Computers & Industrial Engineering 139 (2020) 106200

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Employing revenue sharing strategies when confronted with uncertain and promotion-sensitive demand

Yu-Chung Tsao*, Pei-Ling Lee

Department of Industrial Management, National Taiwan University of Science and Technology, Taiwan

ARTICLE INFO

Keywords:
Operations-marketing interface
Revenue sharing
Promotion
Profit
Uncertain demand

ABSTRACT

Revenue sharing (RS) refers to the distribution of income among stakeholders (e.g., manufacturers and retailers) in a business-to-business environment. This method is a widely employed marketing strategy in the video rental, telecoms, computer, sports, and music industries, among others. Promotional activities are also a popular method of stimulating sales. This study considers a decentralized supply chain, in which a manufacturer sells products to a retailer under conditions of uncertain and promotion-sensitive demand, using either a manufacturer or retailer promotion. The study aims to determine how RS affects the decision making and profits of both manufacturers and retailers. It seeks to uncover whether RS benefits channel members in the cases of manufacturer and retailer promotions. The results show that when the manufacturer promotes products to end customers, RS is an effective incentive for increasing the profits of manufacturers and retailers. However, retailers may be unwilling to share revenue when they undertake their own promotional activities based on different demand function settings. The findings add novel insights to the literature of management, which can be used by business decision makers.

1. Introduction

Throughout most of the 1990s, video rental companies (e.g., Blockbuster) would spend around $65 on an original videotape from a studio (distributor), which they could then rent out to customers for $3 to $4 a time (Cachon & Lariviere, 2005). However, demand for new releases is always highest when they first come out (there is always an audience for the newest releases) and drops dramatically after the first few weeks. Thus, video retailers had a very hard time making money on the rentals. Consequently, video rental companies could only afford to buy a few cassettes to accommodate the initial surge in demand. In 1998, Sumner Redstone, the CEO of Blockbuster's parent company Viacom, approached the studios and procured a revenue-sharing deal to address this issue. According to the agreement, instead of the video rental store (Blockbuster) paying the studio $65 per tape and keeping all the subsequent revenues, the retailer would pay around $8 per tape and give close to half the rental revenues to the studio. The video rental company agreed to pay the studios a portion (somewhere between 30% and 45% of its rental income) in exchange for a reduction in the initial price of a tape from $65 to $8. Arrangements like this are common in the business-to-business (B2B) environment.

Promotional activities have also become a popular way of stimulating sales. For example, consumers of tablet computers received a power bank or a protective cover for free when they bought the new Nexus 7 (Google & Asus) or Galaxy Note Pro 12.2 (Samsung) (Tsao, 2015a). JC Penney provided up to 20% off to consumers when they used their JCP card, while Costco provided free food samples to consumers in the store. Manufacturers or retailers may also promote products simultaneously. Wal-Mart and Costco do this whenever they provide price discounts or additional shelf space for specific electrical equipment, in order to stimulate demand. Other promotional activities include offering free goods, advertising, and displays. Narasimhan (1990) explored the important factors that managers consider when allocating trade and consumer promotions. Fig. 1(a) and (b) below show the respective promotion forms of a sporting goods manufacturer and retailer. Thus, it is interesting to discuss how RS policies can affect channel members' decisions and profits in the context of promotional efforts.

Uncertainty and limitations are inherent in supply chains. Demand can never be forecasted exactly, maintenance quality is stochastic (Duan, Deng, Gharaei, Wu, & Wang, 2018), and warehouse space is limited (Hoseini Shekarabi, Gharaei, & Karimi, in press), among other things. In practice, consumer demand is very difficult to forecast accurately. Therefore, it is crucial that companies apply a strategy using probability to make informed business decisions, particularly in companies that sell innovative products, like those found in the fashion or

* Corresponding author.
E-mail address: yctsao@mail.ntust.edu.tw (Y.-C. Tsao).

https://doi.org/10.1016/j.cie.2019.106200
Received 18 August 2018; Received in revised form 4 November 2019; Accepted 21 November 2019
Available online 25 November 2019

(a) Manufacturer Promotion (b) Retailer Promotion

(Source: http://www.couponpromos.org/printable-coupons/nike/) (Source: http://encouponcodes.net/dicks-sporting-goods-printable-coupons-2013/)

Fig. 1. Promotions by sporting goods manufacturer and retailer.

high-tech industries. Promotional efforts and demand uncertainty have rarely been simultaneously considered in previous research on RS. An example of promotion occurring together with RS is the film industry. The film supply chain generally adopts revenue-sharing contracts that specify how the box office receipts will be split between film distributors and exhibitors, while film distributors or exhibitors may adopt promotional activities (advertising or free gifts) to stimulate box office returns. To fill this gap in the research, this study considers a decentralized supply chain, in which a manufacturer sells products to a retailer under the condition of uncertain and promotion-sensitive demand. We consider cases of both manufacturer and retailer promotions. The key question is whether RS benefits channel members in the cases of manufacturer or retailer promotions when confronted with uncertain and promotion-sensitive demand. To answer the key question, we consider the following:

1. In the case of a manufacturer promotion, the manufacturer determines the optimal wholesale price and the promotional activities, while the retailer determines the optimal order quantity, and then, both seek to maximize their own profits.
2. In the case of a retailer promotion, the manufacturer determines the optimal wholesale price, while the retailer determines the optimal order quantity and promotional efforts to maximize their own profits.
3. How does RS affect manufacturers' and retailers' decision making and profits? Additionally, we seek to ascertain methods that would benefit channel members in both promotion cases.

Our findings uncovered new management insights that can be used as guidance points by business managers.

The remainder of this paper is organized as follows: In Section 2, we review all the relevant literature. In Section 3, models for a manufacturer promotion with no RS (NRS) are compared with those of a manufacturer promotion with RS. In Section 4, we compare models for a retailer promotion under NRS and RS policies. Finally, Section 5 presents our conclusions and their implications for future managerial decision making.

2. Literature review

2.1. Revenue sharing

RS refers to the distribution of income among stakeholders (e.g., manufacturers and retailers). For example, a retailer may share some of its revenue with the manufacturer in return for a discount on the wholesale price. Video rental industry giant Blockbuster Inc. engaged in this particular strategy. RS has also been heavily utilized in the telecom, computers, and music industries, among others, and a wide array of research exists on the subject. Most studies consider RS to be a contract for channel coordination. Omkar (2013) analyzed supply chain coordination using a revenue-dependent contract (the actual proportion of supply chain revenue shared among the different players depends on the amount of revenue generated). Chen (2013) showed that, in a cooperative setting, the electronics market tends to achieve lower retail prices, larger stock quantity, and improved channel efficiency when operating under a consigned revenue-sharing, vendor-managed inventory contract (the combination of consignment, revenue-sharing, and vendor-managed inventory). Mafakheri and Nasiri (2013) revisited the issue of RS in reverse supply chains to formulate solutions to problems in the coordination of manufacturers and retailers.

Rhee, Schmidt, Veen, and Venugopal (2014) discussed how to use RS to coordinate a multi-stage, multi-party supply chain. Govindan and Popiuc (2014) considered reverse supply chain coordination under a revenue-sharing contract in the personal computer industry. Panda (2014) explored the coordination of a socially responsible manufacturer-retailer chain using revenue-sharing contracts. Chakraborty, Chauhan, and Navneet Vidyarthi (2015) compared wholesale price and RS mechanisms in a channel of two competing brands, and Avinadav, Chernonog, and Perlman (2015) provided a thorough investigation of the RS contract format typically used in the mobile applications industry. Additionally, Zhang, Liu, Zhang, and Bai (2015) and Dye and Yang (2016) showed that RS and cooperative investment (cooperatively investing in preservation technology to reduce deterioration) contracts could be designed to coordinate the supply chain.

A revenue-sharing contract can play an important role in coordinating the distribution of benefits among the upstream and downstream members of a supply chain and improve its overall performance (Song & Gao, 2018). Hou, Wei, Li, Huang, and Ashely (2017) showed that a revenue-sharing contract can coordinate a decentralized supply chain using a simultaneous move game. Raza (2018) discussed supply chain coordination under a revenue-sharing contract with corporate social responsibility and partial demand information. Heydari and Ghasemi (2018) designed a revenue-sharing contract for reverse supply chain coordination under conditions of stochastic quality of returned products and uncertain remanufacturing capacity. Zhao, Chen, and Gong (2019) explored collusion and information sharing in a two-echelon supply chain coordinated by a revenue-sharing contract. Shafig and Savino (2019) determined the manufacturer's capacity procurement decisions through a new commitment-based model with penalties and revenue-sharing.

2.2. Promotion decision

Most quantitative studies investigating promotions have focused on how customers respond to the promotional efforts of retailers and how retailers make promotion decisions. Tsao and Sheen (2008) studied the problem of dynamic pricing, promotion, and replenishment of deteriorating items by considering the trade credit of suppliers and the promotional efforts of retailers. Zhang, Chen, and Lee (2008) provided an analytical model for making jointly optimal decisions on pricing, promotion, and inventory control. Grewal et al. (2011) indicated that price promotions are a key marketing instrument that retailers use to generate sales and increase their market share. Su and Geunes (2012) analyzed the impact of price promotions on profit levels in a two-stage supply chain. Tsao and Sheen (2012) considered a two-echelon multiple-retailer distribution channel in conjunction with the promotional efforts of retailers and the sales learning curve (the more times a sales process is repeated, the lower its cost). Giri, Bardhan, and Maiti (2013) developed and analyzed a two-echelon supply chain where market demand depends on both retail price and sales effort.

In terms of application, Gong, Smith, and Telang (2014) analyzed the impact of price discounts on self- and cross-channel sales. They claimed that digital movie consumers are highly sensitive to price promotions. Hutter and Hoffman (2014) considered ambient media as a promotional tool and explored its effects on advertising efficacy. Haans and Gijsbrechts (2011) conceptualized why and how store size influences the sales effectiveness of four promotional indicators: depth of promotional discount, display support, feature support, and whether the promotion is quantity-based. Tsao (2015a) considered cooperative promotions (the practice of a retailer sharing the manufacturer's promotional cost to stimulate sales) under uncertain demand. He showed that the retailer is only willing to share the manufacturer's promotional cost when the retail price is internal.

Recently, Yang, Liao, Shi, and Li (2015) investigated the joint optimization of ordering and promotional strategies, focusing on the choice between rebates and everyday low price. Huang, Nie, and Zhang (2018) considered that the promotional efforts of retailers have a positive impact on demand, but a negative impact on the manufacturer's brand image. Zhu, Jiao, and Yuan (in press) determined the optimal product reliability, sales, and promotion decisions under nonrenewable warranties. Malekian and Ratsti-Barzoki (2019) studied the effect of reference price on price promotion considering advertising in a two-echelon supply chain.

Table 1 shows comparisons with other examples of recently related literature. It shows that no study on promotion considered revenue sharing (Huang et al., 2018; Malekian & Ratsti-Barzoki, 2019; Zhu et al., in press). Also, very few studies on revenue sharing considered promotion (Heydari & Ghasemi, 2018; Hou et al., 2017; Raza, 2018; Shafig & Savino, 2019; Song & Gao, 2018; Zhao et al., 2019). Only Bai, Chen, and Xu (2017) considered promotion and revenue sharing simultaneously. Bai et al. (2017) proposed a revenue and promotional cost sharing contract and a two-part tariff contract to coordinate sustainable supply chains with deteriorating items. However, they did not consider manufacturer promotions or uncertain demand.

This paper contributes to the literature in several ways. First, very few studies on RS have simultaneously considered the effects of promotional efforts and demand uncertainty. However, it is worth discussing how RS works under a promotion policy. In this study, we consider a decentralized supply chain, in which a manufacturer sells products to a retailer under uncertain and promotion-sensitive demand. This study seeks to determine whether RS benefits channel members in manufacturer and retailer promotion cases. Second, no previous studies on promotion have incorporated RS policy into their models. This study is the first to consider RS policy in a quantity study about promotion. In this study, we differentiate between manufacturer promotion and retailer promotion. The objective is to determine whether RS benefits channel members through manufacturer promotion or retailer promotion. Third, our results show that when the manufacturer promotes products to end customers, RS is an effective incentive for increasing the profits of manufacturers and retailers. However, the retailer's profit may decrease when it undertakes its own promotional activities. To address this problem, this paper provides a modified RS policy that benefits both the manufacturer and retailer under the condition of a retailer promotion.

3. Manufacturer promotion

We first introduce the following notation used in this paper:

p: retail price
c: manufacturer's marginal cost
ξ: basic demand without any promotional effort
D: demand function $D = f(\rho) + U$, where $f(\rho) = \rho\xi$ is a function of promotional effort ρ, and U is a continuous random variable following uniform distribution on $[-\rho\xi, \rho\xi]$
K: promotional cost parameter
λ: fraction of revenue shared by retailer with manufacturer when manufacturer promotes products
w_λ: wholesale price under RS policy when manufacturer promotes products
q_λ: order quantity under RS policy when manufacturer promotes products
: promotional effort of manufacturer under RS policy
Π_R^λ: retailer profits under RS policy when manufacturer promotes products
Π_M^λ: manufacturer profits under RS policy when manufacturer promotes products
w_α: wholesale price under NRS policy when manufacturer promotes products
q_α: order quantity under NRS policy when manufacturer promotes

Table 1
Comparisons with other recent researches.

Papers	*Situations* Revenue Sharing	Promotion-sensitive Demand	Uncertain Demand	Retailer Promotion	Manufacturer Promotion
Huang et al. (2018)		◎		◎	◎
Zhu et al. (in press)		◎		◎	
Malekian and Ratsti-Barzoki (2019)		◎		◎	◎
Hou et al. (2017)	◎				
Song and Gao (2018)	◎				
Raza (2018)	◎		◎		
Heydari and Ghasemi (2018)	◎		◎		
Zhao et al. (2019)	◎		◎		
Shafig and Savino (2019)	◎		◎		
Bai et al. (2017)	◎	◎		◎	
This study	◎	◎	◎	◎	◎

◎ means that the situation is considered in his/her/their paper.

products
ρ_α: promotional effort of manufacturer under NRS policy when manufacturer promotes products
Π_R^α: retailer profits under NRS policy when manufacturer promotes products
Π_M^α: manufacturer profits under NRS policy when manufacturer promotes products

This paper considers the one-period newsvendor model. Following the assumptions in Krishnan, Kapuscinski, and Butz (2004) and Tsao (2015a, 2015b), the goods have no salvage value and the retailer scraps them after the selling period. Also, the neither the manufacturer nor the retailer incurs any penalty (other than lost revenue) if inventories are insufficient. In this section, we consider a case where a manufacturer promotes the product directly to customers. In practice, manufacturers may promote their products by providing discounts or by upgrading their products. For example, Nike provided $10 discounts on specific products (see Fig. 1). When a retailer shares its revenue with the manufacturer, it is operating under a policy of RS. The retailer profit in the RS case can be written as:

$$\Pi_R^\lambda(q_\lambda) = (1-\lambda)p\int_0^{q_\lambda}\frac{x}{2\rho_\lambda\xi}dx + (1-\lambda)p\int_{q_\lambda}^{2\rho_\lambda\xi}\frac{q_\lambda}{2\rho_\lambda\xi}dx - w_\lambda q_\lambda \quad (1)$$

The manufacturer's profit in the RS case can be written as:

$$\Pi_M^\lambda(w_\lambda,\rho_\lambda) = \lambda p\int_0^{q_\lambda}\frac{x}{2\rho_\lambda\xi}dx + \lambda p\int_{q_\lambda}^{2\rho_\lambda\xi}\frac{q_\lambda}{2\rho_\lambda\xi}dx + (w_\lambda - c)q_\lambda - K(\rho_\lambda - 1)^2 \quad (2)$$

In Eq. (1), $(1-\lambda)p\int_0^{q_\lambda}\frac{x}{2\rho_\lambda\xi}dx$ is the expected revenue when the order quantity is larger the demand, and $(1-\lambda)p\int_{q_\lambda}^{2\rho_\lambda\xi}\frac{q_\lambda}{2\rho_\lambda\xi}dx$ is the expected revenue when the demand is larger the order quantity. In Eq. (2), $\lambda p\int_0^{q_\lambda}\frac{x}{2\rho_\lambda\xi}dx + \lambda p\int_{q_\lambda}^{2\rho_\lambda\xi}\frac{q_\lambda}{2\rho_\lambda\xi}dx$ is the expected revenue shared by the retailer. $K(\rho_\lambda - 1)^2$ represents the cost of promotional efforts, which is assumed to be reasonably convex in promotional effort (Krishnan et al., 2004). The idea of promotion cost setting is referred to in Krishnan et al. (2004), Tsao (2015a, 2015b), and Bai et al. (2017). Promotional effort could encompass types of price cuts, displays, free goods, coupons, advertising, direct mail, point-of-sale information, targeted discounts, free gift wrapping or delivery, or a host of other measures (Krishnan et al., 2004). Following the assumption in Krishnan et al. (2004), we consider a promotional effort to be a decision other than the retail price. The promotional effort ρ_λ does not affect the basic demand ξ, but affects the effort-induced demand $\rho_\lambda\xi$. The cost of promotional efforts satisfies the law of decreasing marginal value. That is, the cost of the first unit of promotional effort yields higher demand than the second and subsequent units, with a continuing reduction for greater amounts.

In the case of a manufacturer promotion, the manufacturer first determines the optimal wholesale price and promotional effort to maximize its own profit. Then, the retailer determines the optimal order quantity to maximize its own profit based on the manufacturer's decisions. To solve the problem, we first determine the closed form of the retailer's order quantity. Then, we determine the wholesale price and promotional effort after substituting the order quantity function into the manufacturer's profit function.

Lemma 1 shows the manufacturer's optimal wholesale price and promotional effort and the retailer's optimal order quantity.

Lemma 1. *Considering the case of manufacturer promotion and revenue sharing:*

(a) *The manufacturer's optimal wholesale price* $w_\lambda^* = \frac{(1-\lambda)[(1-\lambda)p+c]}{2-\lambda}$; *the manufacturer's optimal promotional effort* $\rho_\lambda^* = \frac{(p-c)^2\xi}{2K(2-\lambda)p} + 1$.
(b) *The retailer's optimal order quantity* $q_\lambda^* = \frac{(p-c)[(p-c)^2\xi + 2K(2-\lambda)p]\xi}{K(2-\lambda)^2p^2}$.

To determine the effects of RS, we compare the models with RS and no RS (NRS). By letting $\lambda = 0$ in Eqs. (1) and (2), we can obtain the retailer's and manufacturer's respective profits under an NRS policy. We then solve the profit maximization problem and determine the optimal wholesale price, promotional effort, and order quantity in Lemma 2.

Lemma 2. *Considering the case of manufacturer promotion and no revenue sharing:*

(a) *The manufacturer's optimal wholesale price* $w_\alpha^* = \frac{p+c}{2}$; *the manufacturer's optimal promotional effort* $\rho_\alpha^* = \frac{(p-c)^2\xi}{4Kp} + 1$.
(b) *The retailer's optimal order quantity* $q_\alpha^* = \frac{(p-c)[(p-c)^2\xi + 4Kp]\xi}{4Kp^2}$.

Based on the aforementioned models, Proposition 1 compares the decisions made under an NRS policy to those made under an RS policy. It argues that the wholesale price decreases when the retailer shares its revenue with the manufacturer. However, the manufacturer makes more promotional efforts and the retailer orders more products.

Proposition 1.

(a) *The wholesale price decreases when the retailer shares revenue with the manufacturer, i.e.,* $w_\lambda^* < w_\alpha^*$.
(b) *The manufacturer makes more promotional effort when the retailer shares revenue with the manufacturer, i.e.,* $\rho_\lambda^* > \rho_\alpha^*$.
(c) *The order quantity increases when the retailer shares revenue with the manufacturer, i.e.,* $q_\lambda^* \geqslant q_\alpha^*$.

Proposition 2(a) argues that the manufacturer benefits under RS. In Proposition 2(b), we find the threshold of λ, where the retailer benefits under RS. When the fraction of revenue shared by the retailer with the manufacturer is smaller than the upper bound ($\lambda < \frac{4Kp + 3(p-c)^2\xi + \sqrt{16K^2p^2 + 8K(p-c)^2p\xi + 5(p-c)^4\xi^2}}{4Kp + (p-c)^2\xi}$), the retailer is willing to share revenue with the manufacturer because this will lead to an increase in the retailer's profit. Therefore, RS could be an effective incentive policy for increasing channel members' profits in a decentralized supply chain under a manufacturer promotion.

Proposition 2.

(a) *The manufacturer's profit is higher when the retailer shares revenue with the manufacturer; i.e.,* $\Pi_M^\lambda > \Pi_M^\alpha$.
(b) *When* $\lambda < \frac{4Kp + 3(p-c)^2\xi + \sqrt{16K^2p^2 + 8K(p-c)^2p\xi + 5(p-c)^4\xi^2}}{4Kp + (p-c)^2\xi}$, *the retailer's profit is higher when the retailer shares revenue with the manufacturer; i.e.,* $\Pi_R^\lambda > \Pi_R^\alpha$.

Table 2 presents a summary of the predictions for the manufacturer's and retailer's decision making and profits for a given λ. The table indicates that the wholesale price decreases, but promotional efforts, order quantity, and manufacturer's profit grow with an increase in the fraction of revenue shared λ. It also indicates that the retailer's profit is concave in λ. That means we can find a value of λ, for which the retailer's profit is higher under an RS policy than under a no RS policy. In this example, the retailer's profit is optimal ($\Pi_R^\alpha = 97.71$) when $\lambda = 0.1315$. These results verify Propositions 1 and 2. They show that RS policy under a manufacturer promotion can benefit manufacturers, as well as retailers.

Table 2 also incurs an interesting problem, whereby the fraction of revenue shared λ could be a decision variable. Substituting Eqs. (4), (6), and (7) into Eq. (2), we obtain the retailer profit $\Pi_R^\lambda(\lambda)$ with only one decision variable λ. From the retailer's perspective, the optimal fraction of revenue shared λ_R^* could be determined to maximize $\Pi_R^\lambda(\lambda)$:

$$\lambda_R^* = 1 - \frac{(1/2)\sqrt{[2(p-c)^2\xi + 4Kp]^2 - 8Kp(p-c)^2\xi} - (p-c)^2\xi}{2Kp} \quad (3)$$

Proposition 3(a) shows that the value of the optimal fraction of

Table 2
Influences of λ on decisions and profits ($K = 500$; $\xi = 50$; $p = 15$; $c = 5$).

λ	Wholesale price		Promotion effort		Order quantity		Manufacturer Profits		Retailer Profits	
	no RS	RS	no RS	RS	no RS	RS	no RS	RS	no RS	RS
0.00	10	10	1.17	1.17	38.89	38.89	180.56	180.56	97.22	97.22
0.10	10	8.76	1.17	1.18	38.89	41.24	180.56	190.83	97.22	97.68
0.20	10	7.56	1.17	1.19	38.89	43.90	180.56	202.33	97.22	97.55
0.30	10	6.38	1.17	1.20	38.89	46.91	180.56	215.30	97.22	96.57
0.40	10	5.25	1.17	1.21	38.89	50.35	180.56	230.04	97.22	94.40
0.50	10	4.17	1.17	1.22	38.89	54.32	180.56	246.91	97.22	90.54

revenue shared λ_R^* should be within (0,1). Proposition 3(b) and (c) indicate that the optimal fraction of revenue shared increases as basic demand increases or as the manufacturer's marginal cost decreases.

Proposition 3.

(a) $0 < \lambda_R^* < 1$.
(b) *The optimal fraction of revenue shared λ_R^* increases as the basic demand ξ increases.*
(c) *The optimal fraction of revenue shared λ_R^* increases as the manufacturer's marginal cost c decreases.*

4. Retailer promotion

The following notations are used in this section.

ϕ: fraction of revenue shared by retailer with manufacturer when retailer promotes products
w_ϕ: wholesale price under RS policy when retailer promotes products
q_ϕ: order quantity under RS policy when retailer promotes products
ρ_ϕ: promotional effort of retailer under RS policy
Π_R^ϕ: retailer's profit under RS policy when retailer promotes products
Π_M^ϕ: manufacturer's profit under RS policy when retailer promotes products
w_β: wholesale price under NRS policy when retailer promotes products
q_β: order quantity under NRS policy when retailer promotes products
ρ_β: promotional effort of manufacturer under NRS policy when retailer promotes products
Π_R^β: retailer's profit under NRS policy when retailer promotes products
Π_M^β: manufacturer's profit under NRS policy when retailer promotes products

This section considers the case of a retailer promoting products directly to consumers. Retailers may promote products by providing discount coupons or free samples. For example, Dick's Sporting Goods took 20% off some of its products (see Fig. 1). Under an RS policy, the retailer's profit can be written as:

$$\Pi_R^\phi(\rho_\phi, q_\phi) = (1-\phi)p\int_0^{q_\phi}\frac{x}{2\rho_\phi\xi}dx + (1-\phi)p\int_{q_\phi}^{2\rho_\phi\xi}\frac{q_\phi}{2\rho_\phi\xi}dx - w_\phi q_\phi - K(\rho_\phi - 1)^2 \tag{4}$$

The manufacturer's profit under an RS policy can be written as:

$$\Pi_M^\phi(w_\phi) = \phi p\int_0^{q_\phi}\frac{x}{2\rho_\phi\xi}dx + \phi p\int_{q_\phi}^{2\rho_\phi\xi}\frac{q_\phi}{2\rho_\phi\xi}dx + (w_\phi - c)q_\phi \tag{5}$$

In Eq. (4), $(1-\phi)p\int_0^{q_\phi}\frac{x}{2\rho_\phi\xi}dx$ is the expected revenue when the order quantity is larger than the demand and $(1-\phi)p\int_{q_\phi}^{2\rho_\phi\xi}\frac{q_\phi}{2\rho_\phi\xi}dx$ is the expected revenue when the demand is larger than the order quantity. In Eq. (5), $\phi p\int_0^{q_\phi}\frac{x}{2\rho_\phi\xi}dx + \phi p\int_{q_\phi}^{2\rho_\phi\xi}\frac{q_\phi}{2\rho_\phi\xi}dx$ is the expected revenue shared by the retailer.

In the case of a retailer promotion, the manufacturer first determines the optimal wholesale price to maximize its own profit. Then, the retailer determines the optimal order quantity and promotional effort to maximize its own profit based on the manufacturer's decision. To solve the problem, we first determined the respective closed forms of the retailer's order quantity and promotional effort. Then, we determined the wholesale price after substituting the order quantity and promotional effort functions into the manufacturer's profit function. Lemma 3 shows the manufacturer's optimal wholesale price and promotional effort and the retailer's optimal order quantity.

Lemma 3. *Considering the case of retailer promotion and revenue sharing:*

(a) *The manufacturer's optimal wholesale price* $w_\phi^* = \frac{3(1-\phi)[(3-2\phi)p+c]\xi + \Upsilon/\Omega^{1/3} + (3\Omega)^{1/3}}{6(2-\phi)\xi}$, *where* $\Upsilon = 3^{2/3}(1-\phi)\xi\{2p[3(1-\phi)c\xi + 2K(2-\phi)^2] - 3(1-\phi)\xi(p^2+c^2)\}$ *and* $\Omega = 18K(\phi^2-3\phi+2)^2p(p-c)\xi^2 + 9(1-\phi)^3(p-c)^3\xi^3 + 2\sqrt{3}\sqrt{K(2-\phi)^2(1-\phi)^3p\xi^3\begin{bmatrix}16K^2(2-\phi)^4p^2 - 9K(2-\phi)^2(1-\phi)(p-c)^2p\xi + \\ 54(1-\phi)^2(p-c)^4\xi^2\end{bmatrix}}$.
(b) *The retailer's optimal order quantity* $q_\phi^*(w_\phi) = \frac{[p(1-\phi)-w_\phi]\{[p(1-\phi)-w_\phi]^2\xi + 2K(1-\phi)p\}\xi}{K(1-\phi)^2p^2}$; *the retailer's optimal promotional effort* $\rho_\phi^*(w_\phi) = \frac{[p(1-\phi)-w_\phi]^2\xi}{2K(1-\phi)p} + 1$.

Substituting w_ϕ^* into $\rho_\phi^*(w_\phi)$ and $q_\phi^*(w_\phi)$, we obtain the optimal values of ρ_ϕ^* and q_ϕ^*. We can obtain the retailer's and manufacturer's profits under an NRS policy by allowing $\phi = 0$ in Eqs. (4) and (5), respectively. Then, the optimal wholesale price, promotional effort, and order quantity are determined in Lemma 4.

Lemma 4. *Considering the case of retailer promotion and no revenue sharing:*

(a) *The manufacturer's optimal wholesale price* $w_\beta^* = \frac{3(3p+c)\xi + 3^{2/3}\xi[3\xi(p-c)^2 - 16Kp]/\Gamma^{1/3} + (3\Gamma)^{1/3}}{12\xi}$, *where* $\Gamma = 4\sqrt{6}\sqrt{Kp\xi^3\begin{bmatrix}128K^2p^2 - 18K(p-c)^2p\xi \\ + 27(p-c)^4\xi^2\end{bmatrix}} - 9(p-c)\xi^2[8Kp + (p-c)^2\xi]$.
(b) *The retailer's optimal order quantity* $q_\beta^*(w_\beta) = \frac{(p-w_\beta)[(p-w_\beta)^2\xi + 2Kp]\xi}{Kp^2}$; *the retailer's optimal promotional effort* $\rho_\beta^*(w_\beta) = \frac{(p-w_\beta)^2\xi}{2Kp} + 1$.

Due to the complexity of w_ϕ^* and w_β^*, it is easiest to compare them using a numerical example, such as the one shown in Table 3. Under retailer promotion, Table 3 shows that although the manufacturer benefits (an increase in profits), the retailer suffers (a decrease in profits) under RS. Therefore, RS is not an effective incentive policy in a decentralized supply chain under a retailer promotion. Cachon and Lariviere (2005) also found that RS contracts should be avoided when demand is sufficiently influenced by retail effort. In addition, Table 3

Table 3
Influences of ϕ on decisions and profits ($K = 500$; $\xi = 50$; $p = 15$; $c = 5$).

ϕ	Wholesale price		Promotion effort		Order quantity		Manufacturer Profits		Retailer Profits	
	no RS	RS	no RS	RS	no RS	RS	not RS	RS	no RS	RS
0.00	9.59	9.59	1.10	1.098	39.57	39.57	181.70	181.70	102.24	102.24
0.10	9.59	8.38	1.10	1.097	39.57	41.63	181.70	191.21	102.24	101.90
0.20	9.59	7.20	1.10	1.096	39.57	43.87	181.70	201.66	102.24	100.73
0.30	9.59	6.06	1.10	1.094	39.57	46.30	181.70	213.17	102.24	98.44
0.40	9.59	4.96	1.10	1.091	39.57	48.92	181.70	225.87	102.24	94.67
0.50	9.59	3.92	1.10	1.085	39.57	51.75	181.70	239.89	102.24	88.89

shows that as the wholesale price decreases, the retailer makes fewer promotional efforts and orders more products when the retailer shares its revenue with the manufacturer.

5. Modified revenue sharing

In the case of a retailer promotion, the retailer's profit decreases as the fraction of revenue shared increases. However, channel profit ($\Pi_C = \Pi_R + \Pi_M$) grows as the fraction of revenue shared ϕ increases, as shown in Table 3. In this example, the channel profit ($\Pi_C = 343.178$) is optimal when $\phi = 0.807$. This means that the total channel profit may increase under RS. This implies that the manufacturer's profits, as well as those of the retailer, may increase when a reasonable profit-sharing mechanism exists.

To address this problem, we provide a modified RS (MRS) policy that benefits both the manufacturer and retailer under the condition of a retailer promotion. This MRS policy mandates the sharing of retailer revenue, as well as promotional cost. More specifically, the retailer shares a fraction (ϕ) of its revenue with the manufacturer and the manufacturer shares a fraction (ϕ) of its promotional cost with the retailer. The idea is similar to the revenue and promotional cost-sharing contract in Bai et al. (2017). The retailer's profit under an MRS policy can be written as:

$$\Pi_R^{\phi}(\rho_\phi, q_\phi) = (1-\phi)p\int_0^{q_\phi}\frac{x}{2\rho_\phi\xi}dx + (1-\phi)p\int_{q_\phi}^{2\rho_\phi\xi}\frac{q_\phi}{2\rho_\phi\xi}dx - w_\phi q_\phi - (1-\phi)K(\rho_\phi-1)^2 \tag{6}$$

The manufacturer's profit under an MRS policy can be written as:

$$\Pi_M^{\phi}(w_\phi) = \phi p\int_0^{q_\phi}\frac{x}{2\rho_\phi\xi}dx + \phi p\int_{q_\phi}^{2\rho_\phi\xi}\frac{q_\phi}{2\rho_\phi\xi}dx + (w_\phi - c)q_\phi - w_\phi q_\phi - \phi K(\rho_\phi - 1)^2 \tag{7}$$

The optimal wholesale price, promotional effort, and order quantity are determined as follows:

$$\rho_\phi^*(w_\phi) = \frac{[p(1-\phi)-w_\phi]^2\xi}{2K(1-\phi)^2 p} + 1 \tag{8}$$

$$q_\phi^*(w_\phi) = \frac{[p(1-\phi)-w_\phi]\{[p(1-\phi)-w_\phi]^2\xi + 2K(1-\phi)^2 p\}\xi}{K(1-\phi)^3 p^2} \tag{9}$$

$$w_\phi^* = \frac{18(1-\phi)[(3-\phi)p + c]\xi - \Upsilon/\Omega^{1/3} + (3\Omega)^{1/3}}{18(4-\phi)\xi} \tag{10}$$

where $\Upsilon = 6\cdot 3^{2/3}(1-\phi)^2\xi\{2p[3c\xi + K(2-\phi)(4-\phi)] - 3\xi(p^2+c^2)\}$ and

$$\Omega = 9(1-\phi)^3(p-c)\xi^2[2KP(\phi-4) - (p-c)^2\xi]$$

$$\sqrt{6}\sqrt{K(4-\phi)^2(1-\phi)^6 p\xi^3\left[\begin{array}{c}4K^2(4-\phi)(2-\phi)^3p^2 - 18K[1-(4-\phi)\phi](p-c)^2p\xi + \\ 27(p-c)^4\xi^2\end{array}\right]}$$

Table 4 shows that when $\phi = 0.1$ or 0.2, both the profits of the manufacturer and the retailer are higher than when $\phi = 0$. In this manner, the proposed MRS policy benefits both the manufacturer and retailer under the condition of a retailer promotion. The reason for this is that the manufacturer should also burden the promotional cost when it shares the revenue from a retailer promotion.

6. Other demand functions

In the above sections, we consider the linear promotional effort and uncertain demand with uniform distribution on $[0, 2\rho\xi]$, which refers to the demand setting in Tsao (2015a). To test our results in other uncertain demand functions, we consider a demand with another uniform distribution, that is, $D = f(\rho) + U$, where $f(\rho) = \rho\xi$, U is a continuous random variable following uniform distribution on $[-u, u]$, and u could be any given constant. Thus, the variance of the demand with uniform distribution is not affected by ρ. Tables 5 and 6 indicate the predictions for the manufacturer's and retailer's decision making and profits in the case of a manufacturer promotion and a retailer promotion, respectively. They show that RS is an effective incentive for increasing the profits of manufacturers and retailers in both the case of a manufacturer promotion and a retailer promotion when the variance of the demand with uniform distribution is not affected by the promotional effort. The result in the case of a retailer promotion is a little different to the case of a linear promotional effort and uncertain demand with uniform distribution.

We also test our results under another common distribution, that is, the promotion-sensitive effort and uncertain demand with normal distribution ($D\tilde{N}[\mu = \rho\xi, \sigma]$). From Tables 7 and 8, we know that when the manufacturer promotes products to end customers, RS is an effective incentive for increasing the profits of the manufacturer and the retailer. However, the retailer's profit decreases under an RS policy in the case of a retailer promotion. This means that retailers are unwilling to share revenue when the retailer undertakes its own promotional activities. These results are consistent with those under demand with uniform distribution on $[0, 2\rho\xi]$.

7. Conclusion

RS is commonly used as a marketing strategy to increase channel members' profits in a B2B environment. No previous studies on RS have simultaneously considered the effects of promotional efforts and demand uncertainty. Therefore, in this study, we considered a decentralized supply chain, in which a manufacturer sells products to a retailer under uncertain and promotion-sensitive demand. The objective of this study was to determine whether RS benefits channel members under these conditions. We differentiated between manufacturer promotion and retailer promotion. Under a manufacturer promotion, our results showed that the wholesale price decreases, the manufacturer assumes greater responsibility for promotional efforts, and the retailer orders more products when they share revenue with the manufacturer. That is, when the manufacturer promotes the products, RS is an effective incentive policy for simultaneously increasing the profits of the manufacturer and the retailer. Under retailer promotion, our results show that the wholesale price decreases, and the retailer assumes less of the promotional burden and orders more products when they share

Table 4
Influences of ϕ on decisions and profits under MRS policy ($K = 500$; $\xi = 50$; $p = 15$; $c = 5$).

ϕ	Wholesale price		Promotion effort		Order quantity		Manufacturer Profits		Retailer Profits	
	no RS	RS	no RS	RS	no RS	RS	no RS	RS	no RS	RS
0.00	9.59	9.59	1.10	1.098	39.57	39.57	181.70	181.70	102.24	102.24
0.10	9.59	8.36	1.10	1.109	39.57	42.17	181.70	192.51	102.24	102.96
0.20	9.59	7.17	1.10	1.121	39.57	45.09	181.70	204.65	102.24	102.93
0.30	9.59	6.03	1.10	1.136	39.57	48.41	181.70	218.33	102.24	101.80
0.40	9.59	4.93	1.10	1.153	39.57	52.17	181.70	233.83	102.24	99.13
0.50	9.59	3.89	1.10	1.174	39.57	56.45	181.70	251.49	102.24	94.31

Table 5
Influences of λ on decisions and profits ($K = 500$; $\xi = 50$; $p = 15$; $c = 5$; $u = 40$).

λ	Wholesale price		Promotion effort		Order quantity		Manufacturer Profits		Retailer Profits	
	no RS	RS	no RS	RS	no RS	RS	no RS	RS	no RS	RS
0.00	12.76	12.76	1.39	1. 39	41.36	41.36	245.58	245.58	79.41	79.41
0.10	12.76	11.08	1.39	1. 38	41.36	43.30	245.58	254.40	79.41	87.54
0.20	12.76	9.47	1.39	1. 37	41.36	45.56	245.58	265.11	79.41	94.01
0.30	12.76	7.93	1.39	1. 37	41.36	48.18	245.58	278.01	79.41	98.76
0.40	12.76	6.46	1.39	1. 37	41.36	51.24	245.58	293.48	79.41	101.51
0.50	12.76	5.07	1.39	1. 38	41.36	54.84	245.58	312.05	79.41	101.75

Table 6
Influences of ϕ on decisions and profits ($K = 500$; $\xi = 50$; $p = 15$; $c = 5$; $u = 40$).

ϕ	Wholesale price		Promotion effort		Order quantity		Manufacturer Profits		Retailer Profits	
	no RS	RS	no RS	RS	no RS	RS	not RS	RS	no RS	RS
0.00	10.64	10.64	1.22	1.22	44.17	44.17	249.03	249.03	118.13	118.13
0.10	10.64	9.26	1.22	1.21	44.17	45.73	249.03	257.46	118.13	118.18
0.20	10.64	7.92	1.22	1.20	44.17	47.43	249.03	266.74	118.13	117.25
0.30	10.64	6.62	1.22	1.19	44.17	49.26	249.03	276.90	118.13	114.96
0.40	10.64	5.38	1.22	1.18	44.17	51.22	249.03	288.01	118.13	110.79
0.50	10.64	4.21	1.22	1.16	44.17	53.30	249.03	300.04	118.13	104.08

Table 7
Influences of λ on decisions and profits ($K = 500$; $\xi = 50$; $p = 15$; $c = 5$; $\sigma = 20$).

λ	Wholesale price		Promotion effort		Order quantity		Manufacturer Profits		Retailer Profits	
	no RS	RS	no RS	RS	no RS	RS	no RS	RS	no RS	RS
0.00	11.99	11.99	1.00	1. 00	33.22	33.22	232.17	232.17	66.95	66.95
0.10	11.99	10.53	1.00	1. 02	33.22	35.36	232.17	244.72	66.95	71.42
0.20	11.99	9.08	1.00	1. 04	33.22	37.88	232.17	258.98	66.95	76.41
0.30	11.99	7.67	1.00	1. 06	33.22	40.70	232.17	275.33	66.95	80.84
0.40	11.99	6.28	1.00	1. 09	33.22	44.13	232.17	294.25	66.95	85.58
0.50	11.99	4.95	1.00	1. 13	33.22	48.08	232.17	316.37	66.95	88.79

Table 8
Influences of ϕ on decisions and profits ($K = 500$; $\xi = 50$; $p = 15$; $c = 5$; $\sigma = 20$).

ϕ	Wholesale price		Promotion effort		Order quantity		Manufacturer Profits		Retailer Profits	
	no RS	RS	no RS	RS	no RS	RS	not RS	RS	no RS	RS
0.00	11.47	11.47	1.18	1.18	44.34	44.34	286.85	286.85	99.96	99.96
0.10	11.47	10.08	1.18	1.17	44.34	45.21	286.85	292.95	99.96	99.35
0.20	11.47	8.70	1.18	1.16	44.34	46.24	286.85	299.69	99.96	98.66
0.30	11.47	7.35	1.18	1.16	44.34	47.34	286.85	307.17	99.96	97.00
0.40	11.47	6.03	1.18	1.15	44.34	48.58	286.85	315.56	99.96	94.45
0.50	11.47	4.75	1.18	1.14	44.34	50.02	286.85	325.01	99.96	90.59

revenue with the manufacturer. However, based on a different demand function setting, the profits of the retailer may decrease. When retailer profit decreases, we therefore propose a modified RS policy, in which retailer revenue and promotional costs are shared. Modified RS can increase both the profits of manufacturers and retailers.

Finally, it is important to note that this research is subject to several limitations. To make the model tractable theoretically, this paper follows the assumptions in Krishnan et al. (2004) and Tsao (2015a), so that the goods have no salvage value and no shortage cost. However, it is easy to extend our model to consider the inventory cost $h\int_0^q \frac{q-x}{2\rho\xi}dx$ when order quantity is larger than the demand, and the shortage cost $s\int_q^{2\rho\xi} \frac{x-q}{2\rho\xi}dx$ when order quantity is less than the demand, where h is the unit inventory cost and s is the unit shortage cost. One may incorporate these two costs into our model to discuss the RS policies under uncertain and promotion-sensitive demand. Further research could also extend the model to consider competition among brands or retailers, and the multi-product (Gharaei, Hoseini Shekarabi, & Karimi, in press; Gharaei, Karimi, & Hoseini Shekarabi, in press) or sustainable supply chain (Awasthi & Omrani, 2019; Dubey, Gunasekaran, & Singh, 2015; Gharaei, Karimi, & Hoseini Shekarabi, 2019; Hao, Helo, & Shamsuzzoha, 2018; Kazemi, Abdul-Rashid, Ghazilla, Shekarian, & Zanoni, 2018; Rabbani, Foroozesh, Mousavi & Farrokhi-Asl, 2019; Rabbani, Hosseini-Mokhallesun, Ordibazar & Farrokhi-Asl, in press; Sayyadi & Awasthi, 2018, in press; Tsao, 2015b). In addition, this study discusses whether RS benefits channel members, and therefore, an investigation of how other incentive policies affect channel behavior would also be worthwhile.

Appendix A

Proof of Proposition 1.

(a) From $w_\alpha^* - w_\lambda^* = \frac{p+c}{2} - \frac{(1-\lambda)[(1-\lambda)p+c]}{2-\lambda} = \frac{\lambda[p(3-2\lambda)+c]}{2(2-\lambda)}$ and $0<\lambda<1$, we know that $w_\alpha^* - w_\lambda^* > 0$.

(b) From $\rho_\lambda^* - \rho_\alpha^* = \frac{\lambda(p-c)^2\xi}{4K(2-\lambda)p}$ and $0<\lambda<1$, we know that $\rho_\lambda^* - \rho_\alpha^* > 0$.

(c) From $q_\lambda^* - q_\alpha^* = \frac{\lambda(p-c)[(4-\lambda)(p-c)^2\xi + 4K(2-\lambda)p]\xi}{4K(2-\lambda)^2p^2}$ and $0<\lambda<1$, we know that $q_\lambda^* - q_\alpha^* > 0$. □

Proof of Proposition 2.

(a) From $\Pi_M^\lambda - \Pi_M^\alpha = \frac{\lambda(p-c)^2[(4-\lambda)(p-c)^2\xi + 8K(2-\lambda)p]\xi}{16K(2-\lambda)^2p^2}$ and $0<\lambda<1$, we know that $\Pi_M^\lambda > \Pi_M^\alpha$.

(b) From $\Pi_R^\lambda - \Pi_R^\alpha = \frac{\lambda(p-c)^2[(\lambda^2-6\lambda+4)(p-c)^2\xi - 4K(2-\lambda)\lambda p]\xi}{16K(2-\lambda)^3p^2}$, we know that $\Pi_R^\lambda - \Pi_R^\alpha > 0$ when $(\lambda^2-6\lambda+4)(p-c)^2\xi - 4K(2-\lambda)\lambda p > 0$. This means $\Pi_R^\lambda - \Pi_R^\alpha > 0$ when $\frac{4Kp + 3(p-c)^2\xi - \sqrt{16K^2p^2 + 8K(p-c)^2p\xi + 5(p-c)^4\xi^2}}{4Kp + (p-c)^2\xi} < \lambda < \frac{4Kp + 3(p-c)^2\xi + \sqrt{16K^2p^2 + 8K(p-c)^2p\xi + 5(p-c)^4\xi^2}}{4Kp + (p-c)^2\xi}$. Then from $0<\lambda<1$ should be satisfied, we get the range of threshold $0<\lambda<\frac{4Kp + 3(p-c)^2\xi + \sqrt{16K^2p^2 + 8K(p-c)^2p\xi + 5(p-c)^4\xi^2}}{4Kp + (p-c)^2\xi}$ □

Proof of Proposition 3.

(a) Let $\psi_L = \frac{(1/2)\sqrt{[2(p-c)^2\xi + 4Kp]^2 - 8Kp(p-c)^2\xi} - (p-c)^2\xi}{2Kp}$ and $\psi_H = \frac{(1/2)\sqrt{[2(p-c)^2\xi + 4Kp]^2} - (p-c)^2\xi}{2Kp}$. Because $\psi_H \frac{(1/2)\sqrt{[2(p-c)^2\xi + 4Kp]^2} - (p-c)^2\xi}{2Kp} = \frac{(1/2)[2(p-c)^2\xi + 4Kp] - (p-c)^2\xi}{2Kp} = 1$, then from Eq. (11) and $\psi_H > \psi_L$, we have that $\lambda_R^* = 1 - \psi_L > 1 - \psi_H = 0$. Also, from $[(1/2)\sqrt{[2(p-c)^2\xi + 4Kp]^2 - 8Kp(p-c)^2\xi}]^2 - [(p-c)^2\xi]^2 = 2(p-c)^2Kp\xi + 4K^2p^2 > 0$, we know that $\psi_L > 0$. Thus, $\lambda_R^* = 1 - \psi_L < 1$.

(b) $\frac{d\lambda_R^*}{d\xi} = \frac{(p-c)^2\sqrt{[2(p-c)^2\xi + 4Kp]^2 - 8Kp(p-c)^2\xi} - 2[Kp(p-c)^2 + \xi(p-c)^4]}{2Kp\sqrt{[2(p-c)^2\xi + 4Kp]^2 - 8Kp(p-c)^2\xi}}$. From $\{(p-c)^2\sqrt{[2(p-c)^2\xi + 4Kp]^2 - 8Kp(p-c)^2\xi}\}^2 - \{2[Kp(p-c)^2 + \xi(p-c)^4]\}^2 = 12K^2p^2(p-c)^4 > 0$, we know that $d\lambda_R^*/d\xi > 0$. Thus, λ_R^* increases as ξ increases.

(c) $\frac{d\lambda_R^*}{dc} = \frac{(p-c)\{[Kp - (p-c)^2\xi] - \sqrt{[(p-c)^2\xi + 2Kp]^2 - 2Kp(p-c)^2\xi}\}}{Kp\sqrt{[(p-c)^2\xi + 2Kp]^2 - 2Kp(p-c)^2\xi}}$. From $[Kp - (p-c)^2\xi]^2 - (\sqrt{[(p-c)^2\xi + 2Kp]^2 - 2Kp(p-c)^2\xi})^2 = -3K^2p^2 < 0$, we know that $d\lambda_R^*/dc < 0$. Thus, λ_R^* increases as c decreases. □

Proof of Lemma 1. Taking the first-order condition of the retailer profit with respect to order quantity, we have: $\frac{d\Pi_R^\lambda(q_\lambda)}{dq_\lambda} = 0 \Rightarrow q_\lambda = \frac{2[p(1-\lambda) - w_\lambda]\rho_\lambda\xi}{p(1-\lambda)}$.

Then the manufacturer determines the optimal wholesale price and promotional effort as: $\frac{\partial\Pi_M^\lambda(w_\lambda,\rho_\lambda)}{\partial w_\lambda} = 0 \Rightarrow w_\lambda^* = \frac{(1-\lambda)[(1-\lambda)p+c]}{2-\lambda}$ and $\frac{\partial\Pi_M^\lambda(w_\lambda,\rho_\lambda)}{\partial\rho_\lambda} = 0 \Rightarrow \rho_\lambda = \frac{[p(1-\lambda) - w_\lambda][(1-\lambda)(\lambda p - 2c) + (2-\lambda)w_\lambda]\xi}{2Kp(1-\lambda)^2} + 1$. Substituting w_λ^* into ρ_λ, we obtain: $\rho_\lambda^* = \frac{(p-c)^2\xi}{2K(2-\lambda)p} + 1$. Substituting Eqs. w_λ^* and ρ_λ^* into q_λ, we obtain: $q_\lambda^* = \frac{(p-c)[(p-c)^2\xi + 2K(2-\lambda)p]\xi}{K(2-\lambda)^2p^2}$. □

Proof of Lemma 2. Let $\lambda = 0$ in Eqs. (1) and (2) and follow the procedure in proof of Lemma 1, we can get that $w_\alpha^* = \frac{p+c}{2}$, $\rho_\alpha^* = \frac{(p-c)^2\xi}{4Kp} + 1$ and $q_\alpha^* = \frac{(p-c)[(p-c)^2\xi + 4Kp]\xi}{4Kp^2}$. □

Proof of Lemma 3. Solve $d\Pi_R^\phi(\rho_\phi, q_\phi)/d\rho_\phi = 0$ and $d\Pi_R^\phi(\rho_\phi, q_\phi)/dq_\phi = 0$ simultaneously, we have $q_\phi^*(w_\phi) = \frac{[p(1-\phi) - w_\phi]\{[p(1-\phi) - w_\phi]^2\xi + 2K(1-\phi)p\}\xi}{K(1-\phi)^2p^2}$ and $\rho_\phi^*(w_\phi) = \frac{[p(1-\phi) - w_\phi]^2\xi}{2K(1-\phi)p} + 1$. Then substitute $q_\phi^*(w_\phi)$ and $\rho_\phi^*(w_\phi)$ into Eq. (5) and solve $d\Pi_M^\phi(w_\phi)/dw_\phi = 0$, we can get $w_\phi^* = \frac{3(1-\phi)[(3-2\phi)p + c]\xi + \Upsilon/\Omega^{1/3} + (3\Omega)^{1/3}}{6(2-\phi)\xi}$. □

Proof of Lemma 4. Let $\lambda = 0$ in Eqs. (4) and (5) and follow the procedure in proof of Lemma 3, we can get that $w_\beta^* = \frac{3(3p+c)\xi + 3^{2/3}\xi[3\xi(p-c)^2 - 16Kp]/\Gamma^{1/3} + (3\Gamma)^{1/3}}{12\xi}$, $\rho_\beta^*(w_\beta) = \frac{(p-w_\beta)^2\xi}{2Kp} + 1$ and $q_\beta^*(w_\beta) = \frac{(p-w_\beta)[(p-w_\beta)^2\xi + 2Kp]\xi}{Kp^2}$. □

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