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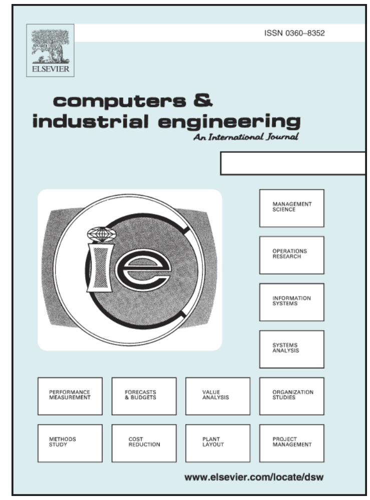

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> We investigate how power structure influences supply chain profitability. > We consider a three-echelon supply chain facing stochastic demand. > We show the next results, which are significantly different from previous insights. > The first-mover advantage is completely lost in a stochastic environment. > The last-mover advantage always arises in a stochastic environment.

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ABSTRACT

In this paper, we investigate how the order in which supply chain members demand their respective margins, which is often called power structure, influences their profitability in a three-echelon supply chain facing stochastic demand. We consider a typical three-echelon supply chain consisting of an upstream firm, a midstream firm, and a downstream firm. Previous supply chain models examining power structure have shown the conventional result that a supply chain member that sets a price or margin earlier generates a higher profit in a deterministic environment; that is, firms achieve higher profits in the order of the first-mover, the second-mover, and the third-mover who set margins in a three-echelon chain. Here, the first-mover, second-mover, and third-mover mean the supply chain member who sets its margin first, second, and third, respectively. Our stochastic model suggests the starkly contrasting result that the expected profit under demand uncertainty is higher in the order: (i) in a wide range of circumstances, the third-mover, the first-mover, and the second-mover who demand margins or (ii) in a narrower range of circumstances, the third-mover, the second-mover, and the first-mover. That is, the range of exogenous parameters leading to the first case is broader than that leading to the second case. The result indicates that the first-mover advantage is completely lost and, instead, the last-mover advantage always arises in a stochastic environment. Currently, power in supply chains tends to shift from upstream firms to downstream firms. However, our results warn a supply chain member in a stochastic environment that if assuming leadership to set its price or margin just because it has the power to do so, the member can ultimately harm itself and reduce its own profit.

Keywords: supply chain management; power structure; stochastic demand; game theory

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1. Introduction

Power structures underlying supply chains involving various firms pursuing respective profits are now becoming an issue of critical importance that attracts the attention of both practitioners and academics. In multi-echelon supply chains distributing consumer products, the order of pricing is usually the same as the order of shipping products; namely, supply chain members sequentially set respective selling prices in order from the upstream to the downstream. Recently, however, large-scale retailers often require margins for products from suppliers before suppliers determine their margins or wholesale prices. Previous empirical studies have also provided real-life cases in which large-scale retailers require *guaranteed profit margins* from various suppliers (e.g., Lee and Rhee, 2008). The price leadership exerted by downstream retailers is also found in agricultural product markets, in which upstream manufacturers are forced to become price takers (Kuiper and Meulenberg, 2004). Such a price leadership change is caused by power shifts from upstream manufacturers to downstream retailers. Huge retailers such as Tesco, Carrefour, and Walmart tend to be stronger competitors, playing more predominant roles than firms in upstream echelons (Ertek and Griffin, 2002).

Given these real-life cases, there exist two types of models involving decision-making by supply chain members in the industrial engineering literature. The first type of model describes a traditional supply chain in which firms from the upstream to the downstream sequentially choose their prices along the chain (e.g., Anderson and Bao, 2010; Jeuland and Shugan, 1983; McGuire and Staelin, 1983). Meanwhile, the second type of model assumes that firms constituting supply chains choose not prices but their margins (e.g., Choi, 1991; Luo et al., 2017; Xia and Gilbert, 2007). In the second type of model, the firm assuming leadership to determine a margin is viewed as having greater power. From this perspective, several studies have examined how the order in which supply chain members demand their respective margins, which is often called power structure, affects the profitability of channel members.

Previously, a two-echelon supply chain, which is typically composed of either a supplier and a manufacturer or a manufacturer and a retailer, has most frequently been considered to examine power structures in the industrial engineering literature. Recently, however, power structure in a three-echelon supply chain has also created interest among both academics and practitioners, because a three-echelon supply chain is often used as a useful distribution system.[1] As a real-life example for such a three-echelon supply chain, the regulation called the *three-tier* system is imposed on companies selling alcoholic beverages in most states of the United States (Gundlach and Bloom, 1998). The basic structure of this system is that producers can sell their products only to wholesale distributors who then sell to retailers, and only retailers may sell to consumers. Moreover, the number of echelons in distribution channels tends to be larger in Asian countries than in the United States. As an example of a three-echelon supply chain, Cheng et al. (2021) refer to Laoganma Flavor Food, which is a prominent processed food company in China distributing products to Walmart. Laoganma distributes its chili sauce to a Walmart supermarket in Shanghai via a local distributor. Hence, Cheng et al. (2021) consider Laoganma, the local distributor, and Walmart as a real-life case of a three-echelon supply chain, modeling the strategic interaction between them as a Stackelberg game. Because such three-echelon supply chains exist in reality, it is worth studying how much total profit generated in a three-echelon supply chain environment—where each supply chain member can decide its timing to demand its margin—is obtained by each supply chain member.

Given that power structures underlying supply chains are rapidly changing, this paper explores how the order in which supply chain members demand their respective margins influences their profitability in a three-echelon supply chain under demand uncertainty. We consider a three-echelon supply chain consisting of an upstream firm, a midstream firm, and a downstream firm. Existing supply chain models examining power structure in the literature have shown the conventional result that a member who sets its price or margin earlier in a supply chain generates a higher profit in a deterministic environment. Namely,

[1] The term *three-echelon supply chain* is often referred to as *three-level supply chain* or *three-tier supply chain* (Lan et al., 2018), all of which have the same meaning.

firms generate higher profits in the order of the first-mover, second-mover, and third-mover who set margins in a three-echelon chain. However, our stochastic model shows the starkly contrasting result that the expected profit in a stochastic environment under demand uncertainty is higher in the order of (i) the third-mover, first-mover, and second-mover who demand margins in a wide range of circumstances, or (ii) in a narrower range of circumstances, the third-mover, second-mover, and first-mover. That is, the range of exogenous parameters leading to the first case is broader than that leading to the second case. The result indicates that the first-mover advantage is completely lost and, instead, the last-mover advantage always arises under demand uncertainty. Although power currently tends to shift from upstream firms to downstream firms along supply chains, our results warn a supply chain member under demand uncertainty that if assuming leadership to set its price or margin just because of having power, the member can ultimately harm itself and reduce its own profit.

The logic behind the results above is laid out as follows. By observing the margin demanded by an earlier decision-maker, a later decision-maker can make a more precise inference about stochastic demand, thereby determining a more appropriate margin. Therefore, the last-mover achieves the highest profit among the three firms. Meanwhile, the rationale for the result that the second-mover achieves the lowest profit in most circumstances can be explained by using insights gained in the game theory literature. In a multi-echelon supply chain where supply chain members set their respective margins, the margins determined by the members have a negative correlation, meaning that one supply chain member decreases its margin if another member increases its margin (Choi, 1991). Moreover, first-mover advantage arises when the decision variables of players in a noncooperative game have a negative correlation (Gal-Or, 1985a). Therefore, when each supply chain member determines its margin, the margins have a negative correlation and hence the first-mover advantage arises in a deterministic environment. Furthermore, it is also known that this negative correlation in a deterministic environment can change to a positive correlation in a stochastic environment. That is, whereas the decision variables between the first-mover and the third-mover maintain their negative correlation, both the decision variables between the first-mover and the second-mover and those between the second-mover and the third-mover change to a positive correlation, respectively, in a stochastic environment (Shinkai, 2000). Hence, the positive correlation of decision variables induces the second-mover to lessen its incentive to increase its margin in our model, because if the second-mover required a high margin, the retail price would be raised too high and the resulting supply quantity would fall substantially from the optimal level, which would in turn reduce its own profit. Consequently, the second-mover becomes unable to demand a high margin in a stochastic environment, which induces its expected profit to fall to the lowest level.

In our model, each of the three supply chain members determines its margin based on its demand forecast. Because we assume these decisions are made sequentially, there are several cases classified by which of the three members constituting a three-echelon supply chain is the first-mover, second-mover, and third-mover, respectively. The major purpose of this paper is to derive the expected profit of each supply chain member in each case (i.e., sequence) and to determine the ranking of the profit by the sequence. As will be elaborated in the next section, this paper is the first to develop a stochastic model involving demand uncertainty to investigate the relationship between the order of decisions and profitability in the context of multi-echelon supply chain management.

The remainder of the paper is organized as follows. Section 2 provides a review of the literature related to game-theoretic models examining power structures underlying supply chains. Section 3 delineates the settings of our three-echelon supply chain model. Section 4 first presents a preliminary model without demand uncertainty, confirming that a first-mover advantage arises in a deterministic environment. Subsequently, we proceed to construct a main stochastic model under the presence of demand uncertainty, deriving supply chain members' optimal strategies and expected profits in equilibrium. In Section 5, we conduct a numerical investigation to confirm that numerical results support the message derived from the analytical model. Section 6 concludes.

2. Literature review

To date, a number of supply chain management studies have investigated the impacts of price leadership, also called power structure, on profitability in supply chain members based on game theory (e.g., Chakraborty et al., 2018; Chen et al., 2016, 2018, 2019; Choi, 1991; Edirisinghe et al., 2011; Li and Chen, 2018; Li et al., 2018a, 2018b, 2019a, 2019b, 2020; Liu and Ke, 2020; Lou et al., 2020; Luo et al., 2017; Matsui, 2017, 2018, 2019, 2020; Pan et al., 2010; Shi et al., 2020; Wei et al., 2013; Xia and Gilbert, 2007; Yan et al., 2017, 2020; Yang et al., 2018; Yu et al., 2017, 2020; Zheng et al., 2019). Choi (1991) investigates pricing decisions made by one retailer and two manufacturers. The following three scenarios describing different power structures are modeled and examined: (i) the manufacturer Stackelberg game, (ii) the retailer Stackelberg game, and (iii) the vertical Nash game. His research is a seminal study examining how supply chain members' profits depend on the power structure, and later studies thereafter consider that a channel member assuming price leadership has the most power. Pan et al. (2010) consider a supply chain involving one manufacturer and two retailers, and one involving two manufacturers and one retailer. They show that the power in the supply chain affects which of a wholesale price contract or a revenue-sharing contract is more profitable under the two respective scenarios. They also compare equilibrium results achieved by the contracts. Edirisinghe et al. (2011) investigate how the stability of a supply chain depends on channel power, in which two asymmetric manufacturers sell differentiated products via one common retailer. They specifically consider eight types of power structures in which the two manufacturers act as a Stackelberg leader and a follower. They show that power imbalance causes substantial reduction in supply chain total profit and that a more stable power balance achieves higher profits. Consequently, they conclude that a structure in which firms have equal power attains the highest supply chain stability and performance. Wei et al. (2013) examine price decisions made by two competing manufacturers and one retailer constituting a supply chain selling two complementary products. They consider three power structures; that is, (i) the vertical Nash game, (ii) the manufacturer Stackelberg game, and (iii) the retailer Stackelberg game. In addition, they assume that the two manufacturers can make sequential decisions in the two Stackelberg games, obtaining wholesale and retail prices in equilibrium. Luo et al. (2017) consider a supply chain in which vertically differentiated products of a superior product and an average product are sold by two manufacturers to a retailer. Here, because the quality of the superior product is higher than that of the average product, the superior product provides a consumer with higher utility than the average product. They describe vertical competition between the retailer and the two manufacturers as well as horizontal competition between the manufacturers. They demonstrate that if one of the two manufacturers is the late-mover to set its price, it can profit from learning the rival's price. While Luo et al. (2017) are especially associated with the current paper because they show the second-mover advantage arising in a supply chain, they assume a deterministic rather than a stochastic environment. Departing from the previous work described, the present paper reveals that the late-mover advantage arises in a supply chain facing stochastic demand by introducing uncertainty into a model involving power structure. Chakraborty et al. (2018) consider a two-echelon supply chain where consumer's demand depends on both retail price and marketing activity expensed by supply chain members. They examine power structure in a supply chain consisting of a manufacturer and a retailer who play a Stackelberg game. They find that while the follower position in the game is more advantageous for a firm to earn a larger profit, the leader position is less advantageous. Moreover, they show that neither a wholesale price contract nor a revenue sharing contract coordinates the supply chain. They finally develop a hybrid revenue and cost sharing contract that coordinates the supply chain. Chen et al. (2018) investigate the order of pricing decisions made by two competing retailers when they allow product returns from consumers. They find that the equilibrium order of decisions depends on a retailer's handling cost and salvage value of a returned product compared with those of the other retailer. Consequently, they show that a returns policy significantly influences the equilibrium power structure between the two retailers. Li et al. (2018b) investigate the influence of product substitutability and channel status on pricing decisions in two distribution channels under different leadership scenarios. They show that channel members always increase their profitability from taking the leader's position regardless of the competition and asymmetric related channel status. Moreover, they show whether the channel leader has the incentive to take the leader's position depends on the asymmetric relative channel status. Li et al. (2019a) investigate the power structure in a dual-channel supply chain consisting of a manufacturer and a retailer that provides services. They find that a showrooming effect enables the supply chain to improve profitability if a retailer determines its service level at a late timing. Moreover, they demonstrate that as the showrooming effect becomes greater, such a late decision of the service allows the supply chain to earn a higher profit. Yan et al. (2020) investigate an e-commerce platform that plays a role as a retail intermediary and provides trade credit to a capital-constrained supplier in the upstream. Their model describes price competition between this e-retailer and the supplier in a dual-channel supply chain. They show that the e-retailer obtains first-mover advantage by announcing its price earlier than the supplier.

Game-theoretic models have also been developed describing multi-echelon supply chains in the industrial engineering literature (e.g., Cheng et at., 2021; Lan et al., 2018; Liu et al., 2013; Panda et al., 2017; Santibanez-Gonzalez and Diabat, 2016; Zhao et al., 2019). Liu et al. (2013) investigate the impact of partial information sharing in a three-echelon supply chain consisting of a manufacturer, a distributor, and a retailer. Partial information sharing means that while information is shared between the distributor and the retailer, it is not shared between the distributor and the manufacturer. Based on these settings, they explore circumstances in which information sharing between the retailer and the distributor improves the profitability of the manufacturer, showing that such partial information sharing does not necessarily benefit the manufacturer. Lan et al. (2018) assume a three-tier supply chain in which a manufacturer distributes products to a retailer under demand uncertainty, through two asymmetric distributors. They derive the optimal ordering policy and equilibrium prices determined in the supply chain. They demonstrate that the dual-channel supply chain benefits both the manufacturer and the retailer if demand uncertainty is sufficiently high and that coordination among the distributors and the retailer is achieved through competition between the two distributors. Zhao et al. (2019) apply a system dynamics approach to design a simulation model describing a three-echelon supply chain consisting of one business unit, one distribution center, and one maintenance outlet. They assume that the business unit and the distribution center are operated by one company and thus regarded as one supplier. They focus on the coordination between the distribution center and the maintenance outlet in order to propose a contract to enhance these two parties' profits. Consequently, they provide a contract through numerical analysis that enhances the profits of both the distribution center and maintenance outlet.

Table 1 summarizes overall information from the related literature, clarifying the main features of the present paper compared with existing studies. The table shows that previous studies related to the issue addressed in the present paper can be classified on several dimensions. First, there are two types of models involving optimal decision-making by supply chain members in response to certain market demand levels, which are classified as deterministic models and stochastic models. Second, there are two types of supply chain models, two-echelon models and three-echelon models. Third, there are models that investigate how the power structure affects supply chain profitability and models that do not focus on the power structure but examine only Nash or Stackelberg games in supply chains. Hence, Table 1 clarifies that only the present paper simultaneously addresses all three issues, namely power structure, stochastic demand, and the three-echelon supply chain.

[Table 1]

The above overview of the literature shows that a number of existing supply chain management papers investigate the issue of power structure underlying supply chains. Nevertheless, to our knowledge, none of the previous work points out that the conventional wisdom—namely, that a supply chain member assuming leadership and hence making its decision earlier achieves higher profitability—significantly changes under the presence of demand uncertainty. It is therefore worth highlighting that, using a rigorous game-theoretic framework, the current paper is the first to show that leadership in a supply chain does not necessarily lead to an improvement in profitability in a stochastic environment.

3. Settings

In this section, we describe settings and assumptions used in our model. Table 2 summarizes the notations used in our model. As illustrated in Figure 1, we consider a three-echelon supply chain consisting of one upstream firm, one midstream firm, and one downstream firm, denoted U, M, and D, respectively.[2] The upstream firm incurs a variable cost denoted by c to produce a unit of its product. The product is sold from the upstream firm to the midstream firm at wholesale price p_U, from the midstream firm to the downstream firm at wholesale price p_M, and from the downstream firm to consumers at retail price p_D. Our model also assumes a make-to-order (MTO) system, in which the price or margin is the only decision variable and supply quantity is thus immediately adjusted in response to order quantity, following previous game-theoretic models in the literature (e.g., Lu et al., 2019; Yue and Liu, 2006).[3]

[Table 2]

[Figure 1]

We assume the next linear demand schedule following supply chain models in the

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[2] There are real-life cases that correspond to a three-echelon supply chain. For example, a supply chain consisting of a manufacturer, a wholesaler, and a retailer corresponds to our model setting. Another example is a chain consisting of a supplier, a manufacturer, and a retailer.

[3] Note that the seminal work on power structure by Choi (1991) demonstrates that the first-mover advantage arises in an MTO system; that is, a firm that determines its margin earlier achieves a higher profit in a deterministic environment. Therefore, our finding that first-mover advantage is completely lost in a stochastic environment with the MTO system is completely the opposite of the conventional insight from the literature.

literature (e.g., Gal-Or, 1985b; Li, 2002; Wang and Zhuo, 2020; Zhang and Zhang, 2020).[4]

$$q = a + e - b\, p_D, \tag{1}$$

where q is quantity and b is a positive constant. The intercept of the demand function consists of a deterministic part, a, and a stochastic part, e. While a is a positive constant, e is a random variable distributed with a mean of 0 and a variance of V.

Because the objective of this paper is to investigate the power structure underlying a supply chain, we assume that the firms choose their respective margins following previous models in the literature (e.g., Choi, 1991; Edirisinghe et al., 2011; Pan et al., 2010).[5] Let m_U, m_M, and m_D represent the margin of the upstream firm, the midstream firm, and the downstream firm, respectively, which are defined as:

$$m_U \equiv p_U - c \tag{2}$$

$$m_M \equiv p_M - p_U \tag{3}$$

$$m_D \equiv p_D - p_M. \tag{4}$$

Eliminating p_U and p_M from Equations (2), (3), and (4) yields the following equation:

$$p_D = m_U + m_M + m_D + c. \tag{5}$$

Substituting Equation (5) into Equation (1), we rewrite Equation (1) as:

$$q = a + e - b(m_U + m_M + m_D + c). \tag{6}$$

Equation (6) indicates that the following single equation describes the profits of the

[4] We derive the linear-form inverse demand function of Equation (1) if the utility function of a consumer denoted by U is specified as:
$U = ((a+e)q - q^2/2)/b$.
Given this function, consumer surplus denoted by S is stated as:
$S = U - p_D q = ((a+e)q - q^2/2)/b - p_D q$.
The consumer maximizes S by solving $\partial S/\partial q = 0$, which gives Equation (1). See Ingene and Parry (2004, Chapter 11) for more details about the derivation process of the demand function from the utility function and the application of the functions to distribution channel issues.

[5] The assumption that margins are decision variables for supply chain members is also congruent with industry practices and insights gained in previous empirical studies. Krishnan and Soni (1997) provide real cases of retailer Stackelberg games, in which large-scale retailers require manufacturers to offer guaranteed profit margins. Moreover, Cotterill and Putsis (2001) provide empirical evidence that a Nash game well describes the strategic interaction across channel members, in which a downstream firm determines a margin simultaneously when an upstream firm does likewise in several product categories.

three channel members:[6]

$$\pi_i = m_i\left(a + e - b\left(m_U + m_M + m_D + c\right)\right) \qquad (i = U, M, \text{ or } D). \tag{7}$$

π_U, π_M, and π_D denote the profit of the upstream, the midstream, and the downstream firm, respectively.

We assume in our Stackelberg game that a firm determines its margin after obtaining a private signal, which is the information concerning the stochastic part (e) of the intercept in the inverse demand function, but before the actual demand (a+e) is realized. Although the true value of the random variable, e, is unknown to all firms, each firm predicts the value by using its information-gathering technology. Specifically, each firm observes the value of its private signal f_i on e (i = 1, 2, 3). Henceforth, let a number or subscript 1, 2, or 3 denote the supply chain member who sets its margin first, second, and third, respectively. Firm 1 initially determines its margin m_1 after observing the private signal f_1. Second, Firm 2 determines its margin m_2 after observing the private signal f_2 and Firm 1's margin m_1. Finally, Firm 3 determines its margin m_3 knowing the private signal f_3, and Firms 1 and 2s' margins m_1 and m_2. Hence, Firm 2 (Firm 3) use the margin of m_1 (m_1 and m_2) as well as its private signal f_2 (f_3) to infer private signal(s) of the firm(s). Therefore, each firm determines its margin as the decision variable and all combinations of the decision sequence, classified by which firm decides earlier and which firm decides later, are considered and examined in our model.

Following the literature (e.g., Li, 2002), we also make an assumption regarding the posterior expectation of e, which is the demand disturbance. Assume that f_i, which is conditional on e, be independent and identically distributed random variables as follows:

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[6] If other expenses like holding cost per unit were considered in our model, then the cost would simply be added and included in the marginal cost because we assume an MTO system. Stated differently, if the upstream firm were assumed to incur holding cost per unit additionally, the variable c would include not only marginal production cost but also holding cost per unit, which means that the variable c would consist of the two cost factors. If the midstream or the downstream firm were assumed to incur such a cost per unit, the cost could simply be deducted from their margin per unit and the main result in this paper on the effect of the decision sequence on profitability ranking would never change due to the assumption of the MTO system.

$$\mathrm{E}(f_i|e) = e,\ \mathrm{var}(f_i|e) = \sigma \quad (i = 1, 2, 3). \tag{8}$$

Therefore, each firm obtains a private signal that is an unbiased value of the demand.

Next, the law of iterated expectations indicates that $\mathrm{E}(f_i) = \mathrm{E}(\mathrm{E}(f_i|e)) = \mathrm{E}(e) = 0$ and $\mathrm{E}(e^2) = \mathrm{var}(e) + (\mathrm{E}(e))^2 = V$ hold. We also have:

$$\mathrm{E}(f_i^2|e) = \mathrm{var}(f_i|e) + (\mathrm{E}(f_i|e))^2 = \sigma \tag{9}$$

$$\mathrm{var}(f_i) = \mathrm{E}(f_i^2) – (\mathrm{E}(f_i))^2 = \mathrm{E}(\mathrm{E}(f_i^2|e)) = \mathrm{E}(\sigma + e^2) = V + \sigma \tag{10}$$

$$\mathrm{E}(f_i f_j) = \mathrm{E}(\mathrm{E}(f_i f_j|e)) = \mathrm{E}(\mathrm{E}(f_i|e)\mathrm{E}(f_j|e)) = \mathrm{E}(e^2) = V \qquad (i \neq j) \tag{11}$$

$$\mathrm{E}(e f_i) = \mathrm{E}(\mathrm{E}(e f_i|e)) = \mathrm{E}(e\ \mathrm{E}(f_i|e)) = \mathrm{E}(e^2) = V. \tag{12}$$

We also assume that there exist constants denoted by Greek letters α, β, χ, δ, and γ, with subscript numbers described by the following linear equations.[7]

$$\mathrm{E}(e|f_1, f_2, f_3) = \alpha_0 + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3 \tag{13}$$

$$\mathrm{E}(e|f_i, f_j) = \beta_{0h} + \beta_{ih} f_i + \beta_{jh} f_j \qquad ((h, i, j) = (1, 2, 3), (2, 3, 1), (3, 1, 2)) \tag{14}$$

$$\mathrm{E}(e|f_i) = \chi_{0i} + \chi_{1i} f_i \qquad (i = 1, 2, 3) \tag{15}$$

$$\mathrm{E}(f_h|\, f_i, f_j) = \delta_{0h} + \delta_{ih} f_i + \delta_{jh} f_j \qquad ((h, i, j) = (1, 2, 3), (2, 3, 1), (3, 1, 2)) \tag{16}$$

$$\mathrm{E}(f_j|\, f_i) = \gamma_{0ji} + \gamma_{ji} f_i \qquad ((i, j) = (1, 2), (2, 3), (3, 1)) \tag{17}$$

We have the posterior expectations of e and the signals given in Equations (13)–(17) in the following lemma. (All proofs are shown in the Appendix.)

Lemma 1. The following equations hold.

$\mathrm{E}(e|f_1, f_2, f_3) = V(f_1+f_2+f_3)/(\sigma+3V)$

$\mathrm{E}(e|f_i, f_j) = V(f_i+f_j)/(\sigma+2V)$ $\qquad ((i, j) = (1, 2), (2, 3), (3, 1))$

$\mathrm{E}(e|f_i) = Vf_i/(\sigma+V)$ $\qquad (i = 1, 2, 3)$

$\mathrm{E}(f_h|\, f_i, f_j) = V(f_i+f_j)/(\sigma+2V)$ $\qquad ((h, i, j) = (1, 2, 3), (2, 3, 1), (3, 1, 2))$

$\mathrm{E}(f_j|\, f_i) = Vf_i/(\sigma+V)$ $\qquad ((i, j) = (1, 2), (2, 3), (3, 1))$

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[7] Examples of the prior-posterior distribution functions satisfying the assumption of the linearity in Equations (13)–(17) include the Normal-Normal, Gamma-Poisson, and Beta-Binomial distributions (DeGroot, 1970). Because we need to restrict the intercept of the demand function, $a+e$, to be nonnegative, the latter two distributions are appropriate, in which both e and f_i ($i = 1, 2, 3$) have a positive support.

4. Analytical results

4.1 Preliminary result: Deterministic environment

Before analyzing the main stochastic model under demand uncertainty, we provide relationships between the order of firms' decisions on margins and their profitability under a deterministic environment. The next theorem summarizes the result.

Theorem 1. In a deterministic environment in which $e = 0$ holds, firms' strategies on margins constituting the equilibrium are:

$m_1^* = (a–bc)/(2b)$, $m_2^* = (a–bc)/(4b)$, $m_3^* = (a–bc)/(8b)$.

The resulting equilibrium profits are:

$\pi_1^* = (a–bc)^2/(16b)$, $\pi_2^* = (a–bc)^2/(32b)$, $\pi_3^* = (a–bc)^2/(64b)$.

The equilibrium profits indicate that the following inequalities hold.

$\pi_1^* > \pi_2^* > \pi_3^*$.

Theorem 1 proves that the ranking of profitability is the same as the order in which firms set their respective margins, which is the conventional result that has been shown in the literature. That is, a firm achieves a higher margin or profit by setting its margin at its earliest time. This theorem gives the reason for why previous studies, focusing on power structures under a deterministic environment, consider that a channel member assuming leadership in determining a margin or price has the most power. In the following section, we will prove that this conventional result is reversed in a stochastic environment.

4.2 Main result: Stochastic environment

In this section, we proceed to analyze the main stochastic model, deriving firms' optimal strategies and expected profits in equilibrium. In general, multiple equilibria can arise in a sequential-move game played by three players under uncertainty. Because this paper aims to investigate how the order in which the margins are set affects optimal actions in the three-echelon supply chain, we employ the Stackelberg perfectly revealing equilibrium as the equilibrium concept. Specifically, Gal-Or (1987) shows that the functions of the equilibrium strategies must be monotonic with respect to signal(s) to ensure that a perfectly revealing equilibrium exists.[8] Because Equation (7) indicates that Firm i's expected profit is the quadratic function with respect to m_i, the function forms of the margins, m_1, m_2, and m_3, must be linear.[9] Given that each strategy is linear with respect to demand signal, we assume that $m_1^* = A_0+A_1f_1$, $m_2^* = B_0+B_1f_2+B_2m_1$, and $m_3^* = C_0+C_1f_3+C_2m_1+C_3m_2$.[10]

Because the model developed in the paper is classified as a dynamic noncooperative incomplete information game, the algorithm of backward induction is used to solve stochastic optimization problems. That is, we sequentially solve optimization problems from a later stage to an earlier stage to derive the total optimal solution, because multiple players who have different objective functions separately choose their respective decisions, which engenders mutual dependence relationships. The following proposition shows the linear (affine) strategies constituting the equilibrium.

Proposition 1. The equilibrium margin strategies are stated as:

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[8] In this paper, an algorithm proposed by Gal-Or (1987) is applied to identify the equilibrium of an incomplete information game. The decision variable controlled by firms in Gal-Or's model is quantity, not price or margin. Nonetheless, the algorithm proposed by Gal-Or (1987) is helpful because she proves that all players perfectly reveal their signals through a linear strategy including a player's private signal, which constitutes a separating equilibrium. Namely, Gal-Or (1987) proves that every player is unable to strategically revise its strategy for the purpose of "fooling" rivals at a pure strategy equilibrium in an incomplete information game.

[9] We focus only on the linear-form strategies, because Gal-Or (1987) proves that all players perfectly reveal their signals through a linear strategy including a player's private signal, which is called a perfect revealing equilibrium.

[10] Because only a linear strategy is considered to derive the perfect revealing equilibrium, each strategy is linear with respect to demand signals and margins. We calculate A_0, A_1, B_0, B_1, B_2, C_0, C_1, C_2, and C_3 by using profit-maximizing conditions for the three supply chain members.

$$m_1^* = \frac{V}{2b(2V+\sigma)}\left(a-bc+\frac{V}{V+\sigma}f_1\right)$$

$$m_2^* = \frac{1}{2(3V+\sigma)}\left(\frac{V^2}{b(2V+\sigma)}(a-bc+f_2)+(V+2\sigma)m_1^*\right)$$

$$m_3^* = \frac{1}{2}\left(\frac{V}{3V+\sigma}\left(\frac{a-bc+f_3}{b}-m_1^*\right)+\frac{3V+2\sigma}{V}m_2^*\right).$$

The following corollary follows from Proposition 1 after a simple calculation.

Corollary 1. The following inequalities hold:

$$\mathrm{E}(m_3^*) > \mathrm{E}(m_1^*) > \mathrm{E}(m_2^*).$$

The next proposition summarizes the firms' ex ante expected profits in equilibrium.

Proposition 2. Firms' expected profits in equilibrium are:

$$\mathrm{E}(\pi_1^*) = \frac{V(3V+2\sigma)(5V+2\sigma)\left(V^2+(V+\sigma)(a-bc)^2\right)}{16b(V+\sigma)(2V+\sigma)^2(3V+\sigma)}$$

$$\mathrm{E}(\pi_2^*) = \frac{V(5V+2\sigma)\left((V+\sigma)(3V+2\sigma)^2(a-bc)^2+V^2\left(9V^2+20V\sigma+8\sigma^2\right)\right)}{32b(V+\sigma)(2V+\sigma)^2(3V+\sigma)^2}$$

$\mathrm{E}(\pi_3^*) =$

$$\frac{(V+\sigma)(3V+2\sigma)^2(5V+2\sigma)^2(a-bc)^2+V^2\left(225V^4+776V^3\sigma+812V^2\sigma^2+336V\sigma^3+48\sigma^4\right)}{64b(V+\sigma)(2V+\sigma)^2(3V+\sigma)^2}.$$

From Proposition 2, we yield the next theorem summarizing the central result in this paper.

Theorem 2. The following inequalities hold depending on exogenous parameters.

Case (i): $\mathrm{E}(\pi_3^*) > \mathrm{E}(\pi_1^*) > \mathrm{E}(\pi_2^*)$ if $(a-bc)^2 > V(-9V^2+2V\sigma+4\sigma^2)/(9V^2+15V\sigma+6\sigma^2)$

Case (ii): $E(\pi_3^*) > E(\pi_2^*) = E(\pi_1^*)$ if $(a-bc)^2 = V(-9V^2+2V\sigma+4\sigma^2)/(9V^2+15V\sigma+6\sigma^2)$

Case (iii): $E(\pi_3^*) > E(\pi_2^*) > E(\pi_1^*)$ if $(a-bc)^2 < V(-9V^2+2V\sigma+4\sigma^2)/(9V^2+15V\sigma+6\sigma^2)$

Observe that Theorem 2 sharply contrasts with the result in Theorem 1. That is, the main result in a stochastic environment shows a significantly different result from the benchmark result in a deterministic environment. Remember that Theorem 1 suggests that the order of the profit earned by the three supply chain members is simply the same as the order of members setting margins under the absence of uncertainty. In contrast, Theorem 2 suggests that under a wide range of circumstances corresponding to Case (i), the expected profit is higher in the order of the third-, the first-, and the second-mover, which is the same as the order of the margins shown in Corollary 1.[11] In a narrow range of circumstances corresponding to Case (iii), the expected profit is higher in the order of the third-, the second-, and the first-mover, which is different from the order of the margins in Corollary 1. That is, the range of exogenous parameters leading to Case (i) is broader than that leading to Case (iii). Theorem 2 suggests that, in any case, the first-mover to demand its own margin at the earliest time never generates the highest profit among the three members. This result sharply contrasts with Theorem 1 in a deterministic environment, which has also been demonstrated in the literature investigating power structures in supply chains.

Because a manufacturer needs to sell products wholesale to a reseller downstream unless it executes direct selling to end consumers, it cannot in general become the last mover in a supply chain. Our result suggests that because the last-moving supply chain member can correctly infer demand information that the first- and second-moving members

[11] The reason why Case (i) in the theorem includes much wider circumstances than Case (iii) is explained as follows. In the condition of the inequality distinguishing the three cases, the left-hand side, $(a-bc)^2$, is positive. Hence, a necessary condition for the existence of the parameters for Case (iii) is that the right-hand side of the inequality, $V(-9V^2+2V\sigma+4\sigma^2)/(9V^2+15V\sigma+6\sigma^2)$, is positive. The necessary and sufficient condition derived from solving this value being positive is calculated approximately as $\sigma > 1.27V$; that is, the variance of the noise of the demand signal is at least larger than the variance of demand itself, which is a quite restrictive condition.

have from observing the margins they have set, the last-moving member achieves the highest margin and profit. Therefore, such an upstream manufacturer should be aware that it is unable to benefit from its price leadership under demand uncertainty. For example, as referred to in the introduction, if a food or beverage manufacturing company faces stochastic demand when selling its products to a retailer like Walmart via a distributor, price leadership is not advantageous to the manufacturer.

Next, we provide the logic behind how our central result described by Theorem 2 is derived. Basically, a later decision-maker can make a more precise inference about stochastic demand by observing the margin demanded by an earlier decision-maker, thereby determining a more appropriate margin. Hence, the third-mover gains an advantage of drawing demand information from the margins set by the first- and second-movers, thereby achieving the highest profit. Next, let us consider a rationale for why the second-mover has a disadvantage in most circumstances as shown in Case (i) in Theorem 2. First, the distribution channel model by Choi (1991) demonstrates that in a multi-echelon supply chain where supply chain members set their respective margins, the margins have the characteristics of *strategic substitutes*. These strategic substitutes mean that if a player increases its decision variable in a noncooperative game, another player decreases its decision variable in response, implying a negative correlation between the decision variables determined by players. At the same time, *strategic complements* are exactly the opposite of strategic substitutes, such that if one player increases its decision variable, another player also increases its decision variable. Therefore, the variables characterized by strategic complements have a positive correlation. Using these concepts, Gal-Or (1985a) proves that first-mover advantage arises when the decision variables are characterized by strategic substitutes. Therefore, in our model context, these previous studies indicate that when each supply chain member determines margins, the margins are characterized by strategic substitutes and therefore the first-mover advantage arises in a deterministic environment.

Based on these basic insights, Shinkai (2000) provides the rationale for why a second-mover does not necessarily generate the second largest payoff in a general incomplete information game, while he considers not a supply chain but a horizontal triopoly in which three firms compete to sell products. Specifically, Shinkai (2000) proves that strategic substitutes in a deterministic environment can change to strategic complements in a stochastic environment. That is, whereas the decision variables between the first-mover and the third-mover remain strategic substitutes, both the decision variables between the first-mover and the second-mover and those between the second-mover and the third-mover change to strategic complements in a stochastic environment. Applying this insight by Shinkai (2000), we understand that only the second-mover lessens its incentive to increase its margin in our model context, because if the second-mover set a high margin, the retail price would be raised too high and the resulting supply quantity would substantially fall from the optimal level, which would reduce its own profit. Consequently, the second-mover becomes unable to demand a high margin in a stochastic environment, which in turn induces its expected profit to fall to the lowest level in our power structure model where each firm sets its own margin.

5. Numerical investigation

So far, we have derived basic results of our model in analytical forms. Given the analytical results, we conduct numerical investigation in this section to confirm that they are numerically supported. Specifically, we substitute certain values into the exogenous parameters in the condition of $(a–bc)^2 \gtreqless V(–9V^2+2V\sigma+4\sigma^2)/(9V^2+15V\sigma+6\sigma^2)$ that classifies the three equilibrium cases in Theorem 2, thereby examining under which sets of exogenous parameters leads to which of the three cases.

Figure 2 illustrates the numerical results. Specifically, it shows the region that leads to each case in Theorem 2 by fixing the value of $(a–bc)^2$ on the left-hand side of the condition of $(a–bc)^2 \gtreqless V(–9V^2+2V\sigma+4\sigma^2)/(9V^2+15V\sigma+6\sigma^2)$ and drawing the implicit function against the horizontal axis and the vertical axis of V and σ, respectively. Hence,

Figure 2 shows which specific values of V and σ lead to Case (i): $E(\pi_3^*) > E(\pi_1^*) > E(\pi_2^*)$ or Case (iii): $E(\pi_3^*) > E(\pi_2^*) > E(\pi_1^*)$ in Theorem 2. The figure contains three panels in which $(a–bc)^2$ is set as 1, 10, and 100 in Panels A, B, and C, respectively. $(a–bc)^2$ is interpreted as the profitability for the entire supply chain, because the demand functional form of Equation (1) suggests that if a is larger and b or c is smaller, the supply chain can charge a higher price with incurring lower cost given a fixed amount of demand. Hence, the profitability for the supply chain is lower in the order of Panel A, B, and C.

[Figure 2]

Let us first look at the result shown in Panel A. The panel shows that Case (i) arises in the region below the curve while Case (iii) arises in the region above the curve. We also observe that the area below the curve is larger and hence there exist more possible combinations of σ and V leading to Case (i) than those leading to Case (iii).[12] In addition, Case (iii) never arises in the region of $V < 1.5$ because the vertical line of $V = 1.5$ is an asymptotic line of the curve. Hence, Panel A suggests that for Case (iii) to arise, the variance of the stochastic part of the intercept in the demand function (i.e., V) must be approximately at least larger than the squared deterministic part of the intercept in the demand function (i.e., a^2), which is a strict condition. Furthermore, Panel A also suggests that when V is equal to, for example, 3, Case (iii) arises only when $\sigma > 9.93$, which is also a quite strict condition because the variance of the information obtained by a firm (σ) must be significantly higher than the variance of the demand itself (V). Namely, there are two strict conditions for Case (iii) to arise in equilibrium.

Figure 2 also suggests that as the profitability of $(a–bc)^2$ increases from Panel A to B and then to C, the curve that separates the two regions of Cases (i) and (iii) shifts to the upper right. Nevertheless, every panel shows that, in any case, it is a necessary condition for Case (iii) to arise that V must be large enough to exceed $(a–bc)^2$ and σ must be large enough to exceed V. Since these conditions are quite strict as discussed above, the

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[12] We confirm this result because the whole region of Case (iii) fits completely at least above the 45 degree line of $\sigma = V$ in the figure.

numerical results suggest that Case (iii) rarely arises, which is consistent with the argument in footnote 11.

To summarize the numerical results in this section, first, Case (i) in Theorem 2 arises in most circumstances whereas Case (iii) rarely arises. That is, we confirm the result that the second-mover has the lowest profit among the three firms in most circumstances. Moreover, the first-mover advantage is completely lost whereas the last-mover advantage instead always arises in all the range of exogenous parameters in a stochastic environment. Consequently, we conclude that our analytical results are supported by the numerical results.

6. Conclusion

This paper investigates how the leadership or followership of firms constituting a three-echelon supply chain to demand margins influences their profitability. Conventional wisdom from preceding research examining power structure suggests that when multiple firms constitute a simple multi-echelon supply chain, the firm assuming leadership to determine its margin achieves a higher profit in a deterministic environment (e.g., Choi, 1991; Luo et al., 2017). Stated differently, it has traditionally been shown that a channel member can make a greater profit by determining its margin as early as possible. This paper has demonstrated that this conventional result never holds in a stochastic environment under demand uncertainty. In particular, we have demonstrated that in a three-echelon supply chain, firms have higher expected profit in the order of the last mover, the first mover, and the second mover under most circumstances in a stochastic environment. Under exceptional circumstances, firms earn higher expected profit in the order of the third-mover, the second-mover, and the first-mover. Hence, the first-mover never generates the highest profit in any case under uncertainty.

This result based on the rigorous game-theoretic approach yields managerial implications useful to multi-echelon supply chain members. For example, let us assume a realistic three-echelon supply chain in which the upstream, the midstream, and the downstream firms are a supplier, a manufacturer, and a retailer, respectively. If the three firms set their respective prices or margins in the order of shipping the product, the manufacturer that intermediates between an upstream supplier and a downstream retailer faces the risk that its margin is the lowest among the supply chain members under a wide range of environments. To avoid a reduction in its profit, the manufacturer can use the insight gained from our model; namely, the manufacturer should demand its margin later or earlier than the other two firms.

Compared with existing deterministic models, our proposed stochastic model also yields theoretical contributions. As the number of echelons increases within a supply chain, the variance of information on uncertain demand tends to rise as it moves up the supply chain from bottom to top, which is known as the *bullwhip effect*. Previous research suggests that information sharing among supply chain members plays a role as an effective solution to mitigate the bullwhip effect and improve supply chain performance (e.g., Lee et al., 1997). The existence of the bullwhip effect evidences that how to deal with demand uncertainties has become an important issue facing general multi-echelon supply chains. Our conclusion that the timing of decisions affects profitability under uncertainty is of theoretical use for firms tackling such a practical problem.

It should be noted that our approach has both strengths as well as limitations. One limitation of our game-theoretic approach is that real-life business practices that are very complex cannot be minutely incorporated into an analytical model. That is, if we considered more complex business practices, it would become even more difficult to derive analytical results in the form of explicit mathematical expressions, especially in a stochastic problem involving demand uncertainty. However, there is one strength that outweighs this weakness. As clearly shown in the forms of theorems in this paper, the incomplete information game model provides managerial insights that can be used for practical decision-making under uncertainty. For example, while it is well known that a firm constituting a three-echelon supply chain should determine its margin as early as possible in the absence of demand uncertainty, this paper yields the practical implication that a supply chain member should not heedlessly assume the leadership of decision-making in the presence of demand uncertainty. As such, a game-theoretic model provides implications that support practical decision-making, which is undoubtedly the strength of this approach.

Finally, we explore future possible research directions before closing the paper. While our model assumes a three-echelon supply chain that practically prevails in real-life supply chain environments, we can also consider a more general multi-echelon supply chain composed of more than three layers. Because our results indicate that the second-mover advantage or disadvantage depends on exogenous parameters, whether supply chain members other than the first- and the last-movers in a multi-echelon supply chain have any advantage or not is ambiguous until we formulate and solve such a model. While this extension is interesting in this respect, it would require a substantial reconstruction of the present model, altering and making it a completely new one. Hence, this development is another issue that is reserved for a future study.

Appendix

Proof of Lemma 1. Ericson (1969) proves the following relationship concerning linear posterior expectations.

$$E(e \mid f_i) = \left(\left(1/E(\mathrm{var}(f_i \mid e))\right)/\left(1/E(\mathrm{var}(f_i \mid e)) + 1/\mathrm{var}(e)\right)\right) f_i + \left(R/\left(E(\mathrm{var}(f_i \mid e)) + 1/\mathrm{var}(e)\right)\right) E(e). \quad \text{(A1)}$$

Equation (A1) is restated as follows with use of E(e) = 0, var(e) = V, and Equation (8).

$$E(e \mid f_i) = Vf_i/(\sigma + V) \quad \text{(A2)}$$

Next, we have the following expectations with the use of Equation (17):

$$E(f_i f_j) = E(E(f_i f_j \mid f_i)) = E(f_i E(f_j \mid f_i)) = \gamma_{0ji} E(f_i) + \gamma_{ji} E(f_i^2) \quad \text{(A3)}$$

$$E(f_j) = E(E(f_j \mid f_i)) = \gamma_{0ji} + \gamma_{ji} E(f_i). \quad \text{(A4)}$$

With use of E(f_i) = 0, E($f_i f_j$) = V, and E(f_i^2) = $V+\sigma$ indicated by Equations (10) and (11), we solve Equations (A3) and (A4) for γ_{0ji} and γ_{ji} to yield $\gamma_{0ji} = 0$ and $\gamma_{ji} = V/(V+\sigma)$. Substituting these two values into Equation (17) gives:

$$E(f_j \mid f_i) = Vf_i/(\sigma + V). \tag{A5}$$

Next, the following equation holds because of the law of iterated expectations and Equation (14).

$$E(e \mid f_i) = E(E(e \mid f_i, f_j) \mid f_i) = \beta_{0h} + \beta_{ih} f_i + \beta_{jh} E(f_j \mid f_i) \tag{A6}$$

Substituting Equation (A5) into Equation (A6), we have:

$$E(e \mid f_i) = \beta_{0h} + \beta_{ih} f_i + \beta_{jh} Vf_i/(V + \sigma). \tag{A7}$$

Because Equation (A7) holds for (h, i, j) = (1, 2, 3), (2, 3, 1), (3, 1, 2), we have $\beta_{0h} = 0$ and $\beta_{ih} = \beta_{jh} = V/(\sigma+2V)$ by solving Equations (A2) and (A7) for $(\beta_{0h}, \beta_{ih}, \beta_{jh})$.

Furthermore, the following equation holds because of the law of iterated expectations and Equation (13).

$$E(e \mid f_i) = E(E(e \mid f_1, f_2, f_3) \mid f_i) = \alpha_0 + \alpha_i f_i + \alpha_j E(f_j \mid f_i) + \alpha_h E(f_h \mid f_i) \tag{A8}$$

Substituting Equation (A5) into Equation (A8), we have:

$$E(e \mid f_i) = \alpha_0 + \alpha_i f_i + (\alpha_j + \alpha_h) Vf_i/(V + \sigma). \tag{A9}$$

Because Equation (A9) holds for (h, i, j) = (1, 2, 3), (2, 3, 1), (3, 1, 2), we have $\alpha_0 = 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = V/(\sigma+3V)$ by solving Equations (A2) and (A9) for $(\alpha_0, \alpha_1, \alpha_2, \alpha_3)$.

Finally, $E(f_h \mid f_i) = E(E(f_h \mid f_i, f_j) \mid f_i) = \delta_{0h} + \delta_{ih} f_i + \delta_{jh} E(f_j \mid f_i)$ holds from Equation (16). Substituting Equation (A5) into this equation yields $E(f_h \mid f_i) = E(E(f_h \mid f_i, f_j) \mid f_i) = \delta_{0h} + \delta_{ih} f_i + \delta_{jh} Vf_i/(V + \sigma)$. This equation is equal to $Vf_i/(V + \sigma)$. Noting that this relationship holds for (h, i, j) = (1, 2, 3), (2, 3, 1), (3, 1, 2), we have $\delta_{0h} = 0$ and $\delta_{ih} = \delta_{jh} = V/(\sigma + 2V)$. Substituting these values into Equation (16) gives $E(f_h \mid f_i, f_j)$ shown in this lemma. □

Proof of Theorem 1. Since e is equal to 0 in a deterministic environment, we substitute $e = 0$ into Equation (7), rewriting profits as:

$$\pi_i = m_i(a - b(m_1 + m_2 + m_3 + c)) \qquad (i = 1, 2, 3).$$

Following backward induction, we initially maximize π_3 with respect to m_3 by solving

$\partial\pi_3/\partial m_3 = 0$, yielding:

$$m_3 = a – b(m_1+m_2+c)/(2b). \tag{A10}$$

After substituting Equation (A10) into π_2, we maximize it with respect to m_2 by solving $\partial\pi_2/\partial m_2 = 0$, obtaining:

$$m_2 = a – b(m_1+c)/(2b). \tag{A11}$$

Lastly, we substitute Equations (A10) and (A11) into π_1 and maximize it with respect to m_1 by solving $\partial\pi_1/\partial m_1 = 0$, yielding:

$$m_1 = (a – bc)/(2b). \tag{A12}$$

Substituting Equations (A10)–(A12) into π_1, π_2, and π_3 gives $\pi_1 = (a–bc)^2/(16b)$, $\pi_2 = (a–bc)^2/(32b)$, and $\pi_3 = (a–bc)^2/(64b)$. These equilibrium profits show that $\pi_1 > \pi_2 > \pi_3$ obviously holds. In addition, note that all second-order conditions in this proof are met since objective functions are quadratic and concave with respect to the margins. □

Proof of Proposition 1. Using backward induction, we first consider the maximization of the expected profit of Firm 3. Based on Equation (7), the expected profit conditional on the signal for Firm 3 is:

$$E\big(m_3\big(a + e - b\big(m_1 + m_2 + m_3 + c\big)\big) \mid f_3, m_1, m_2\big). \tag{A13}$$

Note that Firm 3 maximizes its expected profit after observing f_3, m_1 and m_2. The first-order condition is:

$$\begin{aligned} &\frac{\partial}{\partial m_3} E\big(m_3\big(a + e - b\big(m_1 + m_2 + m_3 + c\big)\big) \mid f_3, m_1, m_2\big) \\ &= a - m_1 - m_2 - 2m_3 + E\big(e \mid f_3, m_1, m_2\big) = 0. \end{aligned} \tag{A14}$$

Taking note that Firm 3 correctly infers the signals of f_1 and f_2 by observing m_1 and m_2 set beforehand, we obtain the reaction function from Equation (A14) as:

$$m_3^* = \big(a - m_1 - m_2 + E\big(e \mid f_1, f_2, f_3\big)\big)/2. \tag{A15}$$

Then, the margin m_3 is conjectured as linear with respect to information variables; namely, $m_3^* = C_0+C_1f_3+C_2m_1+C_3m_2$. Inserting this margin into the left-hand side of Equation (A15) yields:

$$C_0+C_1f_3+C_2m_1+C_3m_2 = \left(a-m_1-m_2+E\left(e\,|\,f_1,f_2,f_3\right)\right)/2\,. \tag{A16}$$

Using Lemma 1, we restate Equation (A16) as:

$$C_0+C_1f_3+C_2m_1+C_3m_2 = \left(a-m_1-m_2+V\left(f_1+f_2+f_3\right)/\left(\sigma+3V\right)\right)/2\,. \tag{A17}$$

Because Equation (A17) holds regardless of the values of m_1, m_2, and f_3, the following four equations must be met:

$$C_0 = \frac{1}{2b}\left(a-bc-\frac{V}{3V+\sigma}\left(\frac{A_0}{A_1}+\frac{B_0}{B_1}\right)\right), \tag{A18}$$

$$C_1 = \frac{V}{2b\left(3V+\sigma\right)}, \tag{A19}$$

$$C_2 = \frac{1}{2}\left(-1+\frac{V}{b\left(3V+\sigma\right)}\left(\frac{1}{A_1}-\frac{B_2}{B_1}\right)\right), \tag{A20}$$

$$C_3 = \frac{1}{2}\left(-1+\frac{V}{b\left(3V+\sigma\right)B_1}\right). \tag{A21}$$

As the next step, Firm 2 determines its margin m_2^* to maximize E(π_2(m_1, m_2, S_3(f_3, m_1, m_2), e) | f_2, m_1). Because Lemma 1 and the linearity of S_3 indicate that the expected profit is quadratic in m_2 and linear in m_1 and f_2, Firm 2's first-order condition gives $m_2^* = S_2(f_2 , m_1) = B_0+B_1f_2+B_2m_1$. Because this equation must hold regardless of the values of f_2 and m_1, the next three equations must be met:

$$B_0 = \frac{1}{2b\left(1+C_3\right)}\left(a-b\left(C_0+c\right)-\frac{VA_0\left(1-bC_1\right)}{\left(2V+\sigma\right)A_1}\right) \tag{A22}$$

$$B_1 = \frac{V}{2b\left(1+C_3\right)\left(2V+\sigma\right)}\left(1-bC_1\right) \tag{A23}$$

$$B_2 = \frac{1}{2\left(1+C_3\right)}\left(\frac{V\left(1-bC_1\right)}{b\left(2V+\sigma\right)A_1}-1-C_2\right). \tag{A24}$$

Lastly, Firm 1 maximizes E(π_1(m_1, S_2(f_2 , m_1), S_3(f_3 , m_1, S_2(f_2 , m_1)), e)| f_1). The first-order condition is:

$$\begin{aligned}\partial E\left(\pi_1\right)/\partial m_1 &= a-b\left(B_0\left(1+C_3\right)+C_0+c+2A_0\left(1+C_2+B_2\left(1+C_3\right)\right)\right)\\ &\quad +f_1\left(-2bA_1\left(1+C_2+B_2\left(1+C_3\right)\right)+\left(1-b\left(B_1\left(1+C_3\right)+C_1\right)\right)V/\left(V+\sigma\right)\right) = 0.\end{aligned}$$

Because the first-order condition holds regardless of the realized value of f_1, the next two equations should be met:

$$a - b\left(B_0\left(1 + C_3\right) + C_0 + c + 2A_0\left(1 + C_2 + B_2\left(1 + C_3\right)\right)\right) = 0 \tag{A25}$$

$$-2bA_1\left(1 + C_2 + B_2\left(1 + C_3\right)\right) + \left(1 - b\left(B_1\left(1 + C_3\right) + C_1\right)\right)V/\left(V + \sigma\right) = 0\,. \tag{A26}$$

Solving Equations (A18)–(A26) for A_0, A_1, B_0, B_1, B_2, C_0, C_1, C_2, C_3, we yield:

$A_0 = V(a - bc)/(2b(2V + \sigma))$

$A_1 = V^2 f_1/(2b(2V + \sigma)(V + \sigma))$

$B_0 = V^2(a - bc)/(2b(3V + \sigma)(2V + \sigma))$

$B_1 = V^2 f_2/(2b(3V + \sigma)(2V + \sigma))$

$B_2 = (V + 2\sigma)/(2(3V + \sigma))$

$C_0 = V(a - bc)/(2b(3V + \sigma))$

$C_1 = Vf_3/(2b(3V + \sigma))$

$C_2 = -V/(2(3V + \sigma))$

$C_3 = (3V + 2\sigma)/(2V)$.

Substituting the nine equations into $m_1 = A_0+A_1f_1$, $m_2 = B_0+B_1f_2+B_2m_1$, and $m_3 = C_0+C_1f_3+C_2m_1+C_3m_2$ yields the equilibrium strategies m_1^*, m_2^*, and m_3^* shown in this proposition. □

Proof of Corollary 1. Proposition 1 indicates that the following inequalities hold.

$E(m_3^*)–E(m_1^*) = (3V^2+12V\sigma+4\sigma^2)(a–bc)/(8b(2V+\sigma)(3V+\sigma)) > 0$

$E(m_1^*)–E(m_2^*) = 3V^2(a–bc)/(4b(2V+\sigma)(3V+\sigma)) > 0$. □

Proof of Proposition 2. After inserting the equilibrium margin strategies in Proposition 1 into Equation (7), we substitute $E(e^2) = V$, $E(e) = 0$, and Equations (10)–(12) into the expected profit of the three firms, thereby yielding the equilibrium expected unconditional profits. □

Proof of Theorem 2. From Proposition 2, we have the following:

$E(\pi_3^*) - E(\pi_1^*) =$

$$\Big((V+\sigma)(3V+2\sigma)(5V+2\sigma)\left(3V^2+12V\sigma+4\sigma^2\right)(a-bc)^2$$

$$+V^2\left(45V^4+524V^3\sigma+700V^2\sigma^2+320V\sigma^3+48\sigma^4\right)\Big)/\left(64b(V+\sigma)(2V+\sigma)^2(3V+\sigma)^2\right) > 0$$

$E(\pi_3^*) - E(\pi_2^*) =$

$$\frac{(3V+2\sigma)^3\left(5V^2+7V\sigma+2\sigma^2\right)(a-bc)^2+V^2\left(135V^4+540V^3\sigma+652V^2\sigma^2+304V\sigma^3+48\sigma^4\right)}{64b(V+\sigma)(2V+\sigma)^2(3V+\sigma)^2}$$

> 0

$$E(\pi_1^*) - E(\pi_2^*) = \frac{V^2(5V+2\sigma)\left(\left(9V^2+15V\sigma+6\sigma^2\right)(a-bc)^2+V\left(9V^2-2V\sigma-4\sigma^2\right)\right)}{32b(V+\sigma)(2V+\sigma)^2(3V+\sigma)^2}.$$

The last equation indicates that $E(\pi_1^*) \gtreqless E(\pi_2^*)$ if $(a-bc)^2 \gtreqless V(-9V^2+2V\sigma+4\sigma^2)/(9V^2+15V\sigma+6\sigma^2)$. These relationships prove this theorem. □

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Table 1. Comparison with the literature

literature	power structure	information structure		number of echelons	
		deterministic	stochastic	two-echelon	three-echelon
Choi (1991), Xia and Gilbert (2007), Edirisinghe et al. (2011), Wei et al. (2013), Chen et al. (2016), Luo et al. (2017), Chakraborty et al. (2018), Li and Chen (2018), Li et al. (2018a, 2018b, 2019a, 2019b, 2020), Liu and Ke (2020), Matsui (2017, 2018, 2020), Yu et al. (2017, 2020), Yang et al. (2018), Zheng et al. (2019), Lou et al. (2020), Shi et al. (2020), Yan et al. (2020)	✓	✓		✓	
Yan et al. (2017), Chen et al. (2018, 2019), Matsui (2019)	✓		✓	✓	
Santibanez-Gonzalez and Diabat (2016), Panda et al. (2017), Cheng et al. (2021)		✓			✓
Liu et al. (2013), Lan et al. (2018), Zhao et al. (2019)			✓		✓
This paper	✓		✓		✓

Table 2. Notations.

p_U	wholesale price from the upstream firm to the midstream firm	
p_M	wholesale price from the midstream firm to the downstream firm	
p_D	retail price from the downstream firm to end-consumers	
m_U	margin of the upstream firm	$(m_U \equiv p_U – c)$
m_M	margin of the midstream firm	$(m_M \equiv p_M – p_U)$
m_D	margin of the downstream firm	$(m_D \equiv p_D – p_M)$
q	quantity	
c	marginal production cost	
a	deterministic part of demand intercept	
e	stochastic part of demand intercept	
b	slope of the demand function	
V	variance of e	
f_i	signal of demand information obtained by Firm i	
σ	variance of f_i conditional on e	
π_U	profit of the upstream firm	
π_M	profit of the midstream firm	
π_D	profit of the downstream firm	
U	subscript representing the upstream firm	
M	subscript representing the midstream firm	
D	subscript representing the downstream firm	
1	subscript representing the first-mover	
2	subscript representing the second-mover	
3	subscript representing the third-mover	

Figure 1. Supply chain structure.

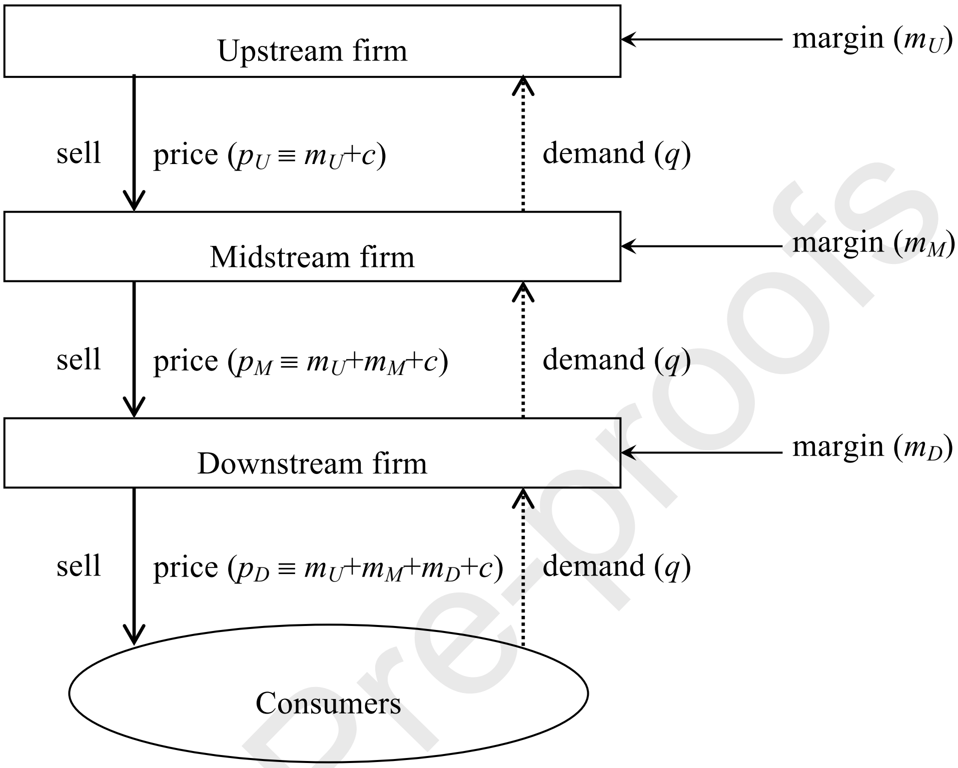
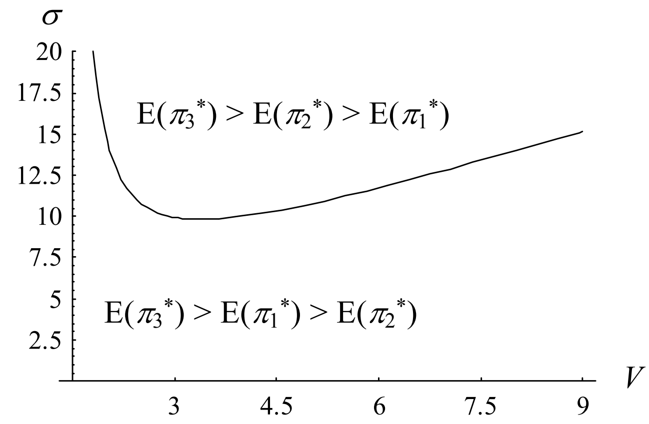
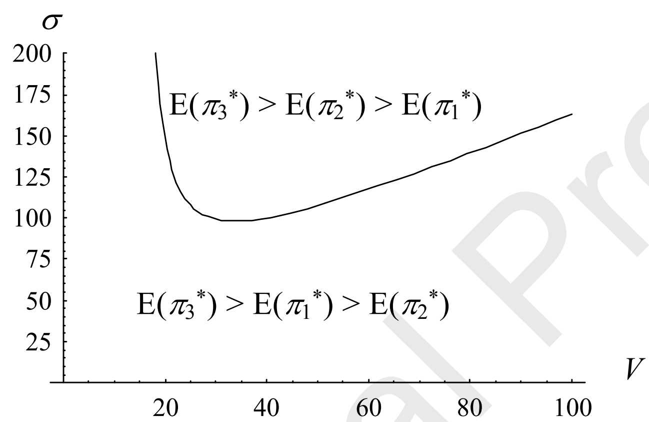


Figure 2. Numerical results.

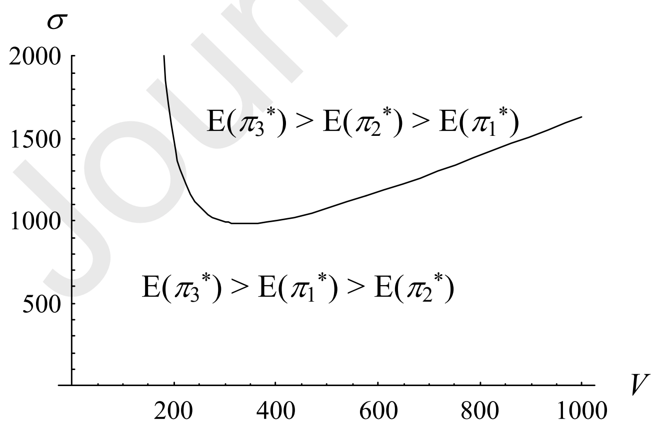
Panel A: $(a–bc)^2 = 1$



Panel B: $(a–bc)^2 = 10$



Panel C: $(a–bc)^2 = 100$



Note: We fix the value of $(a–bc)^2$ at 1, 10, and 100 in Panels A, B, and C, respectively, drawing the curve that separates the equilibrium cases in Theorem 2 against the horizontal and vertical axes of V and σ. $E(\pi_1^*)$, $E(\pi_2^*)$, and $E(\pi_3^*)$ represent the equilibrium expected profits of the first-mover, second-mover, and third-mover, respectively.

Kenji Matsui: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Writing - Original Draft, Writing - Review & Editing, Visualization, Project administration, Funding acquisition

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