Impacts of variable interest rates on the market areas of a spatial duopoly in supply chains operating on the finite horizon

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ABSTRACT

The paper aims to present a theoretical study of inventory allocation in two retailer warehouses that form a spatial duopoly on the finite horizon, where the shortage of goods, and not only prices, determines their market areas, depending on the customer's travel decisions to minimise not only the cost of purchasing the items but also the travel expenses under uncertainties of availability of the products. The customers' decision on which store to visit first is influenced by their estimation of regarding shortages of goods. The demand size required by arrived customers can be one or more units of goods. Delays in the required amount and all flows in the production–inventory–logistic system have varying parameters, simulated by the Network Simulation Method (NSM). The impact of delay in supply chain, which provides the products and consequently creates the shortages on market areas, is studied at varying interest rates and the shortened, unknown length of the time horizon. The annuity stream approach of evaluation of ordering policies is applied in areas of spatial duopoly where the optimal ordering policy depends on the interaction between the prices and the shortages of goods in the studied duopolies. Customer travelling problem (CTP) is defined, which determines the market area for allocated inventories. In our study, various distributions of customers' quantity demand at varying shortages and delays of the provision are supposed, presenting their impact on the market area. NSM helps in describing how to study the impact of varying interest rates and stochastic length of the time horizon in the case of shortages of goods as a consequence of delays in supply system activities and how to improve the estimates of NPV and profit in the case of low and varying interest rates.

1. Introduction

Despite the existence of numerous articles that model the competition among retailers, Web of Science Core Collection lists only two articles formulating the environments where competing retailers sell the same or similar products with different probabilities of shortage of these products at the retailer A_1 and the retailer A_2 in a spatial duopoly (Bogataj, 1999; Bogataj and Bogataj, 2001). These two articles consider profit on the infinite horizon at high enough interest rate so that the net present value (NPV) can be estimated sufficiently well when using Laplace transforms. However, such estimation is not good enough when the interest rate is extremely low and varying, as in today's financial flows, or when the overestimates are too high due to premature obsolescence. Namely, in recent decades, there has been an accelerated rate

of technological evolution and therefore, the obsolescence of the previous products and productions, due to in large part the advances in computer technology (Su et al., 2017) or adaptation of product or process to the ageing of a population (Calzavara et al., 2020). While in the practice of everyday sales, the business environments are rarely characterised by monopoly settings, a large number of studies on inventory control of final products focus on monopoly environments (Chan et al., 2004). To improve their operations to achieve an optimal profit, retailers mainly explore pricing strategies in an effort to improve their operations, but the problem of shortages of items is not focused upon in the spatial studies. The retailers adjust their pricing and supply to demand employing varied tools to learn about customers' demand. The benefits to this can be significant, including not only potential increases in profit but also effective and efficient supply chains, as elaborated in

Chan et al. (2004).

The existence of duopoly affects the behaviour of retailers and customers. Therefore, the results of previous studies on monopoly setting are no longer applicable in duopoly or oligopoly environments. In a spatial duopoly setting, the essential question is the market area of each retailer, since it directly influences quantity of demand and profit.

In our article, the problem of shortages of inventories in a spatial duopoly setting is analysed. Specifically, we consider two competing retailers who get the final product from the same supply chain. We examine pricing and inventory availability in this duopoly setting. The total expenses of the customer looking for such product comprise the selling price of the product and the travel costs to see the product in the store before buying. We assume that if they do not find the product in the first store, they must travel to another store, where they then get the product with certainty. The demand of items at each retailer depends on the market area of the store. The demarcation between these two areas is the set of points (housing units of customers) at which the expected expenditure of purchasing the items in the case of the first trip to retailer A is equal to the expected expenditure if the customer first travels to retailer B. We also assume that the products on sale may soon become obsolete. Therefore, the results obtained on the assumption that such a duopoly situation can take place on the infinite horizon cannot be directly used to study the behaviour of spatial duopoly on a finite, often short horizon with unknown date of obsolescence. This is especially true today when the interest rate is extremely low and is varying on the time horizon of our projections. Also, the time of obsolescence cannot be known in advance. Such environment is motivating our research, presented in this article.

For the purpose of this research, we proceed from the results published by Girlich (1995) on the treatment of spatial duopoly and further developed by Bogataj and Bogataj (2001) and upgrade the previous research with the Network Simulation Method (NSM), which enables the treatment of such duopoly at variable low interest rates and on the finite horizon. We consider the existence of expenditure equilibrium, i.e., demarcate market areas affected by price of items and travel costs of customers. This demarcation line of market areas determines the demand in each store and, consequently, the net present value (NPV) and profit of each retailer operation on the finite horizon at varying interest rates. Our results are applicable in any duopoly situation of retailers who sell the same products in a spatial duopoly on the finite horizon. The NPVs of retailers are considered here based on the profit on the finite horizon that can be appropriately calculated for the retailer W_1 and the retailer W_2 without being overestimated. This is important to know because overestimation could lead even to the bankruptcy of such retailers (Bogataj et al., 2016).

After this introduction, in section 2, the literature review is given. Section 3 presents the methods explored in this paper. Section 4 presents the numerical cases, with the aim to show how to study the impact of delays in supply chain on shortages of final products in the stores of retailers and how to estimate the impact of these influences on the NPV of each retailer, if the retailers operate on the finite horizon at low and varying interest rates. The correction factors for NPV evaluation calculated on the bases of Laplace transforms, as suggested in previous papers of Grubbström (2007), Bogataj and Grubbström (2012) and later demonstrated in Bogataj et al. (2016), is achieved using NSM approach. In section 5, we discuss the results achieved, and section 6 summarises conclusions, generalises the findings, considers limitations, and underscores the scientific value added and the applicability of our contributions. Also, the further studies are suggested.

2. Literature review

2.1. Spatial duopoly operating on the infinite horizon

Spatial duopoly has attracted the attention of scientists for more than 80 years. A total of 233 articles have been indexed in WoS on this topic.

The time series of these publications is provided in Table 1 and shows the growing interest in this topic in the last three decades.

Only four articles dealing with spatial duopoly have been published in the International Journal of Production Economics and some basic ideas were given also in the paper on spatial games (Horvat and Bogataj, 1999). For example, Wan et al. (2020) studied the position of online to offline platforms, enabling consumers to search for information and purchase products or services online and then consume offline through the development of a general duopoly competition model. In our model, we do not aim to search for information and purchase products online. Silveira and Vasconcelos (2020) developed a spatially structured evolutionary game while trying to analyse the competitive behaviour of retail firms in duopolistic markets. In their paper enterprises compete at two strategies, maximising profit or revenues, also introducing the dynamic learning mechanisms: Bogataj (1999), and Bogataj and Bogataj (2001), considering inventory control in such a system operation on the infinite horizon at high enough interest rate.

Among 233 papers describing duopoly positions, which are indexed in WoS, only the last two papers also consider the impact of shortages on the duopoly positions of players. Only the study of Bogataj and Bogataj (2001) observes the impact of delays in a supply chain on the final position of retailers in the spatial duopoly. But this method overestimates the NPV of these operations if the time horizon is short and the interest rate is low. The evaluation of these overestimations is the subject of our article.

Brueckner and Flores-Fillol (2007) consider delays in services due to the time delays in information since collecting and implementing the information in their decision process require some time (Szidarovszky and Matsumoto, 2009) or due to delay entry and a commitment to a location in a Hotelling type of setting (Meza and Tombac, 2009). Delays of activities in supply chains' (SC) nodes influence shortages in the point of delivery of the final products, but shortages could substantially influence the market areas of the final suppliers. By investing in the capacities of the SC, such delays can be mitigated. In that case, it is necessary to weigh between the present value of profit or NPV and the investments in the capacity of the chain. Since the activities in a chain could be only temporal on the finite horizon and the interest rate could be very low, such as in current cases, it is convenient to consider the impact of the unknown time horizon of activities and a variable interest rate on the duopoly position of the final nodes of a chain. For these purposes, the results from the article of Bogataj and Bogataj (2001) should be reconsidered under these new variations and constraints.

The expected demand at a particular location of items in a shop is predicted in terms of the expenditures of travel and the purchase of a predetermined quantity of products when customers travel from their home to the shop and back. The expenditure included in reaching the items from the supply centres depends on their spatial distribution and the probability of them being available at the location when required. We assume that trip chaining occurs when the purchase fails at a particular place in the shop for an advanced determined number of items. In that case, this chain of trips continues until the items are found (Bogataj, 1996, 1999bib_Bogataj_1996bib_Bogataj_1996bib_Bogataj_1999; Girlich, 1995). Knowing the location of supply points and the probability of success of the Poisson-distributed demand, we can establish the chain of trips with minimum expected expenditures of travel and can purchase at a certain level of risk (Bogataj and Bogataj, 1998).

If the final products are assembled at the location of supply shops, components for their production should be assured with an acceptable

Table 1
Articles indexed in WoS showing the growing interest in the study of spatial duopoly.

1937–1980	1981–1990	1991–2000	2001–2010	2011–2020
5	5	38	79	106

shortage level. Their production and supply policy depends on MRP decisions, price, and determined allowable shortage. In the literature, two approaches are known to maximise the supplier's profits: the traditional average cost approach and the more appropriate annuity stream approach, where exposure to risk can also be easily calculated. The second approach is employed. It is based on the skeleton of the MRP theory developed by Grubbström (see the overview in Bogataj and Bogataj, 2019; Grubbström, 2007; Grubbström and Tang, 2000); the numerical example is the extension of the study by Bogataj and Grubbström (2012). How to determine the market areas of each duopoly? How do time delays in supply networks affect shortages and consequently the market area? How do changes in market areas, together with changes in interest rates and the uncertainty of the duration of activities in the supply networks, affect NPV? This paper explores these relations and the consequences in a stochastic environment when the time horizon is finite, and the length of the horizon is not precisely known. Our aim is to show, how the Network Simulation Method (NSM) helps in describing how to study the impact of finite, often stochastic length of the time horizon on the duopoly behaviour. Therefore, Section 2.2 provides a short description of NSM, Section 3 presents the story of a spatial duopoly under consideration, and Sections 4 and 5 lead to the discussion of final results and conclusions, respectively.

2.2. Network Simulation Method

Peusner's (1987) 'Theory of Networks', on which his study of networks in thermodynamics is established, is the first publication that serves as the basis for NSM. This theory is based on the Theory of Circuits from a generalisation of its conjugate variables: electric current and potential difference. For Peusner (1987), the network models are an exact representation of the mathematical characteristics of the processes that they describe. Thus, the characteristic variables of each problem must satisfy Kirchhoff's laws, and their relationships will determine the corresponding circuit elements. In each concrete process, once the conjugated variables have been selected, the information of which circuit elements intervene in the network model and how they connect is obtained from the mathematical model rather than the physical considerations about the role played by the variables. In the study of González-Fernández (2002) and Sánchez-Pérez et al. (2018), detailed descriptions of the fundamentals of NSM and first applications in various fields of science and engineering can be found, such as electrochemical processes, transport through membranes, heat transmission, and so on, but we intend to add the analysis of financial flows based on the consequences of flows of items in SC.

NSM had already been applied successfully in several fields of engineering such as heat transfer, electrochemical reactions, transport through membranes, inverse problems, and ion transport, among others; all describe non-linear transport processes. It has also been implemented to deformable solid mechanics, dry friction problems (Alhama et al., 2011; Marin et al., 2012, 2016bib_Marín_et_al_2012bib_Marín_et_al_2016), and oxidation processes (Sanchez-Perez et al., 2018). Our approach to studying the flows of items and financial flows in SC is based on the theory of circuits.

This article presents a new case of using NSM, now in the problem of production economics of two retailers in a supply chain.

3. Method

3.1. Network simulation in production economics

NSM provides a detailed description of any process. Here, it describes the flow of objects and their present value within the SC, even in the case of discrete functions of current behaviour in the chain and of a combination of discrete and continuous distributed parameters systems that cannot be handled by other software packages. The method is implemented by NGSPICE, an open-source software (see: https://source forge.net/projects/ngspice/). NGSPICE is an efficient tool for simulating electronic devices that control our world today. The application of this method is shown using three easy examples as follows: the stochastic economic process, the deterministic economic process, and the production inventory control. The network model of the selected stochastic economic process is presented in Fig. 1. Figs. 2 and 3 present equivalents for the selected deterministic economic process and the production inventory control. The notation in Figs. 1 and 2 are as follows in Table 2.

The V_{TI} is a piece-wise linear voltage source; R_{TI} is a resistor; E_D is a linear voltage-controlled voltage source with a value equal to instant demand; V_I is an independent source for voltage with a null value, which works as an ammeter; F_1 is a linear current-controlled current source; C_1 is a capacitor; R_{inf} is a resistor with very high resistance.

The time generator circuit equals that of the stochastic model. The circuit implements each function, as presented in Fig. 2. The satisfied demand function is discontinuous, and therefore, two generators are needed to implement this non-linearity, as shown in Fig. 2(c) and (d). The inventory function has the same issue. It also involves the integration of variables. Therefore, an integrator circuit is employed. Every time the inventory reaches zero, integration must be reinitialised.

Total NPV is calculated by adding the integration of demand, production, inventory, and fixed cost; all consider the interest ratio. The circuits calculate each component as presented in Fig. 3. The integration circuit is similar to that used for the stochastic economic process.

The initial conditions are inserted into the specifications of the capacitors' initial conditions (initial voltage value) and coils (initial current value), respectively. The whole network model now runs by employing NGSPICE, an open-source software.

The reliable network model design needs equivalence between the model and process equations, including the initial conditions. Modelling the basic rules is explained in the study of González-Fernández (2002). The first step is to select the equivalence between the physical and electrical variables (different choices give rise to distinct networks). The following equality is established for the problem: V (net present value) ≡ U (voltage) or dV/dt ≡ i (electric current in the network). In other directions, similar equivalences are applied. Each addend of an equation is represented by an electrical potential difference in a circuit element whose constitutive equation is analogous to the added one. Hence, the application of the second Kirchhoff's law in a network equivalent to the equation allows the equation to be solved. Non-linear terms cannot be implemented directly and must be defined by controlled sources or additional circuits.

Most textbooks do not consider this classic electrical analogy. Therefore, the NSM is more interesting and efficient in the numerical computation field. After integrating 'dV/dt', the equivalent to the current in the main circuit, the term 'V' becomes available. The integration is implemented by using a secondary circuit in the models for the NPV. The secondary circuit is implemented by a controlled source F_1 that generates the current 'dV/dt' obtained from the voltage in an ammeter connected in series in the main circuit. A capacitor with $C_1 = 1$ F is used to integrate the current of F_1, and the voltage in this element, $V_{C1} = 1/C_1 \cdot \int (dV/dt) \cdot dt$, is simply the variable 'V'. A resistor with a very high value, R_{INF}, improves algorithm stability. A non-linear voltage source is defined to implement any necessary function (Sánchez-Pérez et al., 2018). The initial conditions corresponding to the NPV at instant t = 0, V0 = 0 are implemented by initial conditions in the capacitor. The NGSPICE allows the network model, as provided by Bogataj and Grubbström (2012), for example, to be simulated in a very straightforward manner. Table 3 shows the equivalence between MRP and NSM notations of procedures running in the electronic components of Figs. 1–3.

3.2. Customer travelling problem and behaviour

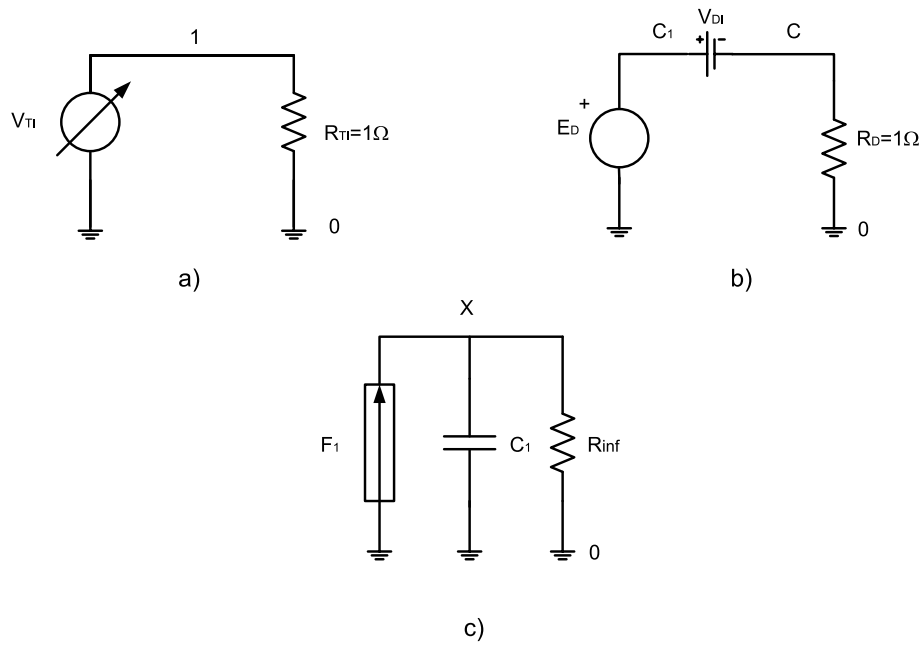This paper analyses a spatial duopoly model in a given market area

Fig. 1. Network model of the stochastic economic process: a) Time circuit, b) Demand function circuit, and c) Net present value circuit.
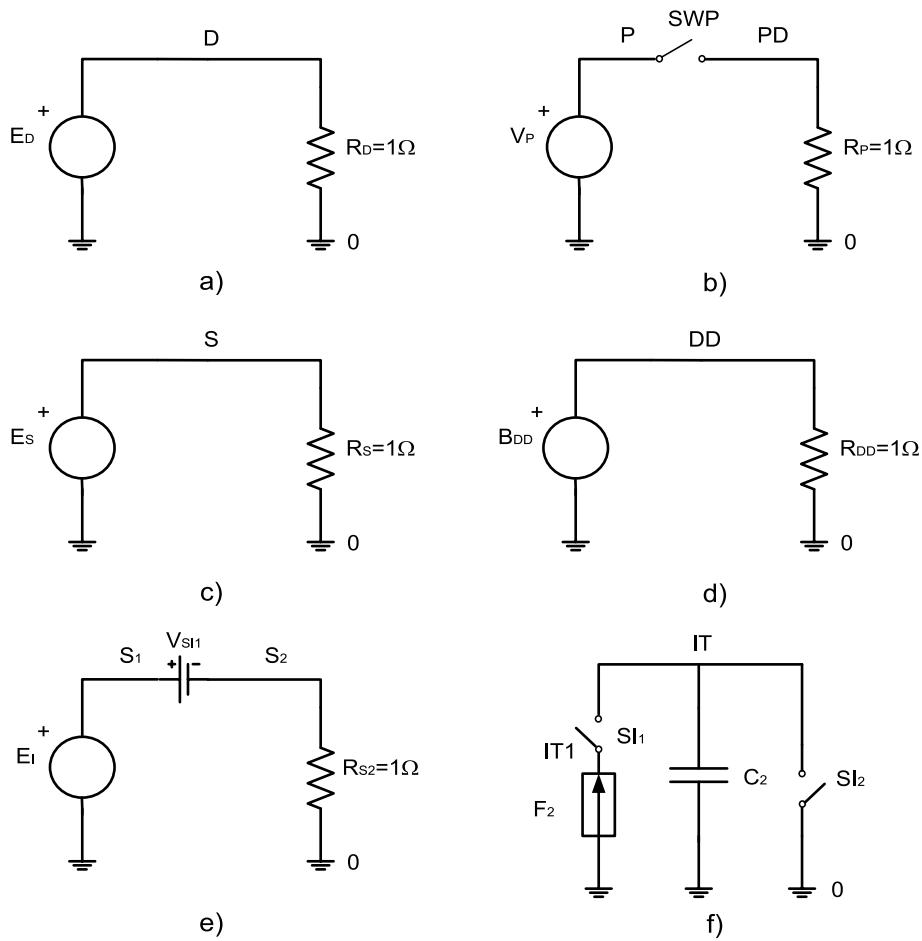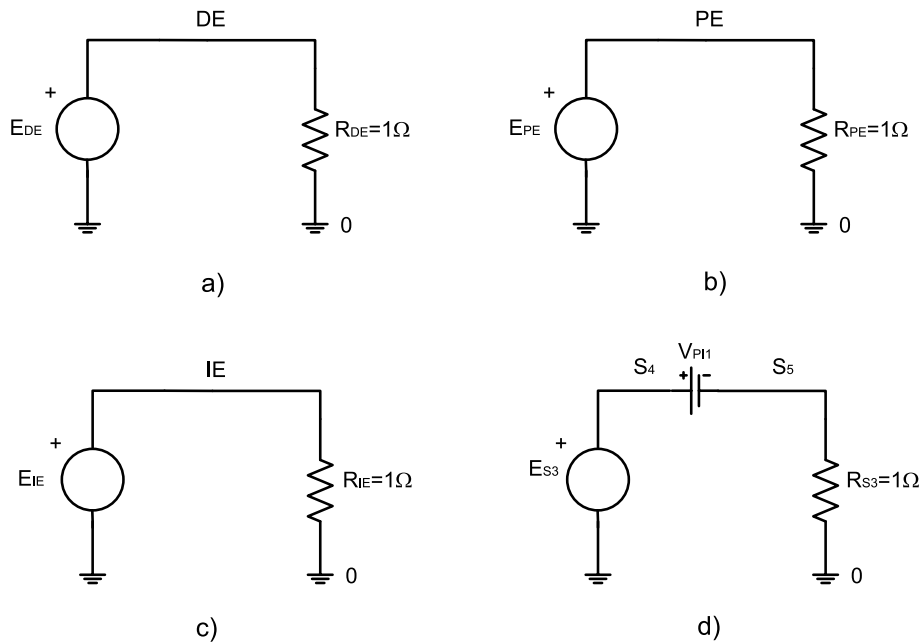

Fig. 2. Network model of the deterministic economic process: a) Demand function circuit, b) Production function circuit, c) and d) Satisfied demand function circuits, and e) and f) Inventory function circuits.

Fig. 3. Network model for the net present value calculation of production inventory control a) Demand considering the interest rate circuit, b) Production considering the interest rate circuit, c) Inventory considering the interest rate circuit, and d) The cash flow circuit.

Table 2
Notations used in electrical circuits (second column) and their correspondence in a supply chain (third column) for the stochastic economic process.

Notations	Electric Components	Physical and economic correspondence
V_{DI}	Ammeter associated with demand	Stream of demand
V_{TI}, V_P	Independent voltage source	Timing t_i, Production intensity: $\mathbf{P}$, p_j
R_{TI}	Resistor associated with time (TI)	Record of timing
E_D, E_I, E_S	Linear voltage-controlled voltage source	Demand, inventory, and satisfied demand
R_D, R_P, R_S, R_{INF}	Resistors associated with demand, production, delivery, and stability, respectively	D(t), P(t), F(t), -
B_{DD}	Non-linear dependent voltage source	Partially satisfied demand
C_1	Capacitor 1	Cumulative demand
F_1	Linear current-controlled current source	Record of demand from an ammeter
SWP	Switch on – switch off	Production controlled as feedback of inventory level
SI_2		Delivery of inventory (to satisfy demand)
VSI	Ammeter	Record of inventory as an inventory stream
F2	Linear current-controlled current source	Record of inventory from an ammeter
SI_1	Switch associated with inventory	Arrival of inventory (orders)
C_2	Capacitor 2	Cumulation of inventories

Table 3
Equivalence between MRP and NSM notations of procedures running in the electronic components of Fig. 3.

Notation	NSM at each activity cell – node $j = 1, 2, .., n$	MRP in *nxn* matrix presentation considering all activity cells simultaneously
E_{DE}	Linear Voltage-Controlled Voltage Source	For example: $d_j(t) = D_j \cdot (1 - cos(\omega \cdot t))$D is the mean of demand intensity, ω is related to the period (T) of a complete season's fluctuation by $2\pi/T$
E_{PE}	Linear Voltage-Controlled Voltage Source	$p(t) = \begin{cases} p_0 \text{ if } R(t) < R_0 \\ 0 \text{ if } R(t) \geq R_0 \end{cases}$ where p(t) is the production capacity, R_0 is the maximum inventory capacity, and R(t) is the accumulated inventory
E_{IE}	Linear Voltage-Controlled Voltage Source	Inventory behaviour $R(t_f) = R_0 + \int\limits_0^{t_f}[p(t) - d(t)] \cdot dt$
E_{S3}	Linear Voltage-Controlled Voltage Source	In this model, not necessarily all the demands d (t) can be satisfied $(d_s(t))$. $\begin{cases} d(t_f) \text{ if } R_0 + \int\limits_0^{t_f}(p(t) - d(t))dt > 0 \\ d_s(t_f) \text{ if } R_0 + \int\limits_0^{t_f}(p(t) - d(t))dt \leq 0 \end{cases}$
R_{DE}, R_{PE}, R_{IE}, R_{S3}	These components are necessary to obtain demand, production, inventory, and satisfied demand values as a voltage respectively	
V_{PI1}	Ammeter associated with satisfied demand	

A, divided between two final suppliers – warehouses with shops, which belong to the same SC. The shortage of goods could appear due to delays in any node or transport between nodes in the SC. Such shortage is supposed to be an essential factor that influences their market areas. The shortage affects the NPV of activities and therefore the profit of these two shops. In order to assess the profitability of these activities in each store, the impact of the shortage on the long-term behaviour of customers must be evaluated. The interest rate and the duration of activities are usually not known in advance. Therefore, we should be able to assess these influences at the forecasted variation of the interest rate and variable duration of the activities on a time horizon in order to consider investing in such activities and their capacities or not.

We shall use the following further notation presented in Table 4.

Let us assume that the customers' locations are continuously, evenly dispersed in the space of equal demand density function *r(z)* on the area A: $A = \{z \in R^2 : r(z) > 0\}$.aa.

Similar to Bogataj and Bogataj (1998), we assume that the retail of production of the i-th firm, located at point z_i of this area, is selling the final product available at the i-th firm at an arbitrarily chosen moment with probability α_i. The set of all customers' locations supplied primarily from z_i is denoted by A_i. It is defined as $A_i = \{z \in A : EC_i(z) \leq EC_j(z),$ $i \neq j \}$ where $C_i(z)$ indicates the random cost of the desired number of items for a customer with residence at z patronising warehouse i. $EC_i(z)$ is its mathematical expectation of expenditures for a desired amount of

Table 4
The basic notation in the spatial duopoly.

Notation	Meaning of the notation
$r(z)$	The demand density function
z_i	Location of the warehouse W with the shop i
A, A_i	The total area and the part i of this area from where the customers arrive
α_i	The probability of shortage of goods in i
p_i	The mill price of items in i
l_i	The distance from z to z_i
λ_i	The transportation costs per distance $l_i = 1$
$C_i(z)$	Expenditures of a customer who is located at z, giving the probability of shortages at z_i
$EC_i(z)$	The mathematical expectation of customer's expenditures to obtain the products
$z(EC_i = EC_j)$	The boundary between A_i and A_j

items.

For the total studied area, we assume that. $A = \cup A_i;\ \ A_i \cap A_j = \varnothing$ for $i \neq j; i = 1, 2, ...n$

We are looking for a logical partition of overall demand density (demand per week or similar demand per time unit) corresponding to market areas A_i. Every customer patronises one retail. His/her decision depends on the mill prices p_i of items, transportation costs per unit of distance t_i, and distance from his location to the warehouse i, denoted l_i.

For simplicity, we shall consider a duopoly situation where only two warehouses at stores A and B with the locations $z_1 = (2,0)$ and $z_2 = (0,0)$, respectively, compete for customers. The policy of delivery in our two-stage system operates in such a way that the following assumption holds (Bogataj, 1999; Girlich, 1995):

The items are always available in the other supply unit if they are not available in the unit visited earlier. If we write $W_i = 0$ in case there is no required quantity of items in i and $W_i = 1$ if there are items as needed there, it holds that for $i, j = \{1, 2\}$, the probability $P(W_i = 1|W_j = 0) = 1$ for $i \neq j$.

For determining the expenditures $C_i(z)$:

$$C_i(z) = \begin{cases} p_i + 2 \cdot \lambda_i \cdot l_i, \ for\ \alpha_i = 0 \\ p_j + \lambda_j \cdot l_j + \lambda_{ij} \cdot l_{ij} + \lambda_i \cdot l_i, \ for\ \alpha_i = 1 \end{cases} \quad (1)$$

here, l_{ij} is the distance between warehouses, and λ_{ij} is its corresponding transportation expenditures per unit of distance.

The boundary between A_1 and A_2 in the spatial duopoly of area A that follows from the condition: $EC_1(z) = EC_2(z)$ and therefore:

$$EC_1(z) = EC_2(z)(1 - \alpha_1)(p_1 + 2\lambda_1 l_1(z)) + \alpha_1(\lambda_1 l_1(z) + \lambda_{12} l_{12} + p_2$$

$$+ \lambda_2 l_2(z)) == (1 - \alpha_2)(p_2 + 2\lambda_2 l_2(z)) + \alpha_2(\lambda_2 l_2(z) + \lambda_{12} l_{12} + p_1 + \lambda_1 l_1(z)) \quad (2)$$

where α_1 and α_2 are probabilities of the shortage of items in warehouses W_1 and W_2, respectively.

In this work, a programme has been developed to plot this boundary in an efficient and fast way. To verify the programme, an example is solved. Considering $p_i = p = 1$, $\lambda_i = \lambda_{ij} = \lambda = 1$, $\alpha_i = \alpha = 0.8$, the expected boundary in the interval $[0, 1]$ is a straight line in x = 1. Then, the effect of the ratio between shortages of items α_2 and α_1 is considered in Fig. 4.

The effect of the ratio between p_2 and p_1 is observed in Fig. 5.

The effect of the ratio between t_2 and t_1 is considered in Fig. 6. In this case, expenditures of the transport between warehouse 1 to 2 is t_1 and the same to warehouse 2 is t_2.

In such a duopoly situation, warehouse W_i can expect the demand density (demand per time, like per week or similar) d_i at z_i.

$$d_i = r(A_i + \alpha_j A_j), i = 1, 2 \quad (3)$$

and the supply of goods meets the demand for

$$d_i^f = r((1 - \alpha_j) A_i + \alpha_j A_j), I = 1, 2 \quad (4)$$

### 3.3. The net present value of a supply chain calculation

3.3.1. Previous results

Let us consider an example of the production part of the SC (Bogataj and Grubbström, 2012) where perturbations have been taken into account as described in Bogataj et al. (2016). Activity cell D assembles two units of E and one unit of F; activity cell B demands three units of D for the production of one unit of B, and at the end, A requires one unit of B and two units of C for the production of one unit of A. The BOM of this example and delivery to the warehouses are presented in Fig. 7. For the previous model explained in detail in Bogataj et al. (2016), we have added the perturbation of production and transportation lead times. Their values can be higher with a certain probability. Fig. 8 presents the average production lead times $\bar{\tau}$ for all nodes, from A to F. The final production is at locations of W_1 and W_2. Therefore, transportation delays from A to W_1 or W_2 are assumed to be negligible.

Table 5 shows the basic notation in the MRP theory.

Now, we try to show the power of NSM by applying it to an SC. Table 6 shows the equivalence between MRP and equations in the time domain used for NSM.

According to Bogataj et al. (2016), we can write the generalised input matrices by considering production and transportation averages of
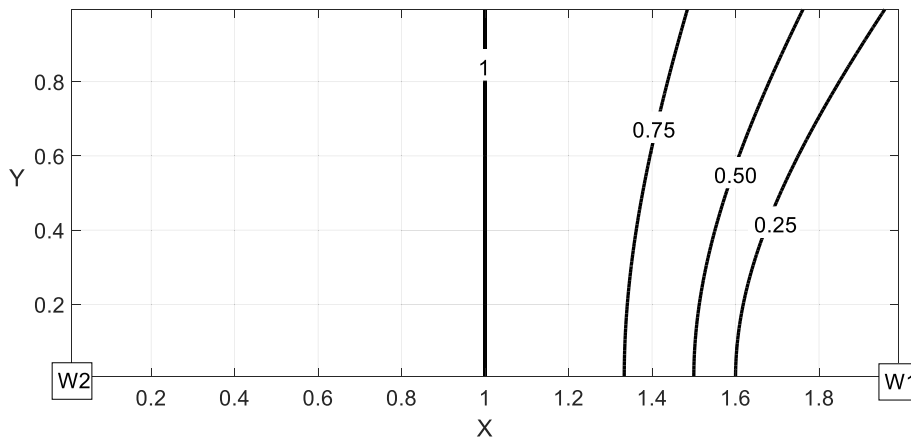
Fig. 4. The boundary of the market area of warehouse 1 (W1) with warehouse 2 (W2) for several ratios α_2/α_1, from 0.25 to 1 and $\alpha_1 = 0.8$, with the rest of the parameters used in Fig. 4.
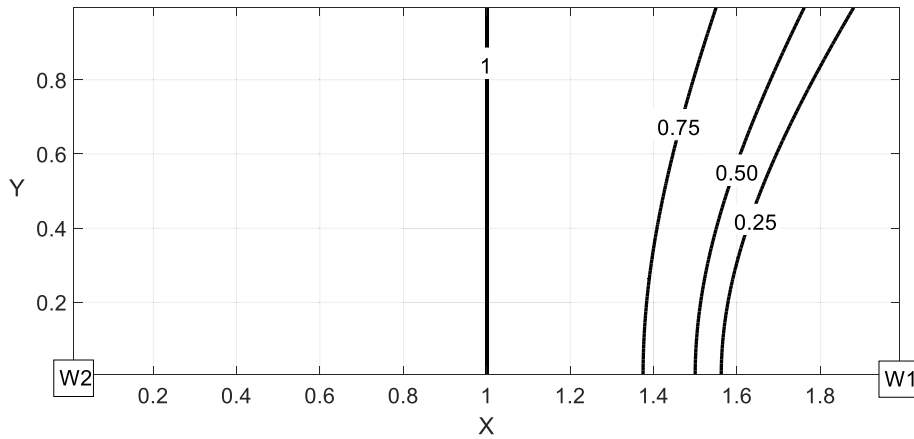
Fig. 5. The boundary of the market area of warehouse 1 (W1) with warehouse 2 (W2). for several p_2/p_1, from 0.25 to 1 and $\alpha_1 = 0.8$, with the rest of the parameters used in Fig. 4.
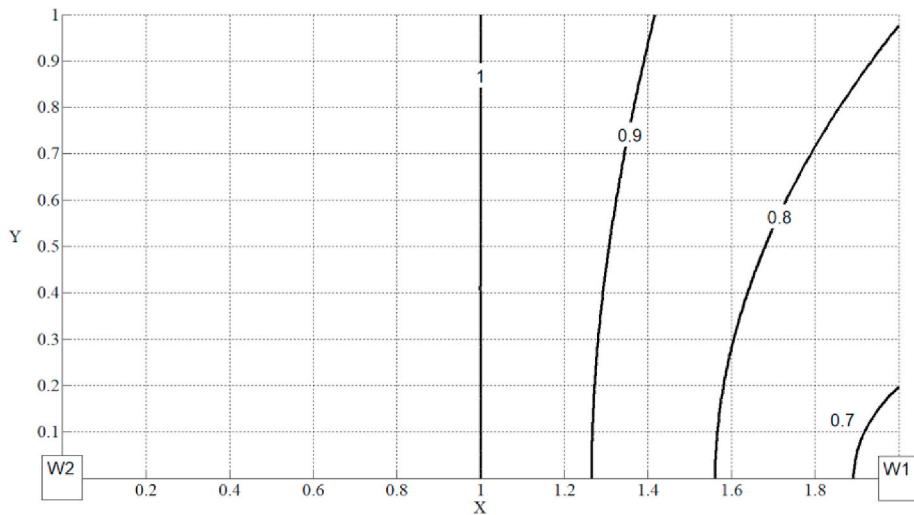

Fig. 6. The boundary of the market area of warehouse 1 (W1) with warehouse 2 (W2) for several λ_2/λ_1, from 0.7 to 1 and $\alpha_1 = 0.8$, with the rest of the parameters used in Fig. 4.

delays described in Fig. 8 when frequency s is equal to the continuous interest rate ρ (Grubbström, 2007) as follows:

$$
\mathbf{H}^\omega \widetilde{\boldsymbol{\tau}}(s=\rho) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1e^{4\rho} & 0 & 0 & 0 & 0 & 0 \\ 2e^{3\rho} & 0 & 0 & 0 & 0 & 0 \\ 0 & 3e^{2\rho} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2e^{3\rho} & 0 & 0 \\ 0 & 0 & 0 & 1e^{1\rho} & 0 & 0 \end{bmatrix} \begin{bmatrix} e^{3\rho} & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{4\rho} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{3\rho} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{2\rho} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{2\rho} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{1\rho} \end{bmatrix} =
$$

$$
= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1e^{7\rho} & 0 & 0 & 0 & 0 & 0 \\ 2e^{6\rho} & 0 & 0 & 0 & 0 & 0 \\ 0 & 3e^{6\rho} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2e^{5\rho} & 0 & 0 \\ 0 & 0 & 0 & 1e^{3\rho} & 0 & 0 \end{bmatrix} \quad (5)
$$

If we start to calculate NPV at the moment when all raw materials, all semi-finished products, and parts for production are ready for production, it will be determined by Eq. (6). Besides, if the prices of the final products in A_1 are equal to those in A_2, the price vector is described for the entire supply network, provided in Fig. 7 as $\mathbf{p} = [560\ 0\ 0\ 0\ 0\ 0]$, and if the transportation costs and delays from the production of B and C to W_1 are equal to those to W_2, the setup cost parameters are identical to

those provided by Bogataj et al. (2016): $\mathbf{K} + \mathbf{\Pi} = [200\ 180\ 210\ 195\ 175\ 215]$, $\widetilde{\mathbf{t}}(s)$ and $\widetilde{\mathbf{T}}(s)$ are also of the same values as in Bogataj et al. (2016) such that for $\widetilde{\mathbf{P}}_0$ realisation $\widetilde{\mathbf{P}}(s)$ and continuous interest rate $\rho = 0.065$, the following is obtained:

$$
\widehat{\mathbf{P}}_0 = \begin{bmatrix} 100 \\ 100 \\ 200 \\ 300 \\ 600 \\ 300 \end{bmatrix}, \widetilde{\mathbf{P}}_0(\rho) = \widetilde{\mathbf{t}}(\rho)\widetilde{\mathbf{T}}(\rho)\widehat{\mathbf{P}}_0 = \begin{bmatrix} 38.4 \\ 54.4 \\ 168.4 \\ 274.5 \\ 849.9 \\ 461.8 \end{bmatrix} \widetilde{\boldsymbol{\nu}}(\rho) = \begin{bmatrix} e^{-\rho t_1}/(1 - e^{-\rho T_1}) \\ \vdots \\ e^{-\rho t_6}/(1 - e^{-\rho T_6}) \end{bmatrix}
$$

$$
= \begin{bmatrix} 0.384 \\ 0.544 \\ 0.842 \\ 0.915 \\ 1.417 \\ 1.539 \end{bmatrix} \quad (6)
$$

Using Eq. (6) for the NPV of production activities, NPV can be calculated when transportation time delays are included as follows:
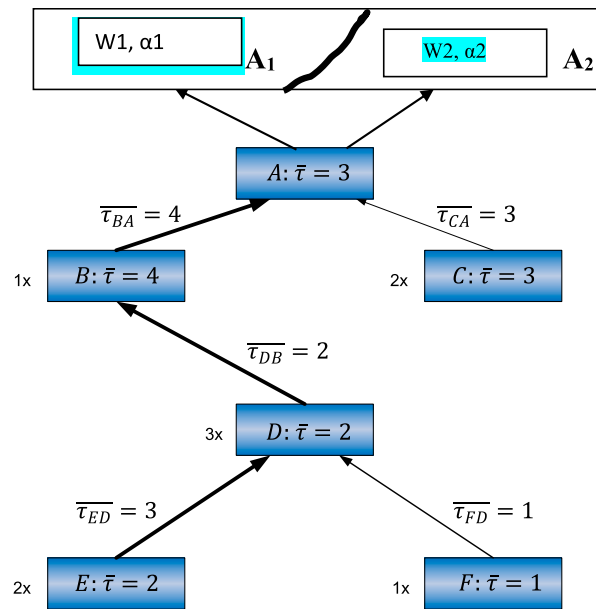
Fig. 7. The supply chain with the final allocation of the semi-products of B and C to the production A close to W_1 and W_2 in the structure so that the shortages of A in W_1 is α_1 and the shortages of A in W_2 is α_2; $\alpha_1 \cdot \alpha_2 \approx 0$
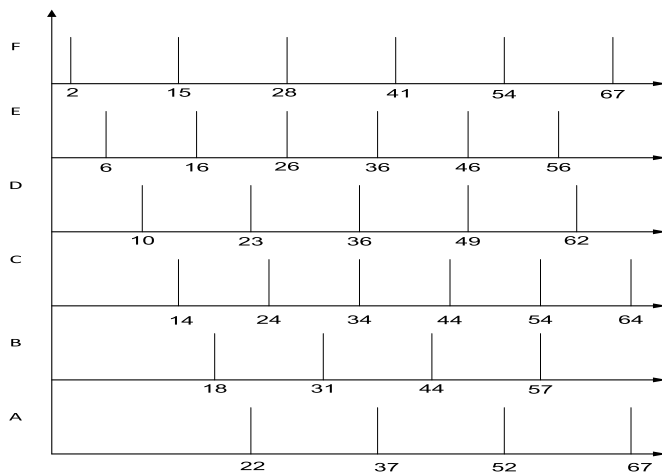
Fig. 8. Production activity cycles used.

Table 5
The basic notation in the MRP theory.

$\widetilde{\mathbf{R}}(s)$	Inventories in the frequency domain
$\widetilde{\mathbf{F}}$ (s(s) delivery	Delivery in frequency space
P	Interest rate
$\mathbf{P}$	Price vector of the prices in individual activity cells of the SC
S	Frequency
$\mathbf{H}$	Generalised input matrix
$\widetilde{\mathbf{H}}$	Generalised delayed transportation-input matrix matrix H, defined in the basic MRP theory.
$\mathbf{K}$	Setup costs
$\mathbf{\Pi}$	Transportation costs
$\widetilde{\tau}$	Matrix of lead times

where $\overrightarrow{\mathbf{X}}$ is $[\mathbf{H}] \cdot \overrightarrow{\mathbf{P}}$ and $\kappa_i \in (\mathbf{K} + \mathbf{\Pi})$ is the setup cost per time unit for the activity i, where the initial moments are distributed on the finite or infinite time horizon and added to the limits of intervals by the NGSPICE

$$NPV = \mathbf{p}\left(\mathbf{I} - \widetilde{\mathbf{H}}^{\omega}(\rho)\right)\widetilde{\mathbf{P}}_0(\rho) - (\mathbf{K} + \mathbf{\Pi})\widetilde{\mathbf{\nu}}(\rho) == \begin{bmatrix} 560 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -e^{7\rho} & 1 & 0 & 0 & 0 & 0 \\ -2e^{6\rho} & 0 & 1 & 0 & 0 & 0 \\ 0 & -3e^{6\rho} & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2e^{5\rho} & 1 & 0 \\ 0 & 0 & 0 & -e^{3\rho} & 0 & 1 \end{bmatrix} \begin{bmatrix} 38.4 \\ 54.4 \\ 168.4 \\ 274.5 \\ 849.9 \\ 461.8 \end{bmatrix} - \quad (7)$$

$\begin{bmatrix} 200 & 180 & 210 & 195 & 175 & 215 \end{bmatrix} = 21,427.20$ monetary units

algorithm. Here, $t_i(u)$ represents the moments of distributed setups on the time horizon for each activity cell as provided in Fig. 8 where the starting moments and the period for the batch production from the BOM (Fig. 7) are shown.

The production cycling is implemented in NGSPICE by a single command. For each activity, this command represents a pulse.

3.3.2. The equivalent version of NPV calculated in the time domain

The equivalent version of NPV in the time domain is as follows:

$$\text{NPV} = \int_0^T e^{-\rho \cdot t} \cdot \overrightarrow{\mathbf{p}}^{\mathrm{T}} \cdot \overrightarrow{\mathbf{P}}_0 \cdot dt - \int_0^T e^{-\rho \cdot t} \cdot \overrightarrow{\mathbf{p}}^{\mathrm{T}} \cdot \overrightarrow{\mathbf{X}} \cdot dt - \sum_{i=1}^N \int_0^T e^{-\rho \cdot t} \cdot \kappa_i \delta_i(t - t_i(u)) \cdot dt$$

$$(8)$$

Table 6
Equivalence between MRP and equations in time domain used for NSM.

NSM at each activity cell – node $j = 1, 2, .., n$	MRP in *nxn* matrix presentation considering all activity cells simultaneously
$\overrightarrow{p}^{T} \cdot ([I] - [H]) \cdot \overrightarrow{P}_0$ $\overrightarrow{P}_0$ is the vector of the production unit, $\overrightarrow{p}$ is a price per unit vector, and **H** is the input matrix	$\widetilde{\mathbf{P}}(s) = \mathbf{I} - \mathbf{H}^{\omega} \cdot \boldsymbol{\tau}\,(s)$ $\widetilde{\mathbf{P}}(s)$ is the production intensity vector in the frequency domain; $\mathbf{H}^{\omega}$ includes transportation delays; $\boldsymbol{\tau}\,(s)$ is the vector of lead times.

4. The NPV IN case of a variation of the interest rate and the length of the horizon – numerical results

The NPV of incomes in the total market area of A$_1$ and A$_2$ on the final stage, calculated in this way, assuming that the market runs on the infinite horizon without perturbations, is 21,427.20 monetary units. In contrast, for the finite time horizon, using the NSM approach, the NPV is lower, especially if the interest rate is lower as in the market of money today.

If the interest rate is only 0.02 and items are on this market only for 200 cycles' long horizon, the NPV will be 2% lower than the results achieved by Eq. (7) using Laplace transforms approach on the infinite horizon. Table 7 provides the required correction factors ψ for some cases of calculation of NPV for production on the finite horizon of different numbers of cycles and various values of the interest rate compared to the infinite horizon.

Our calculation shows that when the product of the interest rate and the number of cycles describing the finite horizon length – $\rho* N_C$ – is higher than 10, the approach with Laplace transforms on the infinite horizon leads to perfect results, but when the product is lower, the correction of NPV using correction factor ψ is advised.

If the interest rate is only 0.03 and items are on this market only for 100 cycles, the same final prices of items in A$_1$ and A$_2$ lead to 0.998* 0.947 = 0.945 of the income, calculated by the MRP model (21,427.20 monetary units), now reduced to 94.5% of it, i.e. to 20,250.975 monetary units. We assumed the probability of shortage of items in A$_1$ as equal to 0.08 ($\alpha_1 = 0.08$) and the probability of shortage of items in A$_2$ as equal to 0.02 ($\alpha_2 = 0.02$), where the market area of W$_1$ is A$_1 = 0,481867$A and the area of W$_2$ is A$_2 = 0,518133$ A. Under assumption (3), the total demand at each unit would create the maximum opportunity for NPV (the monetary unit is assumed as 1000 €) as follows:

$$d_1 : NPV_1 = (0.48186 + 0.02\,0.518133)\,20,250,975\,10^3 € = €9,967.99\,Million$$

$$d_2 : NPV_2 = (0.518133 + 0.08\,0.48186)\,20,250,975\,10^3 € = €11,273.35\,Million$$

but due to shortages in the basic units also, the supply of goods (5), (6) meets this demand, creating the NPV_i^{f}:

$$d_1^{f} : NPV_1^{f} = (0.92\,0.48186 + 0.02\,0.518133)\,20,250,975\,10^3 € = €9,187.34\,Million$$

Table 7
A correction factor ψ for the calculation of NPV for a production on the finite horizon of different numbers of cycles and various values of the interest rate.

Interest rate ρ	Horizon length in number of cycles N$_C$			
	100	200	300	1000
0.01	0.624	0.859	0.947	1.00
0.02	0.858	0.980	0.997	1.00
0.03	0.947	0.997	1.00	1.00
0.04	0.980	1.00	1.00	1.00
0.05	0.992	1.00	1.00	1.00
0.065	0.998	1.00	1.00	1.00

$$d_2^{f} : NPV_2^{f} = (0.98\,0.51833 + 0.08\,0.48186)\,20,250,975\,10^3 € = €11,063.50\,Million$$

5. Discussion

Numerous analyses of the effects of location and prices on the long-term demand for products and profitability in the spatial duopoly can be found in the literature, but the impact of end-user travel costs on demand in these oligopolies or duopolies is not found in articles listed on the Web of Science, except for Bogataj and Bogataj (2001). Grubbström's MRP theory including transforms and analysis in space of complex variables (Laplace transforms) gives excellent support to the analysis of spatial duopoly in which retailers perform their operations on an infinite horizon and where the interest rate considered in NPV valuation is high and stable enough.

From Grubbström's articles (1998, 1999) and articles written together by his students (Grubbström and Ovrin, 1992; Grubbström and Molinder, 1994; Grubbstrom and Tang, 1999), we learned that time delays in supply chain activities affect the profitability of these chains and, thus, the profit or, when viewed over the entire horizon of these chains, the NPV of cash flows that these chains bring. Through the MRP theory, it is possible to observe the time value of cash flows and their volatility in the case of events where flows of goods are delayed. This volatility, however, is most felt by the end sellers. However, the impact of volatility of delays in the chain on the NPV is not the only factor to be observed and taken into account by the end sellers; following factors must be considered, too:

1. These delays create the occasional shortage of goods in the warehouses of retailers operating in spatial oligopolies or duopolies, as in our case. In this way, the delays in supply chains affect the long-term demand of those who know the probability that the product will not be available when they arrive at the store of such a retailer and include the cost of a failed trip to the store in the total purchase price of such a product. This behaviour influences the delineation of market areas in the spatial duopoly, as is explained and calculated in section 3.2, and, consequently, the final demand and profitability of the store.

2. In section 4, we see that the NPV of incomes in the total market area of A1 and A2 on the final stage of a supply chain can be overestimated if it is calculated using the Laplace transform into the space of complex variables, such as in Grubbström (1998,1999) or Bogataj et al. (2016), since it makes it difficult to take into account low interest rate and its volatility over the time horizon and the fact that the time horizons at which spatial duopoly operate can be very short. In this case, the correction factors ψ should be calculated as demonstrated in section 4 and graphically presented for any pair of values of interest rate and length of horizon in Fig. 9.

To know the value of factor ψ is essential for liquidity and solvency management of the SC and retailers, because solvency of each SC node is crucial for solvency of total SC, as considered and proved in Bogataj et al. (2016) and specified for a spatial duopoly by Lambrecht (2001). Therefore, this paper described the use of NSM in the skeleton of a production-supply chain outlined as in the very well developed Grubbström's MRP theory to calculate the NPV of the flows of items in case of variation of the interest rate on the finite horizon, with an uncertain moment of end. We demonstrated how the NSM helps in describing the impact of stochastically shortened length of the time horizon at a low interest rate. The simple approach to calculating NPV using Laplace transforms on the infinite horizon leads to excellent results until the product of the continuous interest rate and the length of the horizon in a number of cycles do not fall under 10. The results influence the expected financial flows of each of the two shopping centres in the duopoly. The shortage of goods allowed in each of them affects the
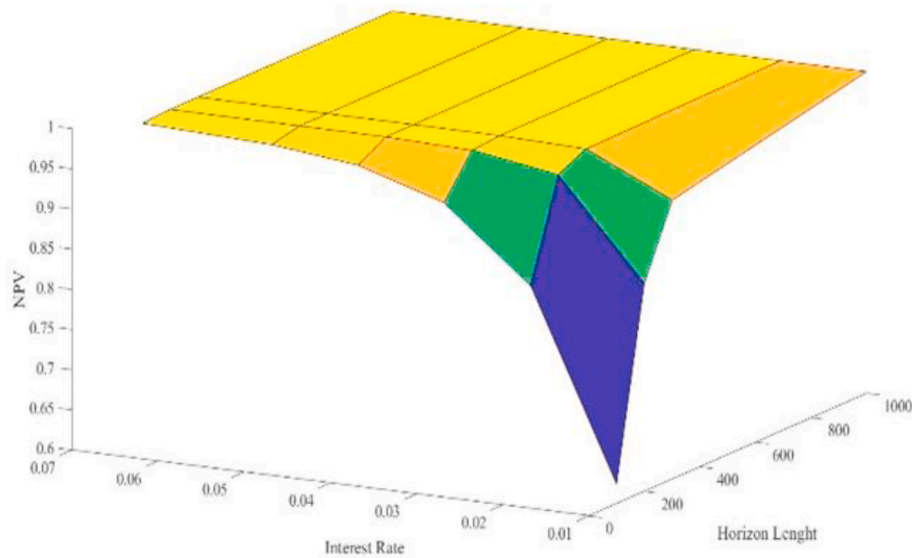
Fig. 9. The reduction of NPV at a final duration of activities and a lower interest rate.

final demand in the equilibrium, and the way in which the income of each market unit is calculated is presented. The NSM enables us to select an arbitrary demand distribution and distribution of shortages accepted by the policymakers in the duopoly. In this way, we obtain a quick answer about the movement of boundaries between market areas and revenues in stochastic supply systems.

Using NSM, we can also successfully study the effects of other parameters' disturbances. The studies can be conducted not only for the infinite horizon but also for the finite horizon where individual perturbations of parameters, including the length of the horizon, and their simultaneous occurrence are even more pronounced.

In our case study, the variation of the time horizon and the interest rate have an impact on the NPV, as shown in Fig. 9. NPV correction factor attains one when the interest rate values exceed 0.04. A similar effect appears with the horizon length above 300 time periods. Thus, lower values of interest rate and initial values of time horizon are decisive due to the high change rate. Consequently, a tool based on the NSM, which provides information about this rate for early values of horizon length, is necessary for making the right decision from NPV correction factor values.

6. Conclusion

Delays of activities in an SC node could influence shortages of goods. Therefore, in the points of delivery of the final products, here available in warehouses at two locations, the duopolies can be affected by these shortages. As presented, the shortages that are different in W1 and W2 could substantially influence the market areas. By investing in the capacities of the SC, such delays can be mitigated. Many authors dealing with production and supply systems, such as Grubbström, suggest evaluating systems by introducing net present value rather than just costs into a criterion function. This approach allows us to weigh between investments in a more reliable supply and the profits brought by such investments. When the NPV of activities and profit achieved in each warehouse are more properly predicted, the managers can better decide whether to invest or not in higher capacities and lower probabilities of shortages. The managers can weigh between the acceptability of non-stock and thus reduce sales or invest in higher capacity and higher stock and hence expand the market area. Nevertheless, sales can take place only on a limited, finite horizon. Short time horizons are a particular characteristic of supply systems with the rapid development

of new technologies and products as witnessed in recent decades.

Furthermore, the interest rate could vary and sometimes be very low. In such cases, the results obtained in the article that describes NPV calculation using the MRP approach should be reconsidered under these variations and constraints. Therefore, in the time of rapid technological changes and decreasing interest rate, in the results obtained with the MRP method, it makes sense to enter correction factors and simulate their variation in the case of an uncertain and volatile environment. All this is enabled by the NSM and the NGSPICE tools.

On increasing the time horizon and the interest rate, the results achieved by using NSM become the same as the results using Laplace transforms in MRP theory, as is evident from Table 3 of the numerical examples. This fact can further convince a reader that this modelling technique leads to reliable and useful results. Therefore, we can see that MRP theory is perfect and gives the results efficiently if the ratio between the time horizon and the interest rate is high enough, as required by Grubbström in his articles. But as mentioned in the abstract, such estimation is not good enough when the interest rate is extremely low and varying, which is the case in today's financial flows. Additionally, the overestimates are too high due to premature obsolescence of products in the case of high frequency of appearance of new, innovative products on the market. These were also the reasons and motivations for writing our article. We see that in this case the NSM which had been better known in engineering also found "fertile ground" in the field of production economics.

Regarding further study on this topic, we should know that same as offline markets in the spatial duopoly, the online market also includes some spatial attributes, significantly influencing NPV and profit of retailers (Ma and Huo, 2020). In this context, the question appears as how the offline market modifies in the case of combination of offline and online purchasing in a spatial duopoly. Wan et al. (2020) analysed the positions of online to offline platforms, enabling consumers to search for information and purchase products online. Therefore, the study of duopolies should also be extended to address the questions of how the partial electronic search for product availability, which differs between customers in A1 and A2, influences the final design of market areas and how to introduce such platforms in further research. The approach described in our paper can also provide us with the answers in a relatively simple way, using Eq. (2) where the percentage of customers who use the electronic search for product availability in each area is known and introduced in this formula.

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