



## Research article

## The systematic risk estimation models: A different perspective

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## ABSTRACT

In practice, the capital asset pricing model (CAPM) using the parametric estimator is almost certainly being used to estimate a firm's systematic risk (beta) and cost of equity as in Eq. (1). However, the parametric estimators, even when data is normal, may not yield better performance compared with the non-parametric estimators when outliers existed. This research argued for the non-parametric Bayes estimator to be employed in the CAPM by applying both advance and basic evaluation criteria such as hypotheses/confidence intervals of the AIC/DIC, model variance, fit, and error, alpha, and beta and its standard deviation. Using all the S&P 500 stocks having monthly data from 07/2007–05/2019 (450 stocks) and the Bayesian inference, we showed the non-parametric Bayes estimator yielded less number of zeroed betas and smaller alpha compared with the parametric Bayes estimator. More importantly, this non-parametric Bayes yielded the statistically significantly smaller AIC/DIC, model variance, and beta standard deviation and higher model fit compared with the parametric Bayes estimator. These findings indicate the CAPM using the non-parametric Bayes estimator is superior compared with the parametric Bayes estimator, a contrast of common practice. Hence, the non-parametric estimator is recommended to be employed in asset pricing work.

## 1. Introduction

Firms are dealing with many different types of risks from environmental, social, and governance (ESG) related risks such as strategic risks, financial risks, operational risks, compliance risks, credit risks, etc. The effective and integrated risk management is one of the keys to help firms survive, grow, and create value for shareholders (Committee of Sponsoring Organizations (COSO), 2018; Bromiley et al., 2015; McShane et al., 2011; Elowe and Nottingham, 2017). Therefore, the systematic or market risk (beta) and the cost of equity ( $k_e$ ) reliable and efficient estimations are very important to the firm's success in areas such as financing, assets allocation, investing, and dividend policy (Gordon et al., 2009; Hoyt and Liebenberg, 2011; Kiselakova et al., 2015; Lai and Shad, 2017; McShane et al., 2011; Shad and Lai, 2015). This reliable and efficient beta estimate becomes more serious now since most firms and industries are depending heavily on equity finance with low debt ratios to support their operations after the financial crisis in 2007–2008 as shown in Figure 1 below.

Theoretically, the managers may employ one of many different available asset pricing models as suggested in Bertomeu and Cheynel (2016) to estimate their firm's beta and cost of equity. However, some

studies (e.g., Association for Financial Professionals, 2011; Bartholdy and Peare, 2005; Da et al., 2012; Jacobs and Shivdasani, 2012; Fama and French, 1996b; Liu et al., 2009; Moore, 2016; Zhang, 2017) showed the capital asset pricing model (CAPM, Black, 1972; Lintner, 1965; Sharpe, 1964, 1966; Jensen, 1967, 1969) is the preferred model in practice. The reasons the CAPM is popular in practice are 1) the returns on the firm's stock and market are easily accessed and available to all investors. 2) More importantly, it shows a very simple linear relationship between the excess return on the stock  $i = 1, 2, \dots, N$  at the time  $t = 1, 2, \dots, n$  and the market risk premium as follows:

$$R_{it} - R_{ft} = \alpha_i + \beta_i [RM_t - R_{ft}] + \varepsilon_{it} \quad (1)$$

where,

- $R_{it}$ : the return on the stock  $i$  at the time  $t$ ,
- $RM_t$ : the return of the market portfolio at the time  $t$ ,
- $R_{ft}$ : the risk-free rate,
- $\alpha_i$ : Jensen's alpha coefficient (alpha) of stock  $i$ ,
- $\beta_i$ : the stock  $i$ 's sensitivity to the market portfolio (beta),
- $\varepsilon_{it}$ : the random error term that has mean zero and variance  $\sigma^2$ .

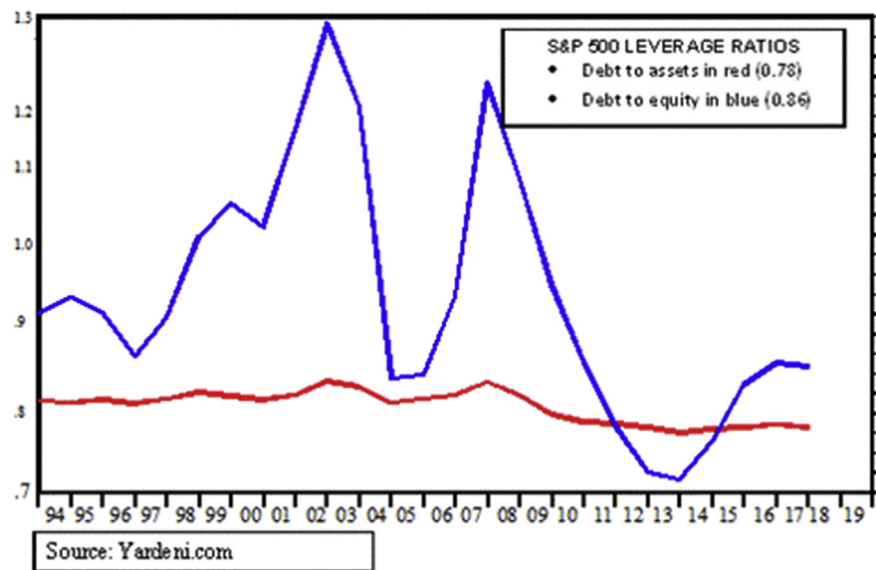
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**Figure 1.** The S&P 500 leverage ratios. Reprinted from Yardeni.com. Retrieved September 20, 2018, from <https://www.yardeni.com/pub/spxratios.pdf>. Copyright 2007 by Yardeni.com. Reprinted with permission.

The firm's beta estimation as in Eq. (1) using different estimators often yield different outcomes and result in different costs of equity; then different financial risks (Martin and Simin, 2003). Therefore, using the poor performance estimator in beta estimation may mislead the managers in their decision makings and the firm performance and shareholder wealth would be affected. In corporate practice, this beta is very often estimated using the ordinary least square (OLS) and parametric Bayes (Bayes) estimators because the OLS estimator is the best linear unbiased estimator (BLUE) and the availability of software packages having the OLS estimator algorithm, for example, the Microsoft Excel. Also, the OLS and Bayes estimators yielded similar performance if the sample size of the data is large (Barry, 1980; Phuoc, 2018; Zellner, 1971). In addition, the monthly/quarterly/annual returns data are preferred in the CAPM because of their normality property and availability of data (Ang and Bekaert, 2007; Fama and French, 1992, 1993, 1996a, 1996b, 2004, 2015, 2016, 2018; Kamara et al., 2016, 2018; Phuoc, 2018; Phuoc et al., 2018; Zhang, 2006). However, the OLS and other parametric estimators are known to be sensitive to outliers or unequal variance data, which are the very common problems in real returns on stocks (Phuoc, 2018), compared with the non-parametric (robust) estimators (Alexander and Chervany, 1980; Baissa and Rainey, 2018; Bian and Dickey, 1996; Bian et al., 2013; Lange et al., 1989; Levy, 1971; Martin and Simin, 2003; Rousseeuw, 1984; Wilcox and Keselman, 2004).

According to our research, studies of beta estimations have been conducted using different estimators in finance and economics. However, all of these studies did not employ any hypothesis tests/confidence intervals or advance information criteria, but used only one or two basic criteria such as the alpha, beta, beta standard deviation, mean square error, or r-squared to compare the performance of used estimators. Sharpe (1971) studied the beta estimation of 30 U.S. stocks and 30 mutual funds using the mean absolute deviation (MAD) and OLS estimators and quarterly returns data for five years period. The empirical results showed these two estimators yielded similar performance in terms of beta and alpha estimates and was confirmed by other research (Cornell and Dietrich, 1978). However, this study employed only beta and alpha estimates in comparison. Using other robust estimators, some studies showed they were more efficient beta estimates than the OLS estimator. Chan and Laskonishok (1992) conducted a study of beta estimation using a simulated and actual monthly returns data of 50 randomly selected NYSE firms for 1983–1985. In this study, these authors employed the minimum absolute deviations, the trimmed regression quantile, and the

Tukey's trimmed and Gastwirth estimators. The empirical results from simulated and actual returns data showed that the robust estimators were more efficient than the OLS estimator. Similarly, other studies (Bowie and Bradfield, 1998; Phuoc, 2018; Phuoc et al., 2018; Serra and Martelanc, 2013; Shalit and Yitzhaki, 2002; Trzpiot, 2013) showed the least absolute deviations, Koenker-Basset trimmed mean, Tukey's biweight, Lp, Gini, weighted Gini, bounded influence version of the Koenker-Basset, bounded influence version of Lp, least trimmed squares, quantile, and maximum likelihood-type M estimators yielded more efficient beta estimates than the OLS estimator. However, these studies employed only estimator efficiency property in comparison. Besides the efficiency property, other studies examined the fit to the data of the robust estimators in the CAPM. Fong (1997) conducted a study of 22 stocks for 6 years from the Straits Times Industrial Index using the Generalized Student-t. Fong found that this estimator provided a significantly better fit to the data than the OLS estimator. This finding was confirmed by later studies (Ho and Naugher, 2000; Shumacker et al., 2002). However, these studies employed only one criterion in comparison. Other studies (Bian et al., 2013; Martin and Simin, 2003; Wong and Bian, 2000) compared the predictive power using the mean square error (MSE) or residual scale estimate (RMSE) of the robust estimators with the OLS estimator in beta estimation using the weighted least squares, robust Bayesian estimator using the Cauchy-type g-prior (Bian and Dickey, 1996), and modified maximum likelihood estimators, respectively. They showed that those estimators outperformed the OLS estimator in terms of predictive power when outliers existed or for the small sample size data. Again, these studies employed only one criterion in comparison.

Using the hypothesis tests/confidence intervals on both the advance information criteria such as the Akaike and deviance information criteria (AIC/DIC), model variance  $\sigma^2$  and mean square error and its standard deviation; the commonly used criteria such as the alpha, beta, beta standard deviation, Bayesian r-squared, this research argued for the robust Bayes (R.Bayes) estimator to be employed in the CAPM.

## 2. Methodologies

### 2.1. The approach and estimators

The practitioners and researchers very often applied the frequentist approach such as Phuoc et al. (2018) and Fama and French (2015); few studies applied the simulations in their asset pricing work. In contrast,

this study applied the Bayesian approach to take advantages of past knowledge of the interested parameters (Cornfield, 1969) as in Eq. (2) as follows:

$$p(\theta|y) \propto p(\theta)p(y|\theta) \quad (2)$$

where ,

- $p(\theta|y)$ : the posterior distribution of the interested parameter theta,
- $p(\theta)$ : the prior distribution of the interested parameter theta,
- $p(y|\theta)$ : the likelihood distribution.

We employed the Gibbs sampling simulation, a Markov chain Monte Carlo (MCMC) algorithm, with 300,000 iterations to obtain MC error E-4 or less on real data to reduce misspecifications (Barillas and Shaken, 2018; Kothari and Warner, 1997, 2001). Besides, it was much easier to derive the posterior distributions of the interested parameters (parameter estimate, standard deviation, and confidence interval) compared with the frequentist approach, especially the model error, variance, and fit.

Two Bayes estimators employed in this study to check for the consistency in results. The first one was the parametric Bayes (Bayes) because it is the most used estimator among Bayesian practitioners. Also, this Bayes estimator yielded a similar performance as the most popular estimator, the OLS, in the CAPM if the sample size is large (Barry, 1980; Phuoc, 2018; Zellner, 1971). The second estimator was the non-parametric, Student's t, Bayes (R.Bayes) with three degrees of freedom because the t-distribution is flatter and handles outliers better than the normal distribution or when the variance of stock returns is unknown. Some studies (e.g., Blattberg and Gonedes, 1974; Bollerslev, 1987; Chan and Laskonishok, 1992; Kon, 1984, 1994) successfully applied this Student's t in practice. The weekly informative normal priors for both alpha and beta for both Bayes and R.Bayes estimators were chosen to match with the frequentist approach and past knowledge. Also, some studies discussed and successfully applied the Bayes and non-parametric-type Bayes estimators in asset pricing models in finance and economics literature (Lange et al., 1989; Phuoc, 2018; Wong and Bian, 2000).

## 2.2. Evaluation criteria

To evaluate the performance of both the Bayes and R.Bayes estimators, we first used the guidelines that Ryan (2007) suggested. Then, we looked at the advance information criteria using the AIC/DIC. These criteria have been successfully applied by other studies (e.g., Berg et al., 2004; Gelman et al., 2014; Spiegelhalter et al., 2002, 2014). We also applied the model precision or variance using  $\sigma^2$  (Sigma2) as suggested by McKean (2004) in our evaluation. Besides, this research employed the model fit using Bayesian r-squared (R2B) as suggested in other studies (e.g., Buse, 1973; Lee, 1991; Trzpiot, 2013), the absolute value of alpha since the CAPM considered non-traded factor model, and beta and its standard deviation (Beta Std) in our evaluation. The alpha and beta and beta standard deviation have been employed in other studies (e.g., Barillas and Shanken, 2017, 2018; Chan and Laskonishok, 1992; Fama and French, 1992, 1993; 1996a, 2015; 2016; Hou et al., 2015; Kozak et al., 2018; Phuoc, 2018; Phuoc et al., 2018; Stambaugh and Yuan, 2016; Trzpiot, 2013). Importantly, we employed the hypothesis/confidence interval of the interested parameter beta and the mean difference of all interested parameters using two estimators since the parameter estimates themselves did not yield enough information in decision makings.

## 2.3. Hypotheses/confidence intervals

The key contribution of the CAPM is the linear relationship between market risk and stock returns through the beta as in Eq. (1). Hence, we first conducted the beta test as follows:

**Hypothesis 1.**  $\beta_i = 0, i = 1, 2, \dots, 450$ .

Then, we conducted tests to compare the mean differences between two AIC/DICs, sigma2s, alphas, betas and their standard deviations, and R2B as follows:

**Hypothesis 2.**  $AIC_{R.Bayes} - AIC_{Bayes} = 0$

**Hypothesis 3.**  $DIC_{R.Bayes} - DIC_{Bayes} = 0$

**Hypothesis 4.**  $\sigma^2_{R.Bayes} - \sigma^2_{Bayes} = 0$

**Hypothesis 5.**  $Apha_{R.Bayes} - Apha_{Bayes} = 0$

**Hypothesis 6.**  $Beta_{R.Bayes} - Beta_{Bayes} = 0$

**Hypothesis 7.**  $Beta.Std_{R.Bayes} - Beta.Std_{Bayes} = 0$

**Hypothesis 8.**  $R2B_{R.Bayes} - R2B_{Bayes} = 0$

To conduct the hypothesis test, we would not use p-value in order to avoid its serious shortcomings. Instead, we would apply the confidence interval to check on these eight hypotheses.

## 3. Data

At first, all 500 U.S. stocks listed on the S&P 500 index were considered in our study because they were known to be very efficient, a needed assumption of the CAPM. If the tests on these stocks confirmed what we proposed, then they were likely to perform as well on the other U.S. stocks and in other less efficient markets. To minimize the data time frame selection bias, we included both the up and down periods in stock performance in our analysis. We also wanted the stability in parameter estimation (Alexander and Chervany, 1980; Levy and Schwarz, 1997; Theobald, 1981) and to avoid the serious size distortions associated with very long/long-horizon data (Ang and Bekaert, 2007; Kothari and Warner, 1997, 2001; Valkanov, 2003). Hence, the medium-horizon (12-year), a balance of both the very long/long and short-horizon data, monthly returns data between July 2007 to May 2019 were collected. If the firms did not have enough 12-year data (such as Facebook and General Motors Company), then we dropped them. In the end, we found 450 firms that met our objectives. The returns were chosen to measure the S&P 500 firms' performance over the logarithmic returns (Shafer and Vovk, 2019) because these firms are big, efficient, and not volatile stocks. This medium-horizon data was consistent with other studies (Campbell and Shiller, 1988; Fama and French, 1988, 1989). In practice, the 10-year U.S. T-Bond yield was often chosen as the risk-free rate (Brotherson et al., 2013). However, our study employed the three-month U.S. T-Bill's secondary market rate because it was stable and consistent with the CAPM as originally derived, returns on stocks and market, and other studies (Fama and French, 1993; Harrington, 1983; Black, 1972; Kothari et al., 1995; Litzenberger and Ramaswamy, 1979; Martin and Simin, 2003; Moore, 2016; Phuoc et al., 2018; Rosenberg et al., 1985; Sharpe, 1971). The S&P 500 index was used as a proxy for the market which was consistent with other studies (Brotherson et al., 2013; Harrington, 1983; Bruner et al., 1998; MacKinlay, 1995; Rosenberg et al., 1985; Sharpe, 1971). The U.S. T-bill was from the Federal Reserve Bank of St. Louis Economic Data. The returns on the S&P 500 firms and index were calculated based on their adjusted close prices listed on the public and firms' websites.

## 4. Empirical results

### 4.1. The beta test using Hypothesis 1

Figure 2 using the Bayes estimator showed the 95% confidence intervals of the beta of stocks 91, 185, 207, 230, 231, 296 and 411 included zero. Hence, we would not reject the Hypothesis 1:  $\beta_i = 0, i = 91, 185, 207, 230, 231, 296, \text{ and } 411$ . It showed that the CAPM as in Eq. (1) using the Bayes estimator did not work for these 7 stocks (1.6% of all stocks in the sample). In contrast, Figure 3 using the R.Bayes estimator showed the

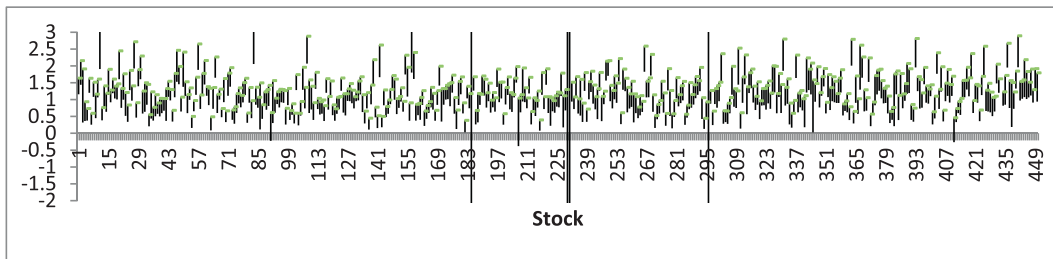


Figure 2. The 95% confidence intervals of the beta using the Bayes estimator.

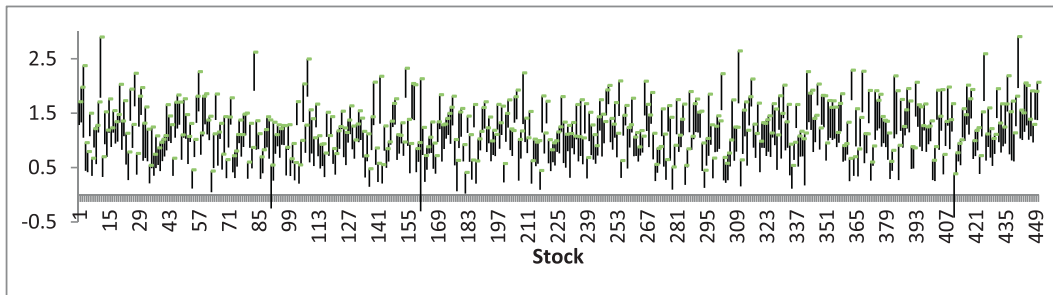


Figure 3. The 95% confidence intervals of the beta using the R.Bayes estimator.

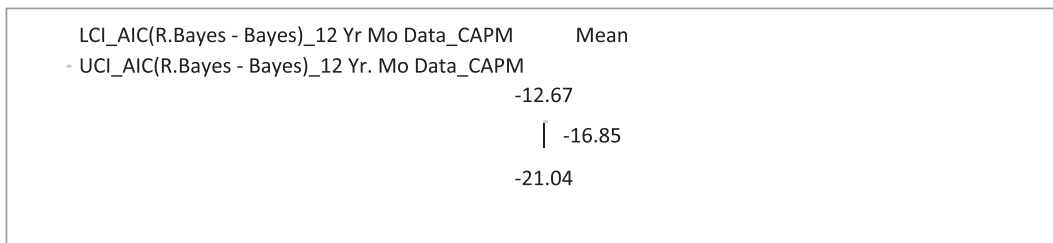


Figure 4. The mean and 95% confidence interval of the mean difference of  $AIC_{R.Bayes}$  and  $AIC_{Bayes}$ .

95% confidence intervals of the beta of stocks 91, 161 and 411 included zero. So, we would not reject *Hypothesis 1*:  $\beta_i = 0$ ,  $i = 91, 161$ , and 411. It showed that the CAPM as in Eq. (1) using the R.Bayes estimator did not work for only 3 stocks (0.7% of all stocks in the sample). These findings showed the CAPM using the R.Bayes estimator yielded a more reliable model in beta estimation in practice compared with the Bayes estimator.

The results of the beta test using both the Bayes and R.Bayes estimators forced us to drop the following firms: 91, 161, 185, 207, 230, 231, 296 and 411 for the next analyses and comparisons. Therefore, there were 442 remaining stocks in the sample.

#### 4.2. The AIC/DIC tests using Hypotheses 2&3

Figure 4 showed the mean and 95% confidence interval of the mean difference of  $AIC_{R.Bayes}$  and  $AIC_{Bayes}$  were -16.85 and (-21.04, -12.67), respectively. Similarly, Figure 5 showed the mean and 95% confidence interval of the mean difference of  $DIC_{R.Bayes}$  and  $DIC_{Bayes}$  were -16.83 and (-21.01, -12.65), respectively. Hence, we would reject Hypotheses 2&3. Besides, Figures 6 and 7 both showed the majority of stocks in the sample (66%) with  $AIC_{R.Bayes}$  and  $DIC_{R.Bayes}$  smaller than that of  $AIC_{Bayes}$  and  $DIC_{Bayes}$ , respectively. These findings

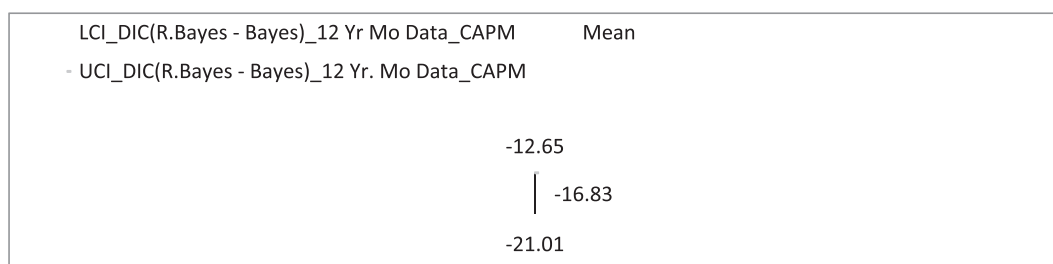


Figure 5. The mean and 95% confidence interval of the mean difference of  $DIC_{R.Bayes}$  and  $DIC_{Bayes}$ .

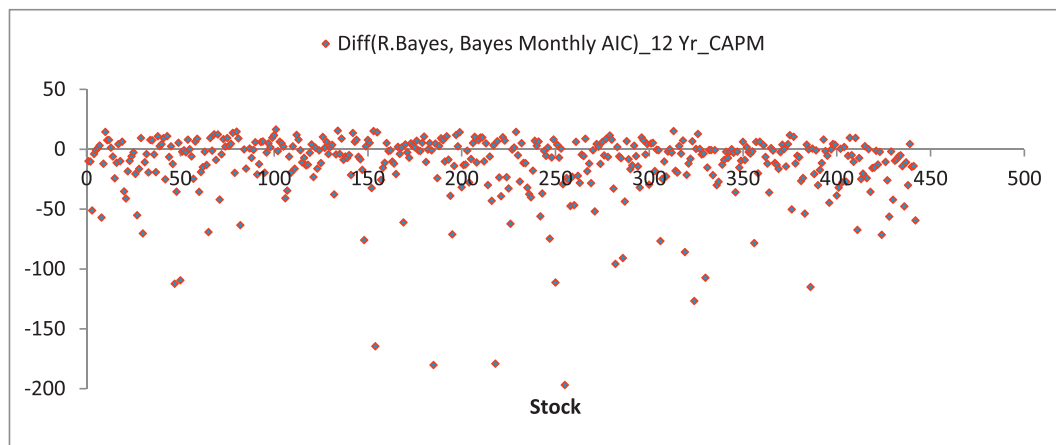


Figure 6. The differences of  $AIC_{R.Bayes}$  and  $AIC_{Bayes}$ .

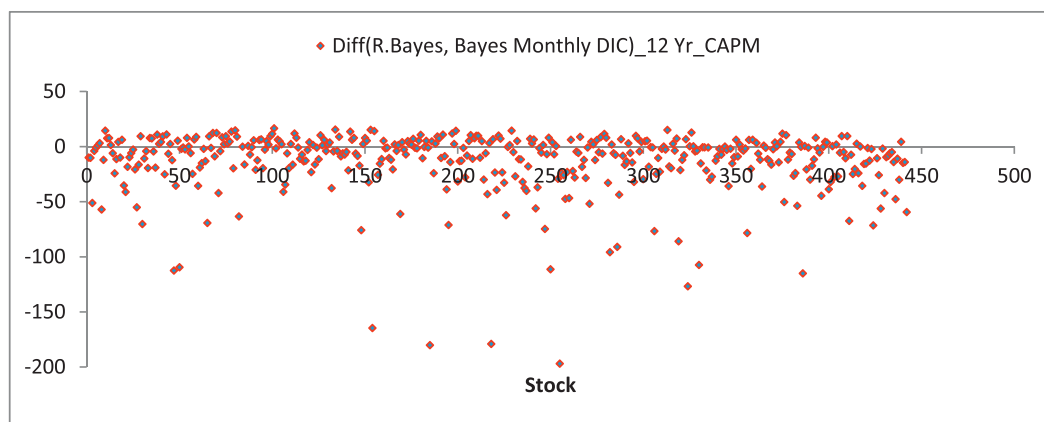


Figure 7. The differences of  $DIC_{R.Bayes}$  and  $DIC_{Bayes}$ .

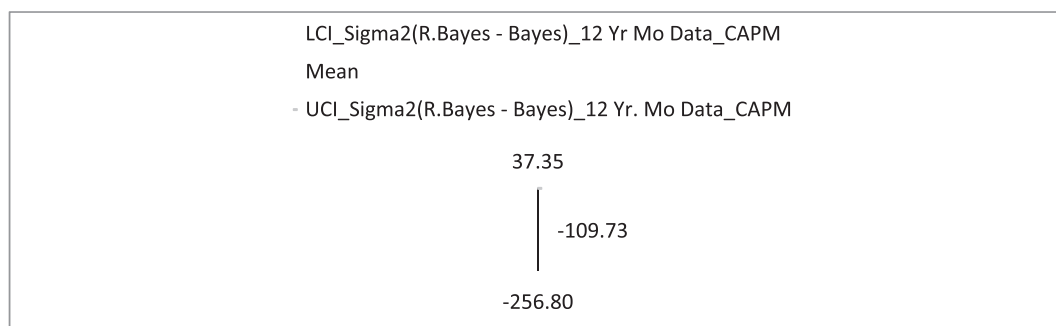


Figure 8. The mean and 95% confidence interval of the mean difference of  $\sigma^2_{R.Bayes}$  and  $\sigma^2_{Bayes}$ .

showed the CAPM using the R.Bayes estimator yielded statistically significant smaller AIC/DIC and more explanation power compared with the Bayes estimator.

#### 4.3. The Sigma2 test using Hypothesis 4

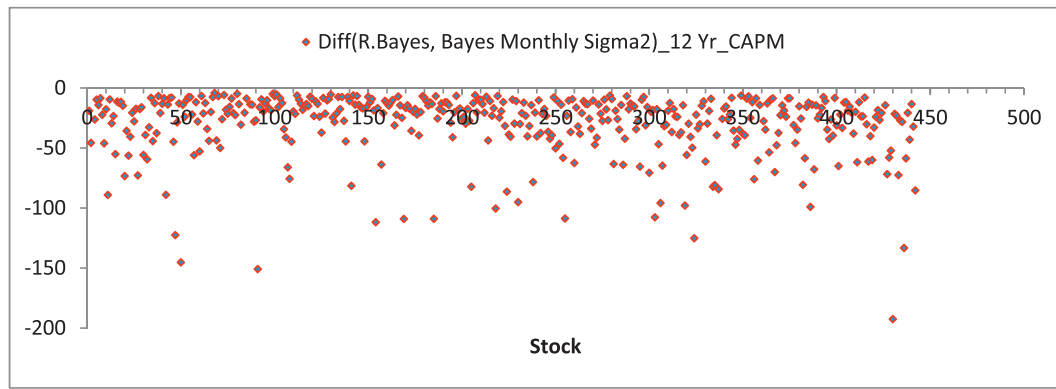
Figure 8 showed the mean and 95% confidence interval of the mean difference of  $\sigma^2_{R.Bayes}$  and  $\sigma^2_{Bayes}$  were -109.73 and (-256.80, 37.35). Hence, we would not reject Hypotheses 4. However, Figure 9, showed all 442 firms in the sample (100%) with  $\sigma^2_{R.Bayes}$  smaller than that of  $\sigma^2_{Bayes}$ , a solid result. These findings showed the CAPM using the R.Bayes estimator yielded and more precision model compared with the Bayes estimator.

#### 4.4. The alpha test using Hypothesis 5

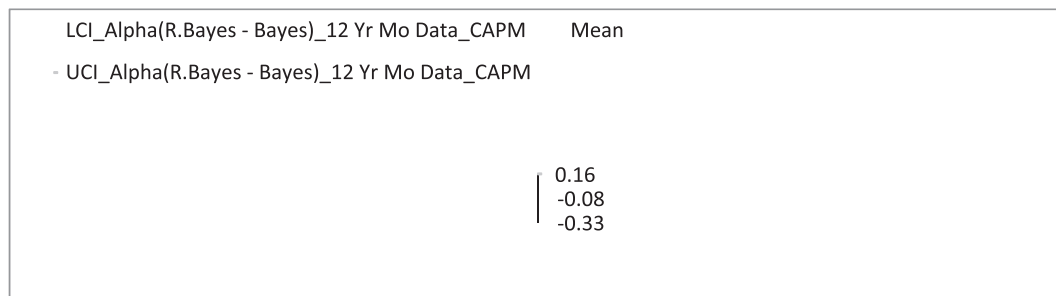
Figure 10 showed the mean and 95% confidence interval of the mean difference of absolute value  $Apha_{R.Bayes}$  and  $Apha_{Bayes}$  were -0.08 and (-0.33, 0.16), respectively. Even though we would not reject Hypothesis 5, but we still had evidence (mean equaled to -0.08) to claim the CAPM using the R.Bayes estimator satisfied the CAPM's assumption of market efficiency better than that of the Bayes estimator.

#### 4.5. The beta and beta standard deviation tests using Hypotheses 6 & 7

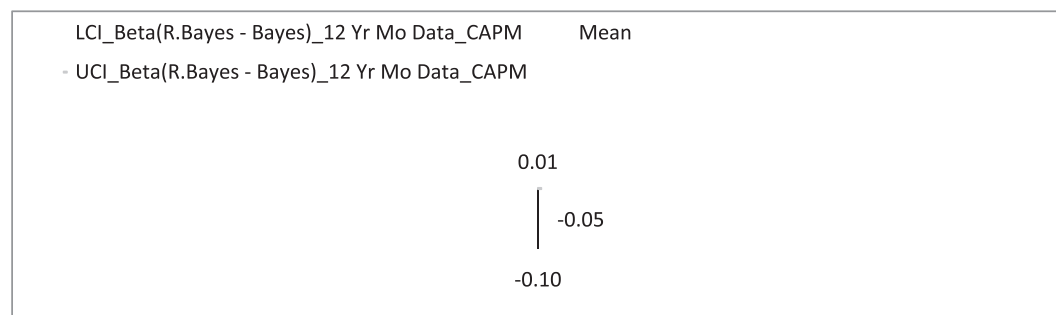
Figure 11 showed the mean and 95% confidence interval of the mean difference of  $Beta_{R.Bayes}$  and  $Beta_{Bayes}$  were -0.05 and (-0.10,



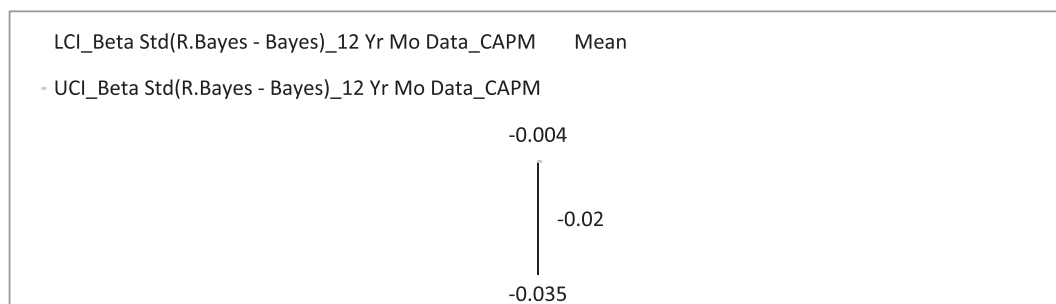
**Figure 9.** The differences of  $\sigma^2_{R.Bayes}$  and  $\sigma^2_{Bayes}$ .



**Figure 10.** The mean and 95% confidence interval of the mean difference of absolute value  $\text{Alpha}_{R.Bayes}$  and  $\text{Alpha}_{Bayes}$ .



**Figure 11.** The mean and 95% confidence interval of the mean difference of  $\text{Beta}_{R.Bayes}$  and  $\text{Beta}_{Bayes}$ .



**Figure 12.** The mean and 95% confidence interval of the mean difference of  $\text{Beta.Std}_{R.Bayes}$  and  $\text{Beta.Std}_{Bayes}$ .

0.01), respectively. However, Figure 12 showed the mean and 95% confidence interval of the mean difference of  $\text{Beta.Std}_{R.Bayes}$  and  $\text{Beta.Std}_{Bayes}$  were -0.02 and (-0.035, -0.004), respectively. Besides, Figure 13 showed the majority of firms in the sample (65%) with  $\text{Beta.Std}_{R.Bayes}$  smaller than that of  $\text{Beta.Std}_{Bayes}$ . Hence, we would not reject Hypothesis 8, but rejected Hypothesis 9. These findings showed the CAPM using the R.Bayes estimator yielded similar beta

estimates, but more stable and efficient compared with the Bayes estimator.

#### 4.6. The R2B test using Hypothesis 8

Figure 14 showed the mean and 95% confidence interval of the mean difference of  $R2B_{R.Bayes}$  and  $R2B_{Bayes}$  were 0.35 and (0.34, 0.36),



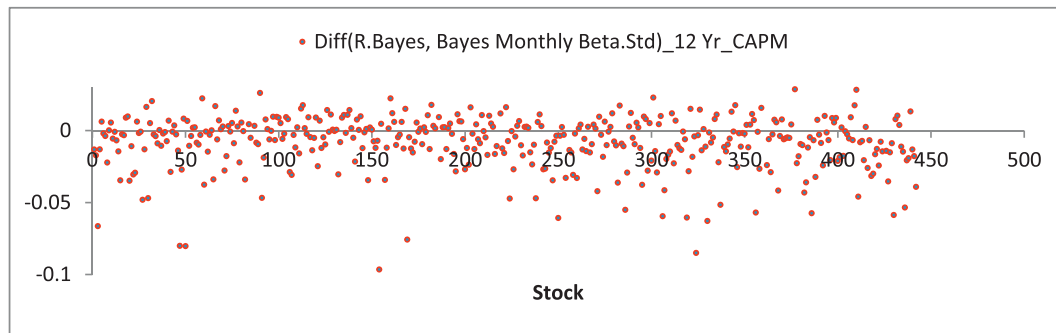


Figure 13. The differences of  $Beta.Std_{R.Bayes}$  and  $Beta.Std_{Bayes}$ .

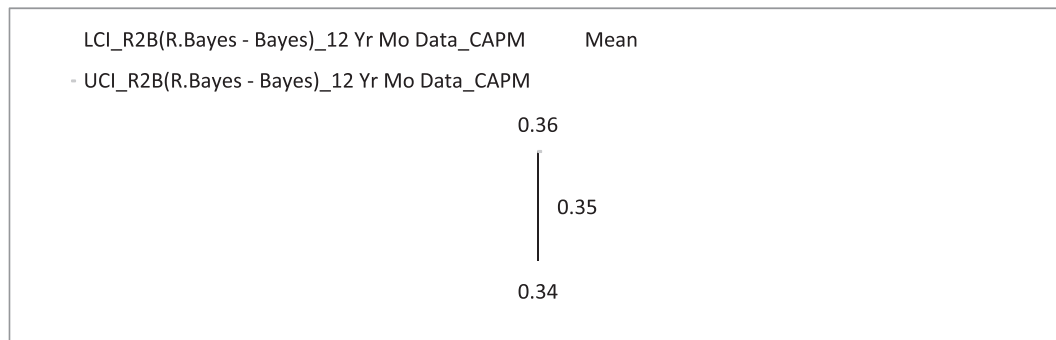


Figure 14. The mean and 95% confidence intervals of the mean difference of  $R2B_{R.Bayes}$  and  $R2B_{Bayes}$ .

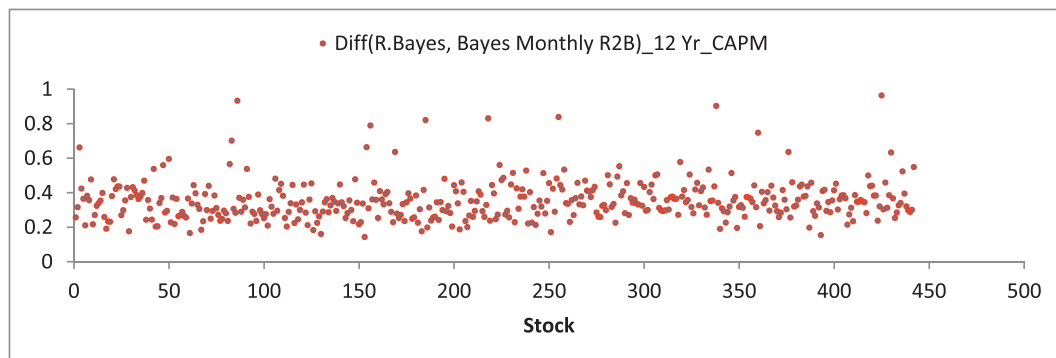


Figure 15. The differences of  $R2B_{R.Bayes}$  and  $R2B_{Bayes}$ .

respectively. Hence, we would reject Hypothesis 8. Besides, Figure 15 showed all 442 firms (100%) in the sample with  $R2B_{R.Bayes}$  higher than that of  $R2B_{Bayes}$ , a solid result. These findings showed the CAPM using the R.Bayes obviously yielded higher model fit and explanation power compared with the Bayes estimator.

## 5. Discussion and conclusions

In corporate finance, the managers very often employed the CAPM using the parametric estimator to estimate their firm's beta and cost of equity. However, the parametric estimators yielded unreliable results compared with the non-parametric estimators when the outliers existed in the data. Phuoc (2018) showed the outliers existed in returns on all stocks in the sample of the S&P 500 index. Hence, this research tried to argue for the CAPM using the non-parametric estimator through the use of Bayesian inference and a Gibbs sampler, a MCMC simulation with 300,000 iterations; the hypothesis tests/confidence intervals and analyses of the beta, advance information criteria AIC/DIC, model variance, alpha,

beta, beta standard deviation, and model fit. Using all the S&P 500 stocks having monthly data from 07/2007–05/2019 (450 stocks) and both parametric and non-parametric Bayes estimators, the empirical results showed: 1) the CAPM using the R.Bayes yielded only three (0.7% of all firms in the sample) compared with seven (1.6% of all firms in the sample) zeroed betas of Bayes estimator. This finding reinforces the application of the CAPM itself and the CAPM using the non-parametric estimator in a firm's beta and cost of equity estimations in practice. This conclusion matches with the findings of previous studies (e.g., Association for Financial Professionals, 2011; Brotherson et al., 2013; Bruner et al., 1998; Da et al., 2012; Jacobs and Shivdasani, 2012; Liu et al., 2009; Moore, 2016; Phuoc, 2018; Phuoc et al., 2018; Zhang, 2017). 2) The CAPM using the R.Bayes yielded the statistically significant much smaller information criteria AIC/DIC compared with the Bayes estimator. These findings indicate the R.Bayes is a more parsimony estimator and possess more explanation power compared with the Bayes estimator. This conclusion contradicts the common practice of using the parametric estimators in asset pricing work, but supports the findings of other

studies (e.g., Fong, 1997; Ho and Naugher, 2000; Shumacker et al., 2002). 3) The CAPM using the R.Bayes yielded smaller model variance compared with the Bayes estimator for all 442 firms in the sample. This finding supports what McKean (2004) tried to argue for the application of the non-parametric estimator in practice. 4) CAPM using the R.Bayes yielded a smaller absolute value of alpha compared with the Bayes estimator. This finding indicates the R.Bayes estimator fits more with the efficient market assumption of the CAPM and the common belief of efficiency of the S&P 500 stocks and its index. This conclusion contradicts the common practice of using the parametric estimators in asset pricing works as well as other studies (e.g., Cornell and Dietrich, 1978; Sharpe, 1971). 5) The CAPM using the R.Bayes yielded a statistically significant smaller beta standard deviation compared with the Bayes estimator. This finding implies the non-parametric estimator is more efficient estimator and hence, a preferred choice in the CAPM as well as other asset pricing models. This conclusion supports the findings of previous studies (e.g., Bowie and Bradfield, 1998; Chan and Laskonishok, 1992; Phuoc, 2018; Phuoc et al., 2018; Serra and Martelanc, 2013; Shalit and Yitzhaki, 2002; Trzpiot, 2013). 6) The CAPM using the R.Bayes yielded the statistically significantly much higher model fit compared with the Bayes estimator. This finding indicates the R.Bayes holds more explanation power of the relationship between movements of the stock returns based on the market returns compared with the Bayes estimator. Again, this conclusion contradicts the common practice of using the parametric estimators in asset pricing work, but agrees with the finding of other studies (e.g., Ho and Naugher, 2000; Shumacker et al., 2002).

All roads lead to Rome. However, all findings above indicate the CAPM using the R.Bayes is superior compared with the Bayes estimator. Therefore, the CAPM using the non-parametric Bayes estimator is recommended for a firm's beta and cost of equity estimations since it also met Ryan (2007) criteria.

## Declarations

### Author contribution statement

Le Tan Phuoc: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Pham Duc Chinh: Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data.

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### Competing interest statement

The authors declare no conflict of interest.

### Additional information

No additional information is available for this paper.

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