risk}_\omega &\leq \alpha \cdot \text{EDR} \\ \text{EDR} &= \sum_{\omega=1}^{Ns} \rho_\omega \cdot (\text{Target}_{\text{profit}} - \text{profit}_\omega) \end{aligned} \quad (33)$$

In Eq. (33),  $\text{Profit}_\omega^{\text{No Risk}}$  is the profit in each scenario without DRC. Also, in order to control and analyze the risk,  $\alpha$  is risk control parameter (RCP), which in risk-neutral strategy is equal to 1 and in risk-averse strategy, is 0. It should be noted that  $\alpha$  is changed between 0 and 1 by a step of 0.1, to the realization of other strategies [38]. In order to clear understanding of the proposed risk framework, the flowchart of the proposed risk management approach is presented in Fig. 2.

#### 4. Simulation results

In this section, input data and results of the proposed problem are determined. Also, the effects of the proposed risk constrained framework on the retailer risk strategies are evaluated. The introduced stochastic formulation of real-time retail price determination in the smart grid has been modeled using mixed-integer linear programming which is solved using CPLEX solver [39] in the GAMS software [40].

#### 4.1. Input data

In the considered case study, all of the required input data are derived from Ref. [2]. There are various uncertain parameters in the proposed model, including uncertainty of pool-market price, electricity demand, solar temperature, solar irradiation, and wind speed which all of the mentioned uncertainties are considered using properly fitted distribution. The considered amount for standard deviation of uncertain parameters is equal to 10% of the mean value in each time. Hence, using properly fitted distribution, 30 scenarios are generated for each uncertain parameter alone, which led to  $5^5 = 3125$  scenarios for five uncertain parameters. Therefore, due to the high number of generated scenarios and the prolongation of the problem-solving time, fast forward scenario reduction method is used to reduce the number of scenarios to 5 [37].

#### 5. Results

In this paper, obtained results can be represented in two risk strategies, including risk-neutral and risk-averse strategies. The results of the risk-neutral strategy are obtained from solving the base stochastic problem without considering DRC. On the other hand, the risk-averse strategy can be obtained considering DRC for various  $\alpha$  amounts. Therefore, to represent the obtained results from all five scenarios versus RCP ( $\alpha$ ), Table 1 and Fig. 3 can be useful. Also, risk-in-profit amounts in each scenario versus different amounts of RCP is indicated in Table 2 and plotted in Fig. 4.

According to Table 1, in the risk-neutral strategy, which is indicated in the  $\alpha = 1$  associated row, scenarios 1, 2, and 3 are the worst scenarios that have profit less than expected profit (\$1212.19). In order to show the profit reduction in against  $\alpha$  reduction, Fig. 3 can be plotted from Table 1. According to Fig. 3, it can be shown that profit variation in the best scenario 5 is more than in other scenarios. Also from this Fig, it can be shown that worst scenario 1 has the least sensitivity to  $\alpha$  reduction. Therefore, scenarios 1, 2, and 3 are called downside risk scenarios. Risk-in-profit values related to the mentioned worst scenarios are indicated in Table 2 and plotted in Fig. 4. According to Table 2 and Fig. 4, it can be shown that by decreasing  $\alpha$ , risk-in-profit reduces until it reaches zero which is risk-averse strategy. In the risk-averse strategy, it is guaranteed that the total expected profit of the retailer would not be less than

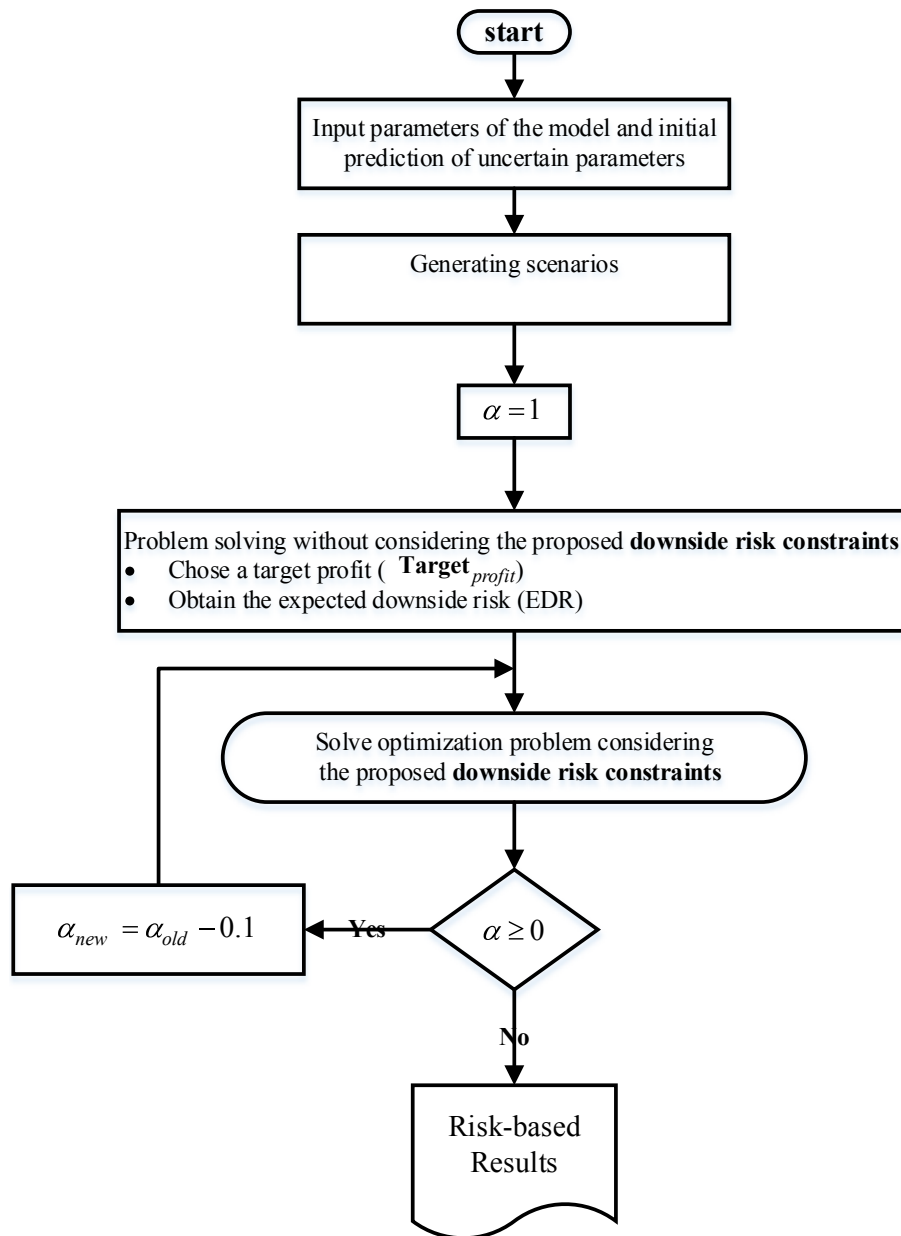
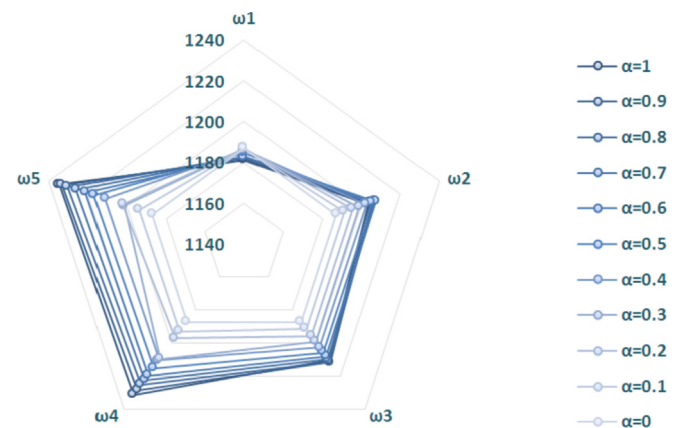


Fig. 2. Flowchart of the proposed Risk framework.

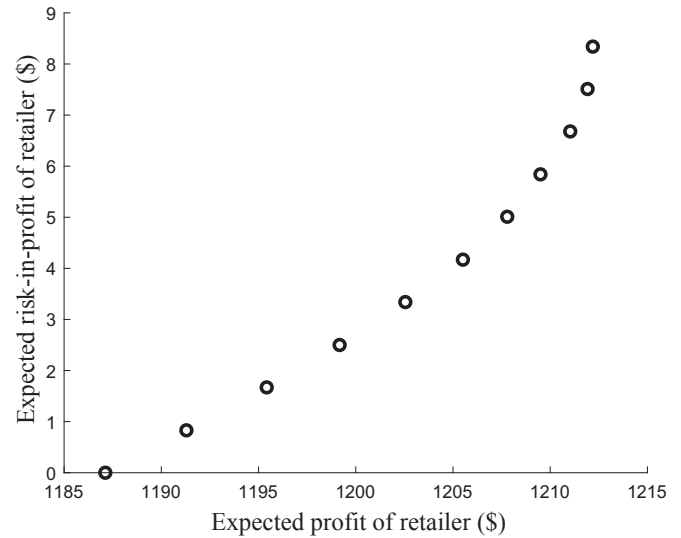
**Table 1**  
Expected profit versus RCP (\$).

	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$	$\omega_5$
$\alpha = 1$	1181.11	1203.46	1210.29	1231.43	1234.68
$\alpha = 0.9$	1182.13	1204.69	1211.39	1228.30	1233.09
$\alpha = 0.8$	1182.70	1205.99	1211.03	1225.30	1230.15
$\alpha = 0.7$	1182.81	1206.99	1209.50	1222.31	1225.90
$\alpha = 0.6$	1182.85	1207.69	1207.79	1219.52	1221.09
$\alpha = 0.5$	1184.65	1205.51	1205.51	1215.01	1216.87
$\alpha = 0.4$ 1185.86	1202.54	1202.55	1210.36	1211.42	
$\alpha = 0.3$	1186.66	1199.17	1199.24	1209.61	1201.18
$\alpha = 0.2$	1187.07	1195.42	1195.42	1196.96	1202.22
$\alpha = 0.1$	1187.12	1191.29	1191.29	1193.14	1193.61
$\alpha = 0$	1187.12	1187.12	1187.12	1187.12	1187.12

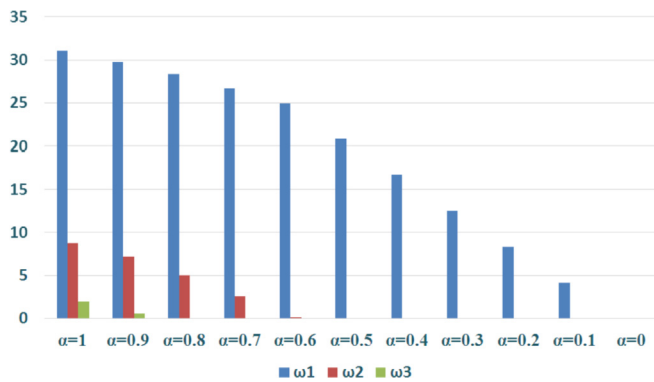
Fig. 3. Variation of profit in each scenario versus  $\alpha$  reduction.

**Table 2**  
Risk in profit versus RCP.

	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$	$\omega_5$
$\alpha = 1$	31.08	8.74	1.91	0.00	0.00
$\alpha = 0.9$	29.79	7.23	0.53	0.00	0.00
$\alpha = 0.8$	28.34	5.04	0.00	0.00	0.00
$\alpha = 0.7$	26.69	2.52	0.00	0.00	0.00
$\alpha = 0.6$	24.94	0.09	0.00	0.00	0.00
$\alpha = 0.5$	20.86	0.00	0.00	0.00	0.00
$\alpha = 0.4$	16.69	0.00	0.00	0.00	0.00
$\alpha = 0.3$	12.52	0.00	0.00	0.00	0.00
$\alpha = 0.2$	8.35	0.00	0.00	0.00	0.00
$\alpha = 0.1$	4.17	0.00	0.00	0.00	0.00
$\alpha = 0$	0.00	0.00	0.00	0.00	0.00



**Fig. 5.** Pareto between average profit versus risk-in-profit.



**Fig. 4.** The decline rate of risk in profit.

related profit in  $\alpha = 0$ . It can be understood from Table 2 that in scenario 1, which is the worst scenario, risk-in-profit slowly, decreases to zero while in the closest scenarios to the expected profit, risk in profit fastly decreases to zero. The risk-in-profit decline rate versus  $\alpha$  reduction in each scenario is indicated in Fig. 4. According to Fig. 4 can be concluded that risk in profit reduction rate in scenario 1 is more than other scenarios. Therefore, according to Figs. 3 and 4 can be shown that scenario 1 which is worst scenario, has a maximum reduction in the risk compared with other scenarios. In the meantime, profit reduction in scenario 1 is minimum compared with other scenarios.

In order to comparison of results and better explain the results of the DRC method, Table 3 is presented. According to Table 3, it can be concluded that to decrease risk-in-profit by 100%, average profit will be decreased by 2.07%. In addition, Table 3 indicates that in the worst scenario, according to the decrement of the RCP, risk-in-profit decreased by 31.19 \$ while the average profit is decreased

**Table 3**  
Comparison of risk results versus RCP.

	Worst Risk-in-profit (\$)	Average Risk-in-profit (\$)	Risk-in-profit Reduction (%)	Average profit (\$)	Average profit reduction (%)
$\alpha = 1$	31.08	8.34	0	1212.19	0.00
$\alpha = 0.9$	29.79	7.51	10	1211.92	0.02
$\alpha = 0.8$	28.34	6.68	20	1211.03	0.10
$\alpha = 0.7$	26.69	5.84	30	1209.50	0.22
$\alpha = 0.6$	24.94	5.01	40	1207.79	0.36
$\alpha = 0.5$	20.86	4.17	50	1205.51	0.55
$\alpha = 0.4$	16.69	3.34	60	1202.55	0.80
$\alpha = 0.3$	12.52	2.50	70	1199.17	1.07
$\alpha = 0.2$	8.35	1.67	80	1195.42	1.38
$\alpha = 0.1$	4.17	0.83	90	1191.29	1.72
$\alpha = 0$	0.00	0.00	100	1187.12	2.07

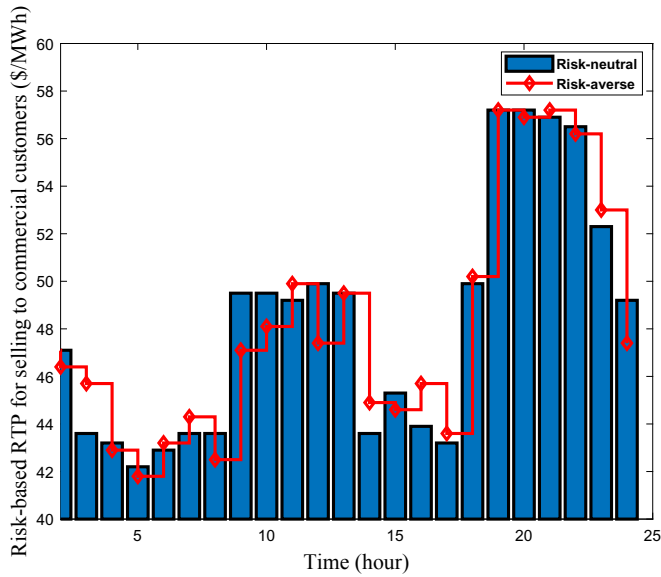


Fig. 6. Risk-based RTP offered to commercial customers.

by 25 \$. Finally, the Pareto solution of risk-in-profit versus total retailer profit is illustrated in Fig. 5.

### 5.1. Risk-based operation

In this part, the power procurement of electricity retailers and offered prices to different customers are presented. First, the risk-based offered price by electricity retailer to three types of consumers is indicated in Figs. 6–8.

According to the mentioned Figs can be shown that the highest offered RTP by the retailer is to the industrial, commercial, and residential customers, respectively. In addition according to Figs. 6–8, it seems that the pattern of offered price to industrial and commercial customers are similar, but carefully in these Figs can be understood that industrial customers accept higher offered price by the retailer. Commercial customers, because of gain utility from consumed electricity, have accepted price more than residential

customers in both risk-neutral and risk-averse strategy, which in the risk-averse strategy is higher than risk-neutral strategy. In addition, according to Fig. 7, it can be concluded that in order to prevent the loss of its customers, industrial customers may accept higher RTP prices which in the risk-averse strategy accepted price by industrial customers is more than risk-neutral customers. Also, it can be concluded from Fig. 8 that residential customers may be more elastic to offered RTP by the electricity retailer. The elastic behavior of residential customers in the risk-averse strategy is more than risk-neutral strategy, which is indicated in Fig. 8. Besides, from Figs. 6–8, it can be shown that risk-averse customers may accept higher RTP than risk-neutral customers. However, it is indicated in the mentioned Figs, the risk-averse customer in peak times accepts lower RTP offered by the electricity retailer.

Supplied demand by electricity retailer in both risk-neutral and risk-averse strategy is illustrated in Fig. 9. According to this Fig, in the peak times, due to the high offered retail price, supplied demand by retailers significantly dropped in both risk-averse and risk-neutral strategies. In addition, it can conclude from Fig. 9 that in risk-averse strategy supplied demand by the retailer is relatively high than the risk-neutral strategy. Also, according to Fig. 9, it can be shown that electricity retailer in the peak times is conservative and has the same demand in the risk-averse and risk-neutral strategies.

In order to procure its required power, Electricity retailer usually uses two market options of the electricity market, includes pool market and bilateral contracts, which are indicated in Figs. 10 and 11. A few minutes before operation, electricity retailers can buy or sell power with the electricity market in the real-time market. Also, it can procure its base required power from the forward contract several days before the operation. Therefore, the risk strategy of retailers has fewer effects on purchasing power from the forward contracts. In Fig. 11, it is illustrated that purchasing power from the forward contracts in risk-averse and risk-neutral strategy is equal. Besides, it can be seen that in both risk-neutral and risk-averse strategies, electricity retailer more rely on the forward contracts at peak times. In addition, the exchanged power of retailer with pool market in risk-averse and risk-neutral strategies is shown in Fig. 10. According to Fig. 10, it can be concluded that in addition to the lack of buying power from the pool market at peak times, electricity retailer sells power to pool market because of the

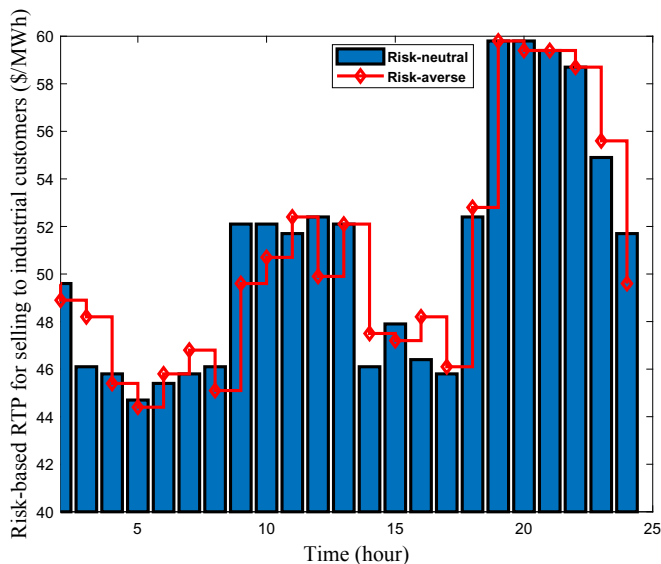


Fig. 7. Risk-based RTP offered to industrial customers.

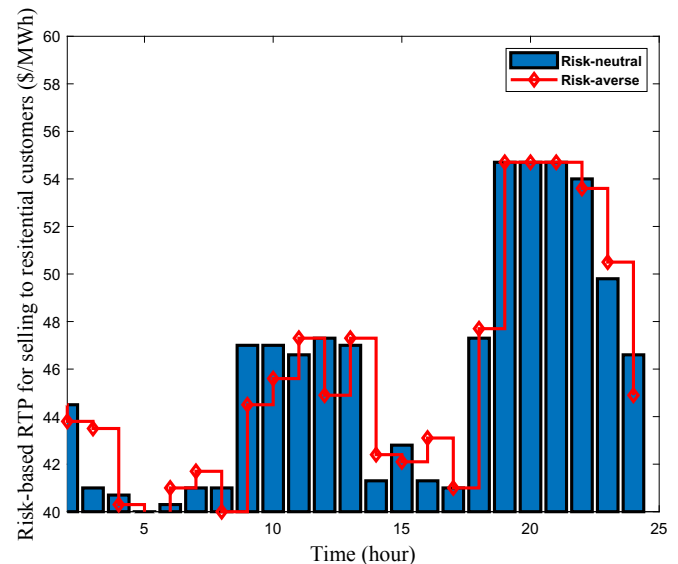


Fig. 8. Risk-based RTP offered to residential customers.

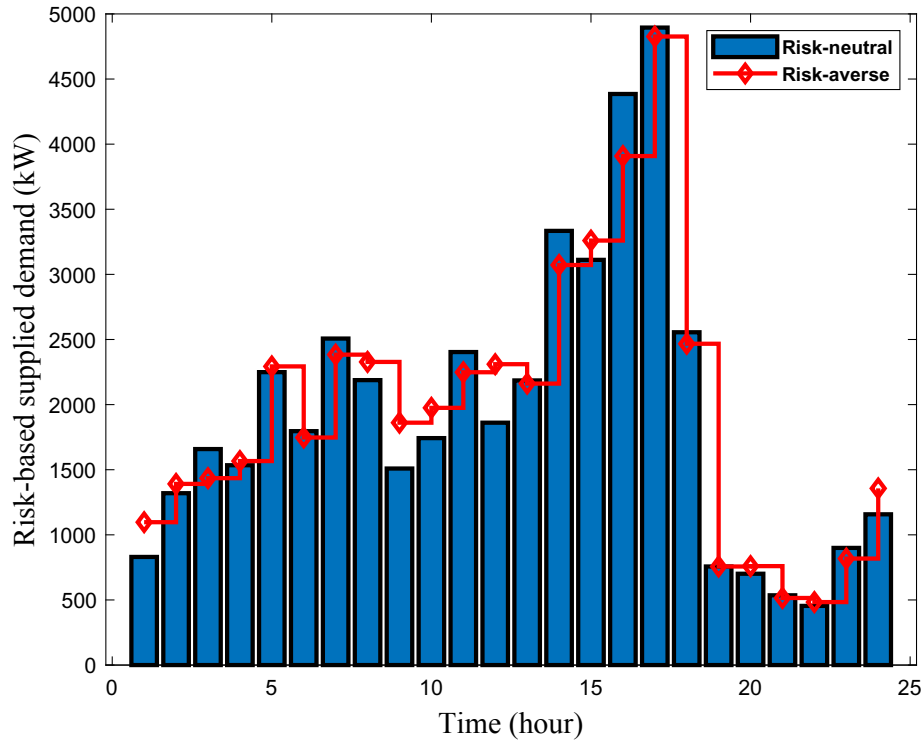


Fig. 9. Risk-based supplied demand by retailer.

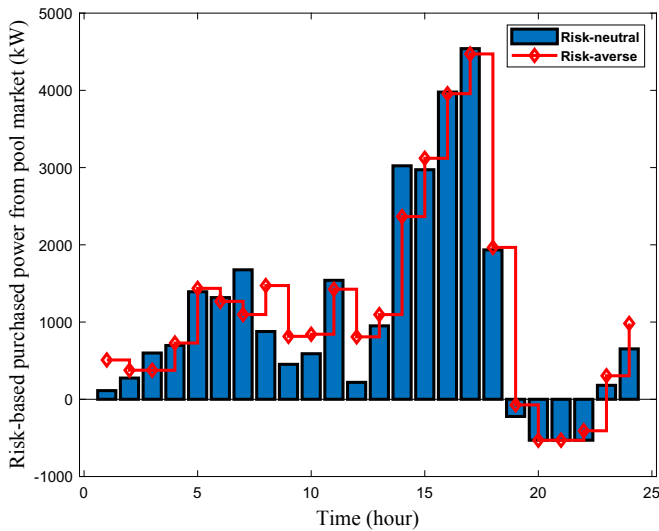


Fig. 10. Risk-based purchased power from the pool market.

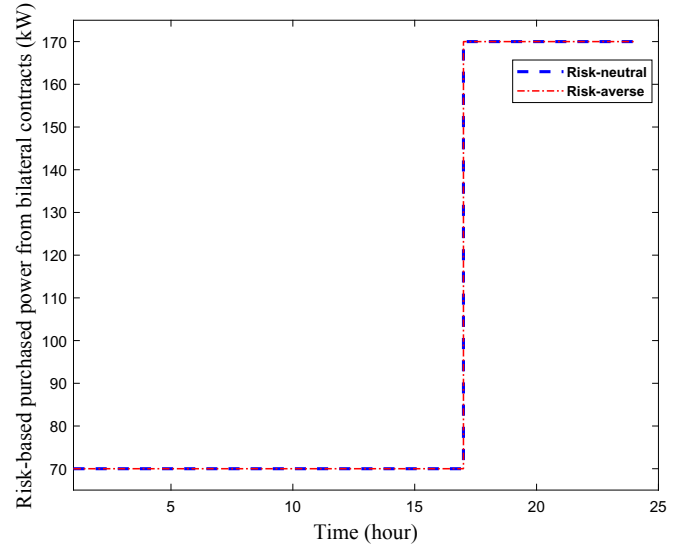


Fig. 11. Risk-based purchased power from the forward contract.

consumer's low demand. Selling power to the market by electricity retailers in the risk-averse and risk-neutral strategies at the peak times is similar which is indicated in Fig. 10. Also, it is illustrated in Fig. 10 that in risk-averse strategy purchased power from the pool market sometimes is more than the risk-neutral strategy.

In addition to mentioned market resources, electricity retailers can be used from self-generation units, including WT, PV, and DG units. Generated power by RERs is depended on wind speed, solar irradiation, and temperature, which stochastically considered in this paper. But DG units are dispatchable units in which electricity retailers can benefit from this feature of DG units. Therefore, the optimal risk-based schedule of DG units is illustrated in Fig. 12.

According to Fig. 12, it can be shown that due to the high price in the pool market, electricity retailer at peak times more rely on the generated power by DG units, which in the risk-neutral strategy is more than the risk-averse strategy. In addition, it can be concluded from Fig. 12 that in risk-averse strategy, electricity retailer less use from DG units that risk-neutral strategy.

Electricity retailers also can be used from energy storage systems to store energy and use from stored energy at the required times. According to Figs. 13 and 14, it can be shown that electricity retailer store electricity in the energy storage system at off-peak times and use from stored energy at peak times. In addition, it is illustrated in Figs. 13 and 14 that the stored energy in the ESS in

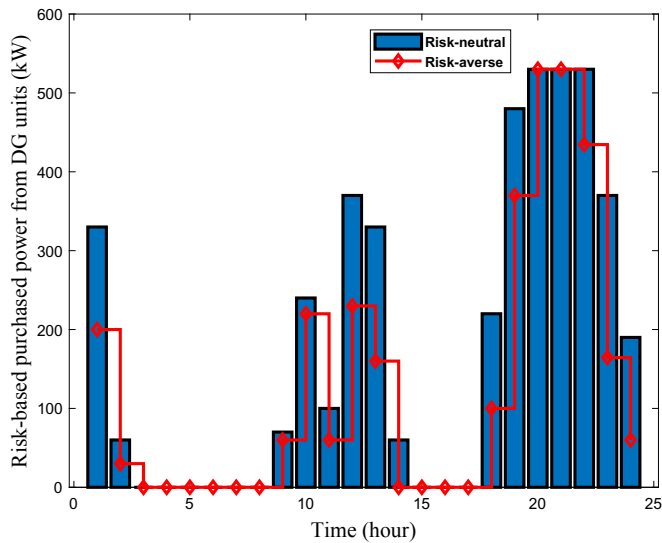


Fig. 12. Risk-based purchased power from the DG units.

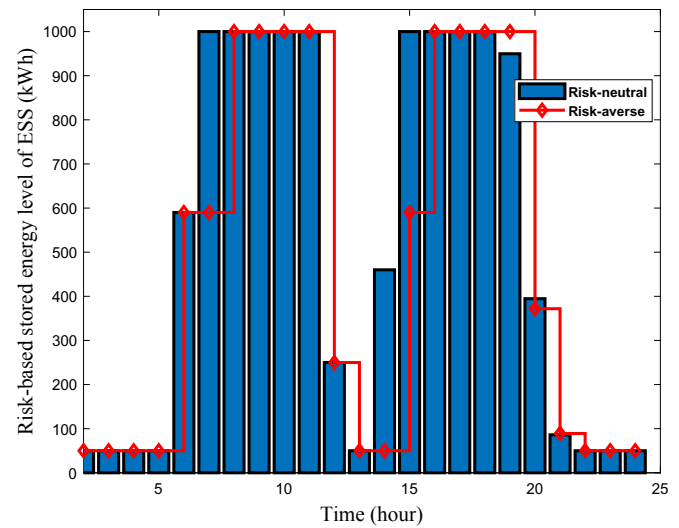


Fig. 14. Risk-based level of stored energy in ESS.

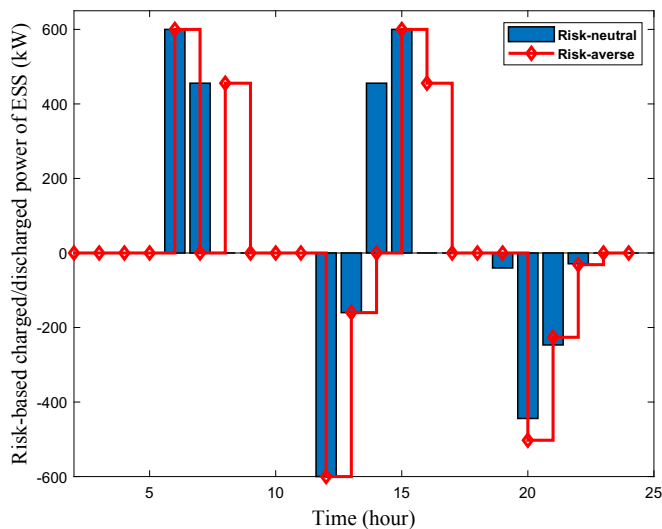


Fig. 13. Risk-based charged/discharged power of ESS.

risk-averse strategy is relatively similar to risk-neutral strategy while how power store in the energy storage are different in risk-averse and risk-neutral strategies.

## 6. Discussion

Electricity retailer offers different prices for residential, commercial and industrial users in the real-time based retail market. In addition, due to the existing uncertainties, real-time pricing in the retail market may be imposed financial risks for electricity retailers in the decision-making problem. Therefore, a proper risk measurement approach is needed to evaluate the faced risks by electricity retailers in the power procurement process. In this paper, we proposed a stochastic based risk evaluation method, which called downside risk constraints approach. Proposed risk-measure is used to analyze the imposed risks from uncertainties in the real-time pricing problem of electricity retailers. Obtained results from the proposed approach are clear and more useful for electricity retailer if wants to select a trade-off between risk and profit. In addition using the proposed risk management approach, electricity retailer

can be obtained risk-averse strategy in a wide range like robust optimization.

## 7. Conclusion

In this paper, a stochastic-based real-time pricing is determined by electricity retailer for three types of customers including industrial, commercial, and residential customers. In addition, uncertainties of solar irradiation, temperatures, wind speed, demand, and pool market price are considered in the proposed model. Therefore, downside risk constraints method is proposed in the real-time pricing problem which leads various risks in the retailer decision-making problem. The proposed risk evaluation method guaranteed a zero risk strategy for electricity retailers unlike other risk measures in the literature. Numerical results represented the various risk-in-profits imposed from various profits amounts in each scenario which helped the retailer in decisions-making in different scenarios and strategies. According to obtained results, in order to decrease amounts of risk-in-profit from 100% to zero, the total profit of electricity retailers should be dropped by 2.07%. In addition, obtained results have been demonstrated that zero risk experience by retailer can be led to imposing \$ 25 more cost to electricity retailer while in the worst scenario reduces risk by \$ 31. Finally, obtained results have been shown that the offered price to industrial consumers is more than and commercial and residential customers because of their desire to pay more price to meet its customers. For future works, the proposed risk method can be applied to model risk related to uncertainties in other energy systems.

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