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Risk-averse supplier selection and order allocation in the centralized supply chains under disruption risks

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ABSTRACT

This paper proposes a mixed-integer non-linear programming (MINLP) model for the integrated supplier selection and order allocation in a centralized supply chain considering the disruption risks and a risk-averse decision-maker. In order to capture a realistic scenario of considering the geographical characteristics of the suppliers, we assume that the suppliers belong to two regions: the buyer's region (domestic suppliers) and outside of the buyer's region (foreign suppliers). Considering this realistic feature, the supply chain might face two types of disruption risk: *first,* local disruption risks which might uniquely occur inside each supplier such as equipment breakdowns, and *second,* regional disruption risks that might occur in the region of the suppliers located in the same geographical region such as natural hazards. We formulate the problem considering a risk-neutral decision-maker as a benchmark, and then a risk-averse model is presented. In the latter case, we apply two types of risk assessment tools introduced in the finance literature to analyze the decision maker's behavior: value-at-risk (VaR) and conditional value-at-risk (CVaR). We show that developed models are non-convex programming, and therefore, we apply the particle swarm optimization (PSO) algorithm as the solution approach. We also compare the developed PSO algorithm with the Genetic algorithm (GA) and the commercial GAMS solver to verify the efficiency of the solution method. The computational experiments indicate the impact of the decision maker's attitude on the supplier selection and the order quantity.

1. Introduction

Companies are interested in optimizing the whole supply chain instead of only maximizing their own profit to achieve competitive advantages. The centralized supply chain has several advantages, such as efficiency improvement and mitigation of the bullwhip effect, and is valuable for managing complex problems, in particular, considering the uncertain environment (Giannoccaro, 2018). In this policy, all members of the supply chain cooperate closely to optimize the integrated supply chain. Thus, full coordination creates between the buyer and suppliers by a single decision-maker who has decision-making power for the total supply chain. In addition, selecting efficient suppliers help the firms significantly to supply the right amount and price of the products. Therefore, the supplier selection and ordering process are introduced as an essential part of the supply chain in the modern production space (Kuo et al., 2010).

On the other hand, the increase in the complexity of the supply chain has led to the occurrence of different types of risks. Supply chain risks are multi-dimensional and can be categorized into two main groups: (a) **operational risks,** which are related to everyday processes in the supply chain operations such as uncertainty in the supply, demand, costs, and lead time and (b) **disruption risks** which are concerned to the low probability and high impact of the disruptions in the supply chain (Govindan et al., 2017; Ivanov, 2020; Ivanov et al., 2018; Tang, 2006; Xu et al., 2020). The latter risks are created by either unpredictable natural hazards or man-made accidents, such as earthquakes, floods, terroristic attacks, labor strikes, and economic crises. For example, a volcanic eruption in Iceland in April 2010 resulted in a shutdown of the factories that supplied the main components of their products from Europe. The BBC news announced that NISSAN wouldn't manufacture three models of its products anymore, and BMW reduced its production in Germany due to the disruption (He et al., 2015). As another example, the March 2011 earthquake in the eastern part of Japan and the October 2011 catastrophic flood in Thailand disrupted the supply chain of the

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electronic industries. The occurrence of these events in a region where many production units are concentrated imposed significant losses on the Japanese companies (Fuller, 2012; Park et al., 2013; Sawik, 2014). Thus, the disruption risks have turned out to be a major challenge for the supply chains, and therefore, supply chain risk management is a fundamental decision for the managers nowadays. Literature studies have developed different mitigation and recovery strategies to reduce the impact of disruption risks. In this paper, we adopt the sourcing strategy by diversifying the orders from the suppliers in different geographical regions to mitigate the supply risks. We note that selecting suppliers from different geographical regions have been considered as a practical strategy among many automotive manufacturing companies after the Japanese Tsunami and earthquakes in 2011 (Hosseini et al., 2019).

According to utility theory, and Its applications in the field of financial risk management, the risk attitude of the decision-maker determines the risk. The subjective realization of the important risk can be divided into three categories: risk-neutral, risk-averse, and risk-seeking. Managers' decision-making processes have entirely dependent on their attitudes of risk (Heckmann et al., 2015). In general, risk (or supply chain risk) is considered as a subjective concept rather than an objective concept, and it relies on the individual's evaluation of potential outcomes (Ellis et al., 2010). Thus, risk attitudes and individual or organization priorities have a critical and major impression on the measurement of future supply chain performance and also co-determine supply chain decisions (Heckmann et al., 2015). Surveying the questionnaires from the executive managers of international companies indicate that the decision-makers are always cautious regarding the risk in the real world (Xinsheng et al., 2015). Therefore, they prefer the risk-avarness attitude toward the risk as they cannot provide financial resources to confront potential losses (Chahar & Taaffe, 2009). In addition, due to the profound impact of managers' behavioral attributes on the decision-making process, risk-averseness has received much more attention from many researchers in recent years. In this study, we use the VaR and CVaR concepts introduced by Rockafellar et al. (2000) that have been used widely in the finance literature to assess the risk. The same unit measure with the underlying random variable is considered as one of the most important advantages of these variables over the other risk measures. In addition, VaR and CVaR have a moderately simple function that decision-makers can apply in real-world decision making and scenario-based problems (Gönsch, 2017).

In this paper, we mainly focus on the supplier disruptions in the supply chains. We investigate a two-echelon supply chain, including the multiple suppliers delivering multiple products to a single buyer. In such situations, it is important that the buyer develop efficient plans for ordering policy during disruptions. In this paper, we investigate the optimal suppliers' and buyer's reactions to the supply disruption. More specifically, we study the optimal decisions regarding the supplier selections and ordering policy of the buyer from the suppliers facing two types of disruptions. Similar to Esmaeili-Najafabadi et al., (2019), the portfolio approach by optimizing the values of VaR and CVaR is enhanced to integrate the decision maker's attitude and the supplier selection problem. In the CVaR measure, we evaluate the cost of the worst-case scenario, and based on the evaluated cost distribution, we select the appropriate supplier and decide the optimal ordering policy. We assume a realistic scenario that the suppliers are located in two different geographical regions: in the buyer's region (domestic suppliers) and outside the buyer's region (foreign suppliers). Thus, the suppliers are involved in two types of random disruptions: local and regional disruption risks. The former might occurs inside any domestic or foreign supplier (such as production line breakdown), and the latter is the result of disruption to all suppliers in the same geographical region (such as natural hazards). The decision-maker selects the optimal suppliers from each region and decides the ordering policy from the selected suppliers. The objective is to minimize the total costs of the suppliers and buyer in the supply chain. We develop a particle swarm optimization (PSO) algorithm as the proposed model is a mixed-integer nonlinear and non-convex programming model, and it is not possible to solve efficiently by the commercial solvers and exact solution methods. We compare the developed solution algorithm with the Genetic algorithm (GA) and GAMS commercial solver to evaluate the efficiency of the PSO algorithm.

The rest of the paper is organized as follows. The next section provides a review of the relevant literature. In section 3, we present the model assumptions as well as the risk-neutral and risk-averse models. The proposed Genetic and particle swarm optimization algorithms are explained in section 4. In Section 5, numerical experiments are carried out to evaluate the models and the solution approach. The performance of the developed algorithms is presented in Section 6. Finally, we offer our conclusions and future research directions in Section 7.

2. Literature review and contributions

The literature related to this study is included in four main areas: supplier selection methods, coordination mechanism, risks of the supply chain, and how to manage and control the potential supply risks. Therefore, we review these issues, specifically those papers which are essential in building up our model.

Recently, the supplier selection and order allocation problem has attracted significant attention among academic and industrial researchers. Thus, a comprehensive effort has been to develop decision techniques and methods for this field. Weber et al. (1991) classified and reviewed about 74 articles in this area between 1966 and 1990 and focused on the analytical methods in supplier selection. Aissaoui et al. (2007) proposed that the supplier selection problem can be categorized into single-sourcing and multiple-sourcing models. In single-sourcing models, one supplier can meet all the demands, and there is only one decision: which supplier is the best? (Chen & Xiao, 2015; Golmohammadi & Mellat-Parast, 2012; Tavana et al., 2016). In multiple-sourcing models, there is a capacity constraint, and one supplier alone is not able to satisfy all the demands. The main decisions are to select the best suppliers and allocate the quantity of orders from each supplier (Alfares & Turnadi, 2018; Kamalahmadi & Parast, 2017; Sawik, 2014). In the supplier selection literature, a variety of methodologies and approaches are used for categorizing methods to solve the supplier selection problem. In one of the important classifieds, models broadly are categorized, two groups (Firouz et al., 2017; Ware et al., 2014): 1) Quantitative models: These models are related to select the best portfolio of suppliers and optimally allocate the buyer's total demand among selected suppliers to meet different purchasing criteria. These category included linear programming (Anthony & Buffa, 1977; Pan, 1989), mixed-integer linear programming (Basnet & Leung, 2005; Chaudhry et al., 1993; Demirtas & Üstün, 2008; Hosseini et al., 2019), mixed-integer non-linear programming (Esmaeili-Najafabadi et al., 2019; Hu et al., 2018; Kamali et al., 2011; Keskin et al., 2010; Mendoza & Ventura, 2010; Rezaei & Davoodi, 2011; Ware et al., 2014), goal programming (Buffa & Jackson, 1983), stochastic programming (Hammami et al., 2014; Kara, 2011; Sawik, 2015, 2017, 2019), dynamic programming (Masella & Rangone, 2000; Mendoza et al., 2008), and multi-objective programming (Demirtas & Üstün, 2008; Kamali et al., 2011; Li et al., 2016; Sodenkamp et al., 2016). and 2) Qualitative models: These models involve the analytical hierarchical process (AHP) (Mendoza et al., 2008), Fuzzy-AHP (Önüt et al., 2009), weighted point method (Hu et al., 2018), analytical network process (ANP), and vendor profile analysis (Thompson, 1990), matrix approach (Gregory, 1986), vendor performance matrix approach (Soukup, 1987), TOPSIS and Fuzzy-TOPSIS (Gupta & Barua, 2017; Önüt et al., 2009), Analytical Network Process (ANP) (Bayazit, 2006; Demirtas & Ustun, 2009; Gencer & Gürpinar, 2007; Hsu & Hu, 2009; Mendoza & Ventura, 2012). As mentioned, MINLP models are a common approach in the literature SS&OA problem. We applied the mathematical programming model. Since the selection problems are recognized as an assignment problem, those have the nature of integer. Also, the allocated quantity of order has a continuous nature. In addition, we used the economic order quantity (EOQ) to estimate the order quantities of each supplier. Therefore, in the final, our model became an MINLP model.

Moreover, in the supplier selection literature, the coordination between supply chain members has been neglected. Buyer and Supplier coordination is defined as an operational plan to coordinate the operations of buyer and suppliers that results in improving the system profit. Li and Wang (2007) reviewed the coordination mechanisms in the supply chain based on decision structures (centralized and decentralized supply chain) and the nature of demand. They concluded that most existing models in this area of literature have considered only single-supplier and single-buyer or single-supplier and multiple-buyer cases. Most of the works on the supplier selection focus on one product and one machine and thereby fail to capture the essence of the real supply chain. In this study, we apply the multi-sourcing strategy and consider the capacity constraint of each supplier in the centralized supply chain producing multiple products.

Supplier selection and order allocation problem is considered as a multi-criteria decision problem in the literature that involves both tangible and intangible criteria (Ho et al., 2010). Literature studies have proposed several supplier selection criteria, such as the greening criteria (Banaeian et al., 2018; Gao et al., 2020; Haeri & Rezaei, 2019; Liu et al., 2019; Phuong Nha Le et al., 2011), resiliency criteria (Davoudabadi et al., 2020; Hassan et al., 2019; Hosseini et al., 2019; Hosseini & Barker, 2016; Sawik, 2013), sustainability criteria (Bai & Sarkis, 2010; Jain & Singh, 2020; Stević et al., 2020; Tirkolaee et al., 2020; Vahidi et al., 2018), and the supply chain risks criteria (Cheraghalipour & Farsad, 2018; Esmaeili-Najafabadi et al., 2019; Pariazar & Sir, 2018; PrasannaVenkatesan & Goh, 2016; Sawik, 2014).

The question of how we can manage and control the potential supply risks has become a vital decision for the companies due to the increased complexity and interactions within the new supply chains as well as the considerable financial losses caused by the disruptions. In addition, the research on supplier selection under disruption risks is limited due to the complexity of the resulted optimization problem (Sawik, 2014). Based on the number of supply chain members involved in a disruption, disruption risks can be classified into local, semi-global (regional), and global disruption. **Local disruption** risks might uniquely occur inside each facility and include only a particular supplier, such as equipment breakdowns. **Semi-global (regional) disruption** risks might happen in the region of the facilities located in the same geographical region. Therefore, all the same, region suppliers would be unavailable simultaneously due to the disruption in their region, such as natural hazards. **Global disruption** risks might result in all facilities disruption simultaneously. Thus, all suppliers in the supply chain become unavailable simultaneously, such as pandemic disease COVID-19. Papers in this area have been reviewed by Hamdi et al. (2018). Tomlin (2006) investigated a single-product supply chain with one buyer and two suppliers; one is unreliable, and the other is reliable but more expensive. Both suppliers have limited capacities; however, a reliable supplier has a flexible capacity. This paper introduced the inventory reduction, multi-sourcing, and acceptance policies to cope with the disruption risks. Yu et al. (2009) studied a two-stage supply chain with two types of non-stationary and price-sensitive demand. They evaluated the effects of supply disruption risks on the single-sourcing and multi-sourcing decisions. They proposed that both sourcing methods can be useful in the occurrence of supply chain disruptions. Schmitt and Singh (2012) focused on the strategies to mitigate the impact of disruption risks. They suggested that considering the quantitative measures of risks enables the managers to make more conscious decisions regarding hazardous conditions. Sawik (2014) proposed a stochastic MINLP model for the supplier selection and customer order scheduling problem under disruption risks. Kamalahmadi and Parast (2017) considered three policies of pre-positioning inventory, backup, and protected suppliers to manage the supply and environmental risks. The authors presented a two-stage mixed-integer programming model to determine the best sourcing strategies by minimizing the total expected cost as the objective function. Hosseini et al. (2019) developed a bi-objective MINLP model for the supplier selection and order allocation problem under disruption risks. They assessed the several reactive and proactive resilience strategies (such as surplus inventory, supplier segregation, supplier reliability, backup supplier contracts, and supplier restoration capability) to manage the disruption risks. Esmaeili-Najafabadi et al. (2019) considered a centralized two-echelon supply chain with one buyer and multiple suppliers. They applied the pre-positioning inventory and protected supplier policies to mitigate the impact of disruption risks. However, in their model, the suppliers deliver the products to the buyer simultaneously, and the decision-maker is risk-neutral. Kaur and Singh (2021, p.) proposed an integrated supplier segmentation and order allocation under disruption risks and disruptive technologies by a multi-stage hybrid model. They used Data Envelopment Analysis (DEA) to evaluate suppliers based on a set of criteria suitable in the industry 4.0 environment and then prioritized suppliers by AHP-TOPSIS. (Alejo-Reyes et al., 2021) presented an MINLP model for the supplier selection and order allocation problem. Due to the complexity of their model, they proposed a new heuristic method. PSO and DE algorithms have been used to evaluate the efficiency of their proposed heuristic.

On the other hand, a McKinsey research report by Koller et al. (2012) surveyed about 1500 executives from 90 countries. The report showed that the decision-makers demonstrate extreme levels of risk aversion in their business processes regardless of the investment size. Therefore, many researchers have recently formulated the supply chains with risk-averse decision-maker (Chahar & Taaffe, 2009; He et al., 2017; Merzifonluoglu, 2015). Gönsch (2017) reviewed different techniques in the finance literature to quantitatively measure the risk, such as: 1) Value-at-risk (VaR) and Conditional value-at-risk (CVaR) (e.g., Chahar & Taaffe, 2009; Madadi et al., 2014; Sawik, 2019). Chahar and Taaffe (2009) considered a company with consecutive quarterly sales and product delivery to different markets with uncertain demand. They applied a profit maximization method based on the newsvendor model and used the concept of CVaR for risk assessment. Madadi et al. (2014) proposed a supply chain design problem for the pharmaceutical industry to manage unreliable capacity. They investigated both risk-neutrality and risk-averseness (CVaR measure) policies. 2) Exponential utility function (e.g., Gönsch, 2017; Sayın et al., 2014; Shu et al., 2015). Sayın et al. (2014) investigated a single-product and single-period inventory problem considering a risk-averse decision-maker. They applied the utility function concept to formulate the risk consequences of the random supply and demand. Shu et al. (2015) investigated an inventory/purchasing strategy in the presence of supply and demand uncertainties. They used an incremental and concave utility function to model the risk-averseness attitude of the retailer. 3) Mean-variance Analysis (Li et al., 2018; Ray & Jenamani, 2016; Xue et al., 2016). (Xue et al., 2016) proposed various strategies for a risk-sensitive manufacturer with unreliable suppliers and used the Markowitz mean–variance concept for the risk assessment. (Q. Li et al., 2018) proposed a two-stage supply chain in which the risk-neutral manufacturer supplies the required products from the point market and sells the final product to the risk-averse retailer. 4) Hybrid techniques (Mahmutoğulları et al., 2018; Ravindran et al., 2010). (Ravindran et al., 2010) proposed a multi-objective method for the risky supplier selection problem. They combined different measures of risk and proposed a price-flexible contract for the manufacturer and retailer. They also applied the mean–variance concept for controlling the retailer's risk-averseness. Based on the studies in the literature, Table 1 shows the different model specifications used in the literature.

Based on the literature analysis, we identify the following research gaps. *First*, the literature studies show that most of the papers have studied the supplier selection and order allocation problem considering the risk-neutral decision-maker. However, as previously mentioned, the lack of research on the impact of the decision maker's behavior on the

Table 1
Summary of the models considering supplier selection and order allocation under disruption risks.

Paper	Multi-Product Problem	Buyer and Supplier Coordination	Product Delivery Strategy			Disruption Risk	Type of disruption risk			Sourcing Strategy			Risk Analysis Technique				Objective Function			Solution Method
			Lumpy Delivery	*Phased Delivery*	*Other delivery policies*		*Local*	*Regional*	*Global*	*SS*	*MSO*	*MSGS*	*VaR*	*CVaR*	*Mean-Variance*	*Utility Function*	*Cost*	*Profit*	*Multi-Objective*	
(Banerjee, 1986)										**							**			Closed-form solution
(Tomlin, 2006)				*		*	*			*	*				**		**	*		Closed-form solution
(Yu et al., 2009)						*	*					*						*		Closed-form solution
(Glock, 2011)		*			*						*						*			Heuristic Algorithm
(Giri, 2011)						*	*			*						*	*			Closed-form solution
(Kamali et al., 2011)		*		*							*								*	Meta-Heuristic Algorithm
(Sawik, 2013)	*					*	*		*		*		*	*			*			Commerical CPLEX solver
(Aliabadi et al., 2013)	*	*		*							*						*			Genetic Algorithm
(Sawik, 2014)	*					*	*	*				*	*	*			*			Commerical CPLEX solver
(Mohammaditabar et al., 2016)		*	*								*						*			Game Theory
(Hosseini & Barker, 2016)	*					*	*				*	*					*			Bayesian Network
(Kamalahmadi & Parast, 2017)						*		*	*			*					*			Commerical CPLEX solver
(Vahidi et al., 2018)	*					*	*				*						*			Hybrid SWOT-QFD
(Alfares & Turnadi, 2018)	*		*								*						*			Genetic Algorithm
(Hosseini et al., 2019)						*	*						*						*	ε-Constraint Method/ Fuzzy c-Mean Clustering Algorithm
(Moheb-Alizadeh & Handfield, 2019)		*									*								*	ε-Constraint Method /Benders Decomposition Algorithm
(Esmaeili-Najafabadi et al., 2019)	*	*	*			*	*				*						*			Commerical CPLEX solver
(Tirkolaee et al., 2020)	*										*								*	Fuzzy DEMATEL/fuzzy ANP/ fuzzy TOPSIS
(Jia et al., 2020)											*								*	Goal Programming
(Kaur & Singh, 2021)	*					*					*						*			FAHP-TOPSIS/ DEA
(Alejo-Reyes et al., 2021)	*										*						*			heuristic method
This study	*	*		*		*	*	*				*	*	*			*			Genetic Algorithm/ Particle swarm optimization

SS: Single-Sourcing, MSO: Multiple Sourcing, MSGS: Multiple Sourcing considering the geographical specifications of the suppliers, etc. Solution Method, Hybrid SWOT-QFD: Strengths, Weaknesses, Opportunities and Threats analysis (so-called SWOT analysis) and Quality Function Deployment (QFD), Fuzzy DEMATEL: Fuzzy Decision-Making Trial and Evaluation Laboratory, Fuzzy ANP: Fuzzy Analytic Network Process, Fuzzy TOPSIS: Fuzzy Technique for Order of Preference by Similarity to the Ideal Solution.

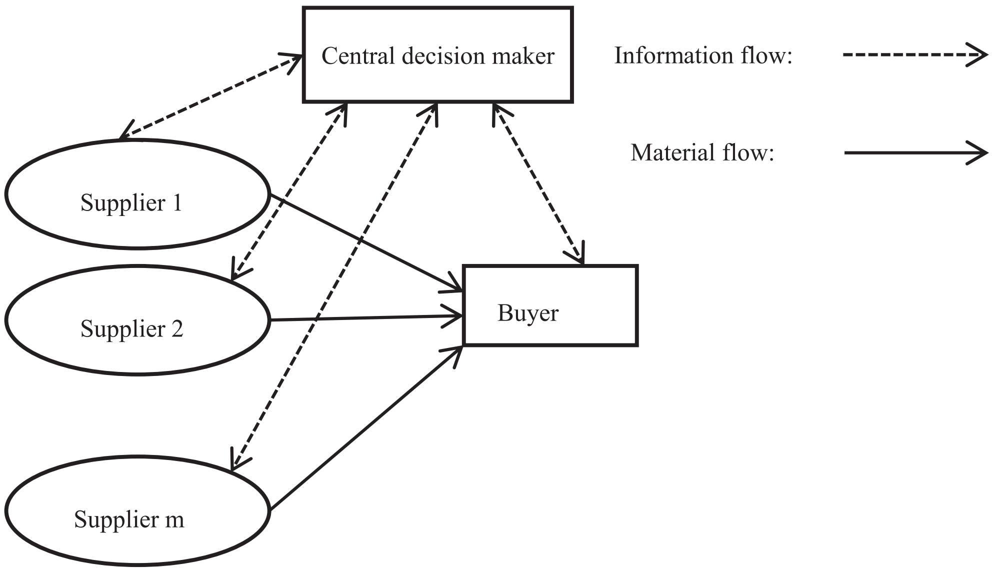

Fig. 1. A single-product centralized supply chain with one buyer and multi suppliers (Gheidar Kheljani, Ghodsypour, & O'Brien, 2009).

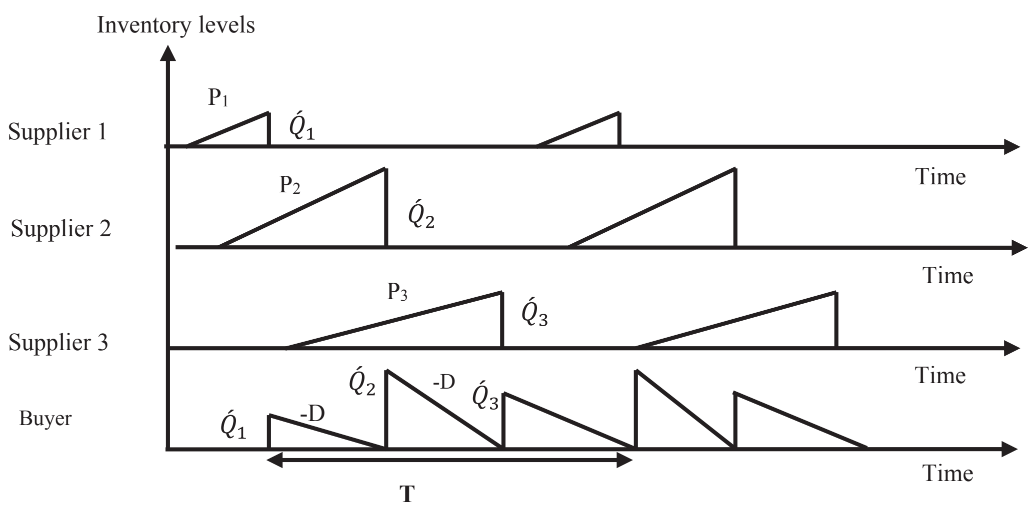

Fig. 2. Inventory levels for a single buyer, multi suppliers and one product (Kamali et al., 2011).

optimal decisions in this area is still under investigation. This paper fills this research gap by analyzing a portfolio selection problem considering the CVaR measure to formulate the total supply chain objective function. In this study, we develop an economic order quantity (EOQ) in a two-echelon centralized supply chain under disruption risks. First, we define the risk-neutral model as a benchmark in which the central decision-maker is indifferent towards the risk. Then, the VaR and CVaR concepts are applied to formulate the risk-averness of the decision-maker. *Second*, the strategy of locating suppliers in different geographical regions is recognized as a practical strategy in the real world to mitigate the impact of disruption risks. In this paper, we assume that the suppliers are located in two geographical regions. Considering this feature, we propose two types of disruption risks in the suppliers: *local disruption,* which might uniquely occur inside each supplier such as equipment breakdowns and *regional disruption* which might occur in the region of the suppliers located in the same geographical region such as natural hazards. *Third*, although the supplier selection and order allocation problem has been studied from different perspectives in the literature, few studies have considered the structure of commodity delivery (e.g., see (Beck et al., 2017; Gheidar Kheljani et al., 2009; Glock, 2011; Kim & Goyal, 2009). In the literature, there are different approaches for the structure of commodity delivery such as: Lumpy delivery: all suppliers ship their lots at the same time to the buyer, Phased delivery: the buyer is supplied from only one supplier whenever its inventory level reaches zero, Other delivery **policies**: such as overlapping delivery cycle, unlimited delivery. In this paper, we assume that the buyer does not receive the $(i+1)^{th}$ supplier's order quantity until all the supplier's order quantities are consumed, such as the phased delivery concept presented by (Kamali et al., 2011). In this delivery policy, the majority of the inventory is held on the suppliers, and this policy can be useful when the buyer's inventory cost is more expensive than the suppliers' inventory cost.

3. Problem description

In this section, we will first list the notations followed by an outline of the assumptions, and then provide the problem statement for risk-neutral and risk-averse decision-maker models.

3.1. Notations

Indices:

i: Index for suppliers $\{1,2,\cdots,n\}$, (I_s: set of non-disrupted suppliers under scenario *s*)

s: Index for disruption scenarios $\{1,2,\cdots,2^n\}$

j: Index for product types $\{1,2,\cdots,m\}$

r : Index for geographical regions $\{1,2\}$

Parameters:

A_{ij}: Fixed ordering cost of the j^{th} product type for the i^{th} supplier

S_{ij}: Setup cost of the j^{th} product type for the i^{th} supplier

C_{ij}: Production cost of the j^{th} product type for the i^{th} supplier

Cap_i: Capacity of the i^{th} supplier

h_i^v: Holding cost per unit of the product for the i^{th} supplier

h_i^b: Buyer's holding cost per unit of the purchased product from the i^{th} supplier

D_j: Market demand for the j^{th} product type

P_j: The selling price of the j^{th} product type in the market

ω_{ij}: Wholesale price of the j^{th} product for the i^{th} supplier

α_i: Probability of the local disruption in the i^{th} supplier

α_r^* : Probability of the regional disruption in the region *r*

β_s: Probability of the s^{th} disruption scenario considering both local and regional disruption risks

B_j: Shortage cost per unit of the j^{th} product type

Decision variables:

τ_s: Tail cost for disruption scenario s^{th}

Q_j: Order quantity of the j^{th} product type $Q_j = \sum_{i\in I} Y_{ij}$

$\hat{Q}_i$: Order quantity from the i^{th} supplier $\hat{Q}_i = \sum_{j\in J} Y_{ij}$

X_i: 1 if the i^{th} supplier is selected, otherwise 0

Y_{ij}: Fraction of the demand for the j^{th} product type that is ordered from the i^{th} supplier

U_j^s: Unfulfilled demand of the j^{th} product type in the disruption scenario s^{th}

VaR: Value-at-Risk value

CVaR: Conditional Value-at-Risk value

In addition, $\boldsymbol{X}^T$ represents the transpose of the matrix $\boldsymbol{X}$, $\otimes$ indicates the Kronecker product (multiplication), * stands for the Hadamard product, and $\oslash$ represents the Hadamard division between the matrices.

3.2. Assumptions

This subsection describes the assumptions of an integrated supplier selection and order quantity allocation problem under supply chain disruption risks. We study a multi-product and two-echelon centralized supply chain with one buyer and multiple suppliers. The buyer can purchase multiple products from multiple suppliers. Fig. 1 shows the structure of the centralized supply chain. In the centralized supply chain, the central decision-maker who has all the information about the supply chain selects the suppliers and allocates the orders to meet the customers' demand under disruption risks. We assume that suppliers have limited capacities, and the customers' demand is deterministic. The policy of commodity delivery from the supplier to the buyer is the phased delivery policy concept that was presented by (Kim & Goyal, 2009).

Similar to (Kim & Goyal, 2009), we assume that in each period, the buyer will not receive the *(i + 1)*th supplier's order as long as the i^{th} supplier's order is not consumed. In this policy, the economic production quantity of the supplier is constant, and that is equal to the economic order quantity of the buyer. Fig. 2 depicts this assumption for a supply chain with one buyer, three suppliers, and a single product.

We also consider two types of suppliers in order to incorporate their geographical characteristics as follows: *domestic suppliers*, which are located in the buyer's region, and *foreign suppliers*, which are located outside of the buyer's region. We assume that the domestic suppliers and buyer are located in a region that is prone to disruptions (ex. due to geographical characteristics or less available technology), and as a result, the buyer seeks to consider the option to procure the products from the suppliers which are located outside of the buyer's region (foreign supplier). On the other hand, foreign suppliers are more reliable but more expensive compared to domestic ones. In addition, we consider a comprehensive scenario for the supplier disruption risks considering their geographical specifications. The first type of disruption risk is the *local disruption* that might happen inside each supplier. Some of the examples are lack of materials, labor strikes, equipment breakdowns, etc. Another type is *regional disruption*, in which all the suppliers in the same region would be unavailable simultaneously due to the disruption in their region. Some examples are earthquakes, floods, hurricanes, etc. Although the probability of such incidents is lower than the local disruption, the negative impact is higher on the supply chain (Ray & Jenamani, 2016; Snyder et al., 2012). We assume that α_i is the probability of the local disruption for the i^{th} supplier, meaning that the supplier *i* can not fulfill the buyer's order due to the disruption with the probability of α_i. As previously mentioned, we assume that domestic suppliers are more prone to local disruption because of economic instability, available technology, and geographical specifications. Thus, the probability of local disruption in domestic suppliers, $i \in I^1$, are higher than that in the foreign suppliers, $i \in I^2$. On the other hand, foreign suppliers are more expensive.

We define α_r^* as the probability of the regional disruption in which all suppliers located in the same region are not available simultaneously due to this type of disruption. For analytical tractability, we assume that regional and local disruption risks are independent. We define β_s as the probability of disruption scenario *s* in which each scenario ($s \in S$) includes a subset of the supplies, i.e., $I_s \subseteq I$, that are non-disrupted and can satisfy the buyer's order. In addition, the number of disruption scenarios depends on the number of suppliers, and it is equal to 2^n. We calculate the probability of each disruption scenario *s* as follows. Please note that I^1 and I^2 are the set of domestic and foreign suppliers, respectively.

$$\beta_s = \begin{cases} \alpha_1^*\alpha_2^* + \alpha_1^*(1-\alpha_2^*)\prod_{i\in I^2}\alpha_i + (1-\alpha_1^*)\alpha_2^*\prod_{i\in I^1}\alpha_i + (1-\alpha_1^*)(1-\alpha_2^*)\prod_{i\in I}\alpha_i \, if I_s = \emptyset \\ (1-\alpha_1^*)\alpha_2^*\prod_{i\in I_s}(1-\alpha_i)\prod_{i\in I^1\setminus I_s}\alpha_i + (1-\alpha_1^*)(1-\alpha_2^*)\eta_s \, if I_s \subseteq I^1 \\ \alpha_1^*(1-\alpha_2^*)\prod_{i\in I_s}(1-\alpha_i)\prod_{i\in I^2\setminus I_s}\alpha_i + (1-\alpha_1^*)(1-\alpha_2^*)\eta_s \, if I_s \subseteq I^2 \\ (1-\alpha_1^*)(1-\alpha_2^*)\eta_s \, if I_s\cap I^1 \neq \emptyset, I_s \cap I^2 \neq \emptyset \end{cases} \tag{1}$$

where η_s is the probability of local disruption for the suppliers under disruption scenario *s* as follows:

$$\eta_s = \prod_{i\in I_s}(1-\alpha_i).\prod_{i\notin I_s}\alpha_i \tag{2}$$

The first term in Eq. (1) investigates the disruption probability when the buyer (he) cannot supply his orders, which is composed of four parts: (i) domestic and foreign suppliers disrupted due to regional disruptions separately, (ii) domestic suppliers disrupted due to regional disruption and foreign suppliers disrupted due to local disruptions, (iii) foreign suppliers disrupted due to regional disruption and domestic suppliers disrupted due to local disruptions, (iv) all of the domestic and foreign suppliers disrupted due to local disruptions in all of them. The second term is the disruption probability when the domestic suppliers deliver parts without disruptions. The third term is the disruption probability when the foreign suppliers deliver parts without disruptions. The last term shows when no disruption occurs.

3.3. Risk-neutral model

As stated before, in the financial risk management literature and supply chain risk literature, risk-neutral and risk-averse concepts are considered as two of the most important attitude of decision-makers with respect to risk. In this subsection, we consider the centralized supply chain in which the decision-maker is indifferent toward the risk (i.e., risk-neutral). We optimize the total supply chain cost, including the total costs of buyer and suppliers. We consider the average inventory of the buyer, defined as $\bar{I}$, to calculate its total cost. As it is shown in Fig. 2, the inventory of the buyer based on the order from the i^{th} supplier continuously changes between zero and $\hat{Q}_i$. In addition, the length of each period is equal to $\frac{Q_j}{D}$, and therefore, the buyer's average inventory for the products ordered from the i^{th} supplier is calculated as follows:

$$\bar{I}_j = \frac{1/2 \times Q_i^{'} \times Q_i^{'}/D}{Q_j/D} = \frac{Q_i^{\text{prime2}}}{2Q_j} \tag{3}$$

In addition, we have $\hat{Q}_i = Y_{ij}{*}Q_j$. The above equation can be rewritten as follows:

$$\bar{I}_j = \frac{\hat{Q}_i^{\,2}}{2Q_j} = \frac{Q_j^{\,2} y_{ij}^2}{2Q_j} = \frac{Q_j Y_{ij}^2}{2} \tag{4}$$

The buyer's annual cost can be formulated as follows:

$$\begin{aligned}\pi^b = &\sum_{j\in J}\sum_{i\in I}\frac{D_j}{Q_j}A_{ij}X_i + \sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s D_j Y_{ij}\omega_{ij} + \sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\frac{\beta_s Q_j Y_{ij}^2 h_i^b}{2}\\ &+\sum_{j\in J}\sum_{s\in S}\beta_s D_j B_j U_j^s - \sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s D_j P_j Y_{ij}\end{aligned} \tag{5}$$

Eq. (5) represents five parts, including the fixed ordering cost, purchasing cost of products from the suppliers, holding cost, shortage cost, and finally, revenue from selling the products, respectively.

The total cost of the i^{th} supplier, $i \in I_s$, will be as follows:

$$\pi_i^v = \sum_{j\in J}\left(Cap_i C_{ij} + \frac{D_j}{Q_j}S_{ij} + \frac{Q_j Cap_i Y_{ij} h_i^v}{2D_j} - Cap_i\omega_{ij}\right)\frac{D_j Y_{ij}}{Cap_i}. \tag{6}$$

Similar to (Mohammaditabar et al., 2016), Eq. (6) includes the production cost, setup cost, cost of holding inventory, and the revenue of supplier i^{th}, respectively.

The total cost of the supply chain is the sum of the buyer's annual cost and the expected annual cost of suppliers considering the probabilities of disruption scenarios as follows:

$$\begin{aligned}\pi^{sc} = \pi^b + \sum_{s\in S}\sum_{i\in I_s}\beta_s\pi_i^v = &\sum_{j\in J}\sum_{i\in I}\frac{D_j}{Q_j}A_{ij}X_i + \sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s D_j Y_{ij}\omega_{ij}\\ &+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\frac{\beta_s Q_j Y_{ij}^2 h_i^b}{2} + \sum_{j\in J}\sum_{s\in S}\beta_s D_j B_j U_j^s - \sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s D_j P_j Y_{ij}\\ &+\sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J}\beta_s\left(Cap_i C_{ij} + \frac{D_j}{Q_j}S_{ij} + \frac{Q_j Cap_i Y_{ij} h_i^v}{2D_j} - Cap_i\omega_{ij}\right)\frac{D_j Y_{ij}}{Cap_i}.\end{aligned} \tag{7}$$

We rewrite the objective function of the supply chain as follows:

$$\begin{aligned}\pi^{sc} = &\sum_{j\in J}\sum_{i\in I}\frac{D_j}{Q_j}A_{ij}X_i + \sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\frac{Q_j}{2}Y_{ij}^2\left(h_i^b + h_i^v\right) + \sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\frac{D_j^2 S_{ij} Y_{ij}}{Q_j Cap_i}\\ &+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s D_j C_{ij} Y_{ij} + \sum_{j\in J}\sum_{s\in S}\beta_s D_j B_j U_j^s - \sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s D_j P_j Y_{ij}.\end{aligned} \tag{8}$$

The function π^{sc} is convex with respect to Q_j. This can be proved through the following Lemma:

Lemma 1:. *The function π^{sc} is convex with respect to Q_j.*

Proofs of lemmas and theorem are given in the Appendix.

Lemma 2:. *The following inequality holds*:

$$\frac{1}{\mu Q_j^{(1)} + (1-\mu)Q_j^{(2)}} \le \left(\frac{\mu}{Q_j^{(1)}} + \frac{(1-\mu)}{Q_j^{(2)}}\right)\forall Q_j^{(1)}, Q_j^{(2)} \ne 0 and \mu \in [0,1]. \tag{9}$$

Based on Lemma 1, the problem can be simplified by solving the problem with respect to Q_j and substituting the optimal value of the variable Q_j in the objective function. As a result, this variable can be eliminated, and the problem becomes more manageable to solve. To do so, we make differentiation of π^{sc} with respect to Q_j, and set the equation equal to the zero as follows:

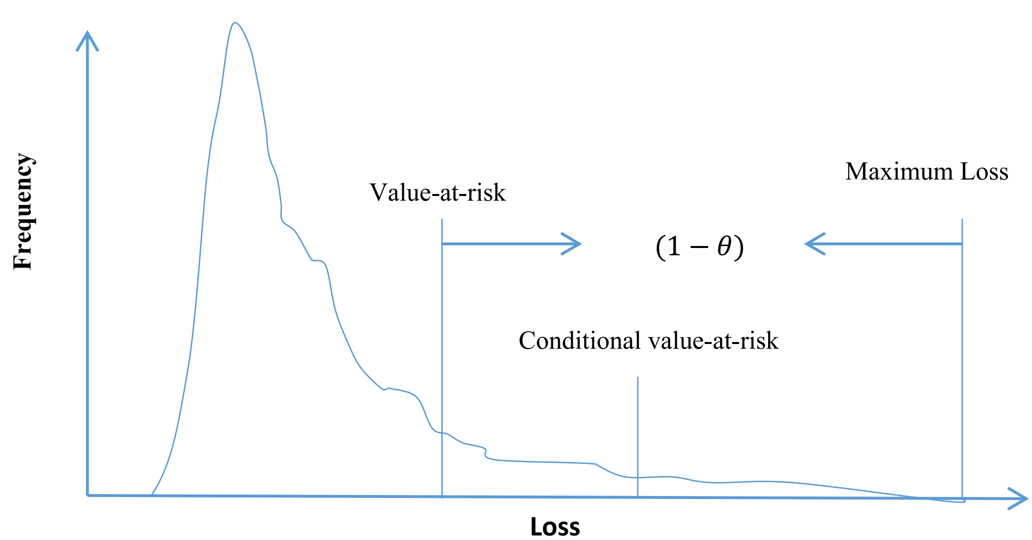

Fig. 3. The VaR and CVaR for portfolio management problem minimizing the worst case losses (Chahar & Taaffe, 2009).

$$\frac{\partial \pi^{sc}}{\partial Q_j} = -\frac{D_j \sum_{i\in I} A_{ij} X_i}{Q_j^2} + \frac{\sum_{s\in S}\sum_{i\in I_s} \beta_s Y_{ij}^2 \left(h_i^b + h_i^v\right)}{2} - \frac{\sum_{s\in S}\sum_{i\in I_s} \beta_s \frac{D_j^2 S_{ij} Y_{ij}}{Cap_i}}{Q_j^2} = 0, \tag{10}$$

By solving the above equation, we have:

$$Q_j = \sqrt{\frac{2D_j\left(\sum\limits_{i\in I} A_{ij} X_i + \sum\limits_{s\in S}\sum\limits_{i\in I_s} \beta_s \frac{D_j S_{ij} Y_{ij}}{Cap_i}\right)}{\sum\limits_{s\in S}\sum\limits_{i\in I_s} \beta_s Y_{ij}^2 \left(h_i^b + h_i^v\right)}}. \tag{11}$$

We substitute the optimal value of Q_j in π^{sc}. Therefore, the model can be formulated as follows:

Model 1- Risk-neutral:

$$\min \pi^{SC} = \sum_{j\in J} \sqrt{2D_j \left(\sum_{i\in I} A_{ij} X_i + \sum_{s\in S}\sum_{i\in I_s} \frac{\beta_s D_j S_{ij} Y_{ij}}{Cap_i}\right)\left(\sum_{s\in S}\sum_{i\in I_s} \beta_s Y_{ij}^2 \left(h_i^b + h_i^v\right)\right)}$$

$$+ \sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s} \beta_s D_j C_{ij} Y_{ij} + \sum_{j\in J}\sum_{s\in S} \beta_s B_j D_j U_j^s - \sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s} \beta_s D_j P_j Y_{ij} \tag{12}$$

s.t

$$\sum_{i\in I_s} Y_{ij} + U_j^s = 1 \forall s \in S, j \in J \tag{13}$$

$$\sum_{j\in J} Y_{ij} D_j \le X_i Cap_i \forall i \in I \tag{14}$$

$$X_i \in \{0,1\}, Y_{ij}, U_j^s \ge 0 \forall i \in I, j \in J, s \in S. \tag{15}$$

The objective function (12) minimizes the total expected annual supply chain cost, including the expected annual costs of the buyer and the expected annual cost of the suppliers. Constraints (13) ensures the demand for each product is satisfied by the non-disrupted suppliers or remains as an unsatisfied demand in each disruption scenario. Constraint (14) addresses the capacity constraint of each supplier in each disruption scenario.

We note that the objective function of the risk-neutral model is a non-convex programming model that is stated in Theorem 1.

Theorem 1. *The objective function of the risk-neutral model is non-convex.*

Supplier disruptions are low probabilities risks, which have a high negative financial impact on the supply chains. Therefore, it is important that the central decision-maker optimizes the supply chain in an efficient way that the high losses due to the supplier disruptions are managed. In the next section, a risk-averse model is developed to investigate how the decision maker's attitude towards the risk would impact the supplier selection and order allocation decisions.

In the supply chain risk literature, there are a few studies that different risk attitudes are explicitly considered simultaneously (Heckmann et al., 2015). To overcome this gap, we proposed a risk-averse model in the next section. Therefore, the risk-neutral model is considered as a benchmark for the risk-averse model.

3.4. Risk-averse model

In this subsection, disruption risks are managed based on two widely measures of risk assessment in the finance literature as follows: Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). The VaR represents the maximum loss for a certain confidence level of outcomes.

It is the probability that the loss of a given portfolio exceeds a certain threshold that is referred to as VaR. VaR is a measure to assess the probable loss of the portfolio due to the market risk. The CVaR considers a portfolio of outcomes so that the losses can exceed the VaR quantity during the given period. Therefore, the CVaR at a confidence level of θ is defined as the expected profit of the supply portfolio within the $(1-\theta)$ % of the worst cases. Fig. 3 represents the relation between VaR and CVaR. Please refer to (Chahar & Taaffe, 2009; Madadi et al., 2014; Merzifonluoglu, 2015) for more information. CVaR measure is applied widely as an alternative measure for the VaR as the VaR complicates the scenario-based optimization model significantly (Merzifonluoglu, 2015). In addition, CVaR minimization optimizes the VaR simultaneously as it is equal or greater than the VaR in the optimization model. Therefore, in the present study, we apply the CVaR measure to control the supplier disruption risks.

In the supplier selection and order allocation problem under disruption risks, the decision-maker controls the high-loss risks resulting from the supply disruptions by selecting the confidence level of θ. The decision-maker accepts only those portfolios in which the loss probability is less than the value of the VaR. Therefore, a higher confidence level indicates the higher risk-averseness of the decision-maker. We define τ_s as the tail cost for the disruption scenario s, and it represents the cost quantity that exceeds the value of VaR in the s^{th} disruption scenario. The supply portfolio is optimized by calculating the VaR and minimizing the CVaR as the objective function as follows:

Model 2: Risk-averse

$$\min CVaR_c = VaR_c + \frac{1}{(1-\theta)} \sum_{s\in S} \beta_s \tau_s \tag{16}$$

s.t.:

Eqs. (13)–(15), and

$$\tau_s \ge \sum_{j\in J} \sqrt{2D_j \left(\sum_{i\in I} A_{ij} X_i + \sum_{i\in I_s} \frac{D_j S_{ij} Y_{ij}}{Cap_i}\right)\left(\sum_{i\in I_s} Y_{ij}^2 \left(h_i^b + h_i^v\right)\right)}$$

$$+ \sum_{j\in J}\sum_{i\in I_s} D_j C_{ij} Y_{ij} + \sum_{j\in J} B_j D_j U_j^s$$

$$- \sum_{j\in J}\sum_{i\in I_s} D_j P_j Y_{ij} - VaR_c \forall s \in S \tag{17}$$

Constraints (17) show the risk constraint in which cost exceeds the value of VaR in each disruption scenario.

Both models 1 and 2 are MINLP and non-convex programming models, and standard commercial solvers may be used to solve these models when dealing with small-size instances. Although this is valuable, the number of variables and constraints in our proposed model for optimizing the supplier selection portfolio is $O(mnh)$. That is mean that it grows linearly in the number h of disruption scenarios and hence exponentially in the number n of suppliers if all $\mathrm{h} = 2^{\mathrm{n}}$ potential scenarios are considered. Pariazar (2013) showed that exact solution approaches such as the integer L-Shaped method are not able to solve and find the optimal solution of these types of problems for more than ten suppliers. Therefore, the *meta*-heuristic algorithms, such as Genetic and Particle Swarm Optimization algorithms, can be an alternative approach to reduce the computational time to solve the large-sized instances (Bianchi et al., 2009; Pariazar & Sir, 2018). Wang et al. (2001) proposed that the Genetic algorithm is an efficient method to select the best combination of the suppliers. Min et al. (2005) suggested that the genetic algorithm is the best population-based algorithm due to the

$X = randi(0,1)_N$	$Y = rand(0,1)_{N\times M}$

Fig. 4. The chromosomes structure.

simultaneous generation of a set of best solutions. On the other hand, (Zhu et al., 2011) showed that the PSO algorithm has high computational efficiency for the nonlinearly constrained portfolio optimization problems. As a result, in the next section, we propose the PSO algorithm to solve the supplier selection and order allocation problem under supplier disruption considering a risk-averse decision-maker. We also apply the GA algorithm as an efficient population-based algorithm, and we show that the proposed PSO algorithm has a better performance compared to the GA algorithm.

4. Solution approach

In this section, we describe the two proposed *meta*-heuristic algorithms. i.e., genetic and particle swarm optimization algorithms to find the supply portfolio in a centralized supply chain. The genetic algorithms are widely applicable in different fields due to their customizability and speed. These algorithms start with an initial population, and then, a final population results with better characteristics. On the other hand, Particle swarm optimization (PSO) is recognized as one of the best evolutionary methods that was developed as an optimization method in 1995 by Kennedy and Eberhart (1995). Compared to other heuristic methods and the conventional mathematical approaches, this algorithm offers advantages such as less sensitive to the nature of the objective function, the ability to escape from the local minimum with a suitable design, and fewer operator settings (Alejo-Reyes et al., 2021). Also, the main advantage of this algorithm is the fast convergence over other global optimization algorithms (Bianchi et al., 2009). The base of the PSO concept is a simulation of the social treatment of birds flock. In the following sub-sections, we present the developed GA and PSO algorithms to solve the models.

4.1. Genetic algorithm

Firstly, (Holand, 1975) introduced the Genetic Algorithm as a bionic optimization algorithm by inspiring from Darwin's evolution theory and Mondel's heredity theory. Based on the evolutions of defined "chromosomes" (a population of solutions) from generation to generation, this algorithm tries to optimize some complex problems. In the GA, the problem codes as binary codes. Then, under the principle of "Survival of the Fittest", the searching processes of the optimal solution are conducted by copy, crossover, and mutation operators. In the following, we introduce chromosome structure as well as selection, crossover, and mutation operators for the GA algorithm.

4.1.1. Chromosomes structure

We create a suitable representation chromosome satisfying all constraints, which is one of the most critical steps in designing a GA. The proposed structure consists of one vector and one matrix in which the vector shows the variable of X, and the matrix specifies the variable of Y. Fig. 4 shows the amount of X and Y in which $randi(0,1)_N$ is a vector of N dimension with random integer number generator within the range $\{0,1\}$ and $rand(0,1)_{N\times M}$ is a matrix of $N \times M$ size from the random number generator within the range [0,1].

4.1.2. Selection operator

According to parents' fitness value, the selection operator chooses the elite individuals from the initial population. The likelihood method with fitness in proportion is recognized as a classical selection operator (Aliabadi et al., 2013). Based on the trial and error experiments, the roulette wheel method is used for selection operator in our problem due to the better performance compared to the other methods. In this paper, we control the diversity of the GA method by selecting the pressure parameter to avoid the quick convergence of the algorithm (Back, 1994).

4.1.3. Crossover operator

The crossover operations role is to generate a new population from two-parent solutions. First, we select both parent solutions for the new population and then generate the offspring solutions by applying the crossover operator. In addition, we use Arithmetic and binary crossover operators for continuous and binary variables, respectively, as there are two different encoding methods. In the binary crossover operator, we choose a single-point or two-points, and in the uniform crossover operator, we choose based on the roulette wheel method.

4.1.4. Mutation operator

The mutation operator is mainly responsible for generating a new chromosome by changing a certain chromosome gene. In addition, both the global search ability and population diversity are improved as this operator is applied to avoid the local optimum. We use binary and continuous mutation methods for binary and continuous variables, respectively.

Generally, the correct selection of parameters significantly affects the effectiveness and operation of the GA algorithm (Naderi et al., 2009). Thus, we adjust the parameters using statistical methods based on the Taguchi method in Section 6.1. In addition, we compare GA performance with the results obtained by the commercial GAMS solver (DICOPT algorithm) in Section 6.2.

4.2. Particle swarm optimization algorithm

The PSO is introduced as one of the population-based evolutionary optimization techniques (Eberhart & Kennedy, 1995). The fundamental aspect of PSO is an iterative approach. In this method, candidate solutions (particles) are improved by moving these particles around the search-space according to position and velocity parameters. This algorithm initializes with a population of random solutions. Each solution (particle) shows a possible solution that can optimize the decision variables. In the iteration, particles (solutions) evolve, and their velocity is updated. In the following, we describe the general PSO procedure.

4.2.1. Initialization

The first step in the algorithm of PSO is initializing the population, where the population shows all the solutions as particles. The input data (decision variables) are the random points that distribute over the objective function's search space. Particles (decision variables) follow a uniform probability distribution. After initializing the particles, the objective function evaluates the decision vector and the local and global particles calculated by the algorithm. Both the velocity and position are updated iteratively. Therefore, particles change.

4.2.2. Velocity of particles

Based on the last velocity value and evaluating the impacts of the best particles' local and global particles, particle velocity and particle position must be updated for a new solution vector. The following equations show the process of updating the position of each individual solution (particle) in each iteration.

Table 2
Parameters input.

Parameters	D_1	D_2	P_j	B_j	S_{ij}	C_{ij}	α_{1^*}	α_{2^*}
Values	*15,000*	*20,000*	*90*	*700*	$100 \times U[1,10]$	$U[1,15]$	0.01	0.001

Table 3
Optimal solutions of risk-neutral model versus the risk-averse model.

$\alpha_i \in [0, 0.06], i \in I$						
	Risk-neutral model	Risk-averse model				
Confidence level θ		0.5	0.75	0.9	0.95	0.99
Total expected cost	−2343690	−2343690	−2343690	−2343690	−2069360	−2045382
CVaR	–	−1982530	−1259820	908292.79	3,322,539	7,059,889
VaR	–	−2705230	−2705230	−2705230	1,007,211	4,930,033
Selected supplier (% of the total allocated orders)	1(31%)	1(31%)	1(31%)	1(31%)	1(15%)	1(8%)
	4(69%)	4(69%)	4(69%)	4(69%)	4(15%)	2(4%)
					6(14%)	3(2%)
					7(14%)	4(11%)
					8(14%)	5(4%)
					9(14%)	6(14%)
					10(14%)	7(14%)
						8(14%)
						9(14%)
						10(15%)
$\alpha_i \in [0.06, 0.15], i \in I$						
	Risk-neutral model	Risk-averse model				
Confidence levelθ		0.5	0.75	0.9	0.95	0.99
Total expected cost	−501005	−501005	−291340	−57568	62,724	137,113
CVaR	–	1,695,668	4,171,445	6,009,119	7,721,653	12,020,238
VaR	–	−2705230	1,811,025	3,254,072	5,367,756	7,729,871
Selected supplier (% of the total allocated orders)	1(14%)	1(14%)	1(16.7%)	1(11.2%)	1(10.1%)	1(7.6%)
	2(17%)	2(17%)	2(16.7%)	2(11.2%)	2(10.1%)	2(7.6%)
	4(69%)	4(69%)	3(16.7%)	3(11.2%)	3(10.1%)	3(7.6%)
			4(16.7%)	4(11.1%)	4(10.1%)	4(7.6%)
			5(16.6%)	5(11.1%)	5(10.1%)	5(7.6%)
			9(16.6%)	6(11%)	6(9.9%)	6(11.8%)
				7(11.1%)	7(9.9%)	7(11.8%)
				9(11%)	8(9.9%)	8(11.7%)
				10(11.1%)	9(9.9%)	9(15%)
					10(9.9%)	10(11.7%)

$$\vec{v}_i \leftarrow \vec{v}_i + c_1 \vec{r}_1 \otimes \left(\vec{p}_i - \vec{x}_i\right) + c_2 \vec{r}_2 \otimes \left(\vec{p}_g - \vec{x}_i\right) \tag{18}$$

$$\vec{x}_i \leftarrow \vec{x}_i + \vec{v}_i \tag{19}$$

where $\vec{x}_i$ and $\vec{v}_i$ are the position and velocity of the i^{th} particle, $\vec{r}_1$ and $\vec{r}_2$ are two randomly generated numbers from $U[0, 1]$, c_1 and c_2 are acceleration coefficients, $\vec{p}_i$ and $\vec{p}_g$ are the best particle in each iteration, and the best particle found so far, respectively. Note that Eq. (19) is the new position of the particle (decision vector) on the current iteration. Based on the new position of particles in the search space, we can explore new possible solutions. We found that the constriction coefficients (Clerc & Kennedy, 2002; Poli et al., 2007) are suitable for our models by the trial and error procedure. Thus the Eq. (18) changes to the following equation:

$$\vec{v}_i \leftarrow \chi\left(\vec{v}_i + \phi_1 \vec{r}_1 \otimes \left(\vec{p}_i - \vec{x}_i\right) + \phi_2 \vec{r}_2 \otimes \left(\vec{p}_g - \vec{x}_i\right)\right) \tag{20}$$

$$\vec{x}_i \leftarrow \vec{x}_i + \vec{v}_i \tag{21}$$

where $\phi = \phi_1 + \phi_2 > 4$ and $\chi = \frac{2}{\phi - 2 + \sqrt{\phi^2 - 4\phi}}$ and the optimal value of ϕ is equal to 4.1, and the values of ϕ_1 and ϕ_2 are identical (Poli et al., 2007). Therefore, we have:

$$\phi_1 = 2.05, \phi_2 = 2.05, \phi = \phi_1 + \phi_2 = 4.1, \chi = \frac{2}{\phi - 2 + \sqrt{\phi^2 - 4\phi}} = 0.7298,$$

Thus, both terms of $(\vec{p} - \vec{x})$ in Eq. (45) are multiplied by a random number within the range of $[0, 1.49618]$. Section 6.2 shows the performance of the PSO algorithm by comparing the obtained result of PSO with GA and commercial GAMS solver.

5. Benefits of the proposed models

This section aims to demonstrate the performance of the proposed risk-neutral and risk-averse models for the supplier selection and order allocation problem in a centralized supply chain under disruption risks. We assume that the number of suppliers is ten that are located in two different geographical regions, and they satisfy two different products. Therefore, the number of disruption scenarios is equal to $2^{10} = 2014$. In addition, we assume that the subset of foreign suppliers is $\{1, 2, 3, 4, 5\}$, and the subset of domestic suppliers is $\{6, 7, 8, 9, 10\}$. Without loss of generality, we also assume that the two products have the same parameters. We use the following parameters in Table 2 for both models, and the other ones have been adopted (Esmaeili-Najafabadi et al., 2019). The algorithms are in MATLAB R2018a and performed on a laptop computer with an Intel Core-i5 2.3 GHz processor and 8 GB of RAM. In addition, we benchmark our results using the DICOPT algorithm in the commercial GAMS solver.

Table 3 provides the comparison results of risk-averse and risk-neutral models with two levels of local disruption probability generated from$\alpha_i \in [0, 0.06]$ and $\alpha_i \in [0.06, 0.15] i \in I$. Considering two different levels of disruption risk allowed us to investigate the impact of disruption risk on the decision-maker decisions. We also consider five different confidence levels, i.e., θ, in the risk-averse model equal to 0.5, 0.75, 0.9, 0.95, and 0.99, meaning that the objective function minimizes the highest 50%, 25%, 10%, 5%, 1% of all disruption scenario outcomes (i.e, expected supply chain costs).

This permits us to study the impact of the risk-averse size of the decision-maker on the supplier selection and order allocation (i.e., the main decisions of our problem). Note that the decision-maker is more risk-averse as the value of θ increases, and therefore, the model focuses on the smaller percentage of the highest outcome.

The optimal risk-neutral supply portfolio for the local disruption probability of $\alpha_i \in [0, 0.06]$ is shown in Fig. 5a. A similar figure for the local disruption probability of $\alpha_i \in [0.06, 0.15]$ is shown in Fig. 5b.

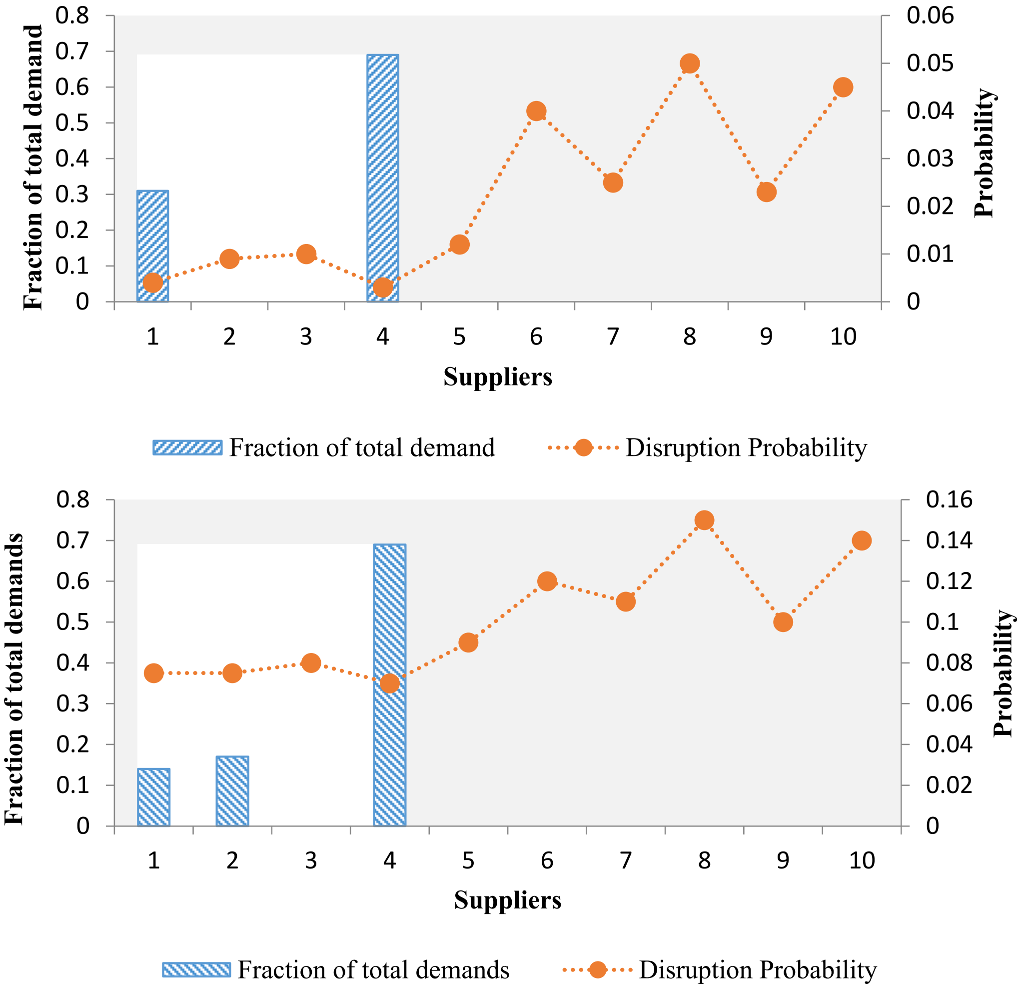

Fig. 5. Optimal supply portfolio for risk-neutral model.

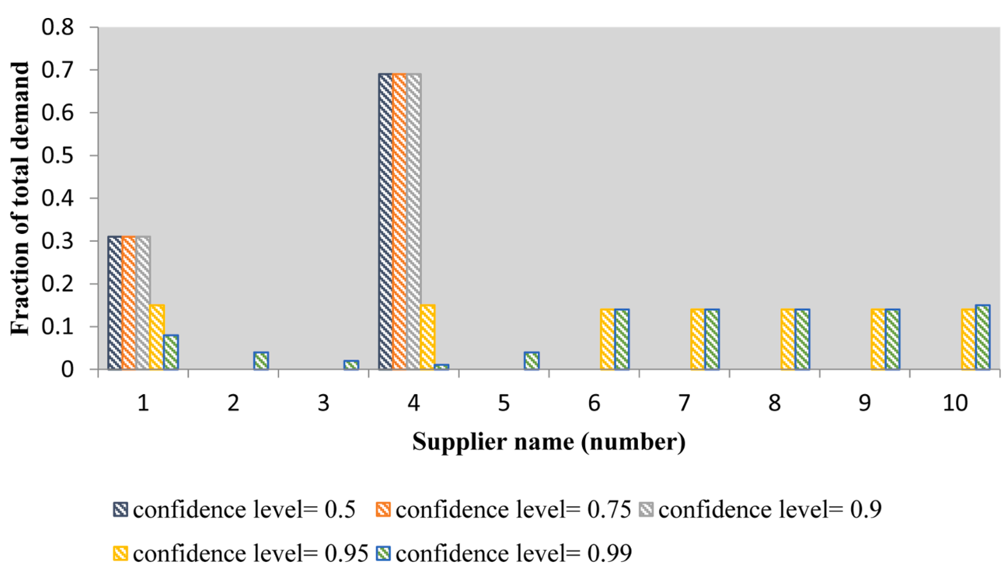

Fig. 6. Risk-averse supply portfolios.

Table 3 indicates that the optimal supply portfolio for $\alpha_i \in [0, 0.06]$ allocates the total demand of customers between the two foreign suppliers with the lowest disruption risks. The lower disruption probability and the higher capacity for a supplier would result in a higher percentage of the total demand allocated. When the production cost is significantly lower than the shortage cost, disruption risk becomes a key component in the supplier selection. Also, the disruption risk and capacity of suppliers are two keys factors in the decision of demand allocation among

Table 4
Effective factors and levels in Taguchi method.

Level			Factor
3	2	1	
500	300	100	Maximum Iteration
50	30	10	Population Size
0.9	0.8	0.7	Mutation Rate
0.3	0.2	0.1	Crossover Rate
15	10	5	Selection Pressure

the selected suppliers to minimize the expected cost of the centralized supply chain. Similarly, when the local disruption probability is high, i. e., $\alpha_i \in [0.06, 0.15]$, the total demand is allocated among three suppliers, with the largest orders placed when disruption probability is low. Therefore, the decision-maker in the risk-neutral model prefers to increase the number of selected suppliers as the probability of local disruption increases. A comparison of two levels of local disruption probability for the risk-neutral model indicates that the allocated demand to the supplier $i = 1$ is divided between two suppliers $i = 1, 2$. Note that the local disruption probability of suppliers $i = 1, 2$ is equal within the range $\alpha_i \in [0.06, 0.15]$. Although domestic suppliers offer low prices, they are not selected in both of optimal risk-neutral supply portfolios. These are evidence that supports reliability importance compared to costs for a multi-sourcing strategy. The importance of interaction between the reliability of suppliers and regions on supplier selection (sourcing decisions) in a multi-sourcing strategy is highlighted in the analysis of this section. This result in line with what (Hosseini et al., 2019; Kamalahmadi & Parast, 2017) discussed about the positive impacts of regionalized supply chains on risk management.

The optimal risk-averse supply portfolio for two levels of disruption probability, i.e., $\alpha_i \in [0, 0.06]$ and $\alpha_i \in [0.06, 0.15]$, and the five confidence levels, i.e., 0.5, 0.75, 0.9, 0.95, and 0.99, are illustrated in the Fig. 6a and b. Table 3 and Fig. 6a show that when the probability of local disruption is low, and θ is between 0.5 and 0.9, the obtained risk-neutral results are identical with the corresponding risk-averse results. This indicates that when the disruption probability is not high enough, the risk-averse decision-maker is not interested in selecting a large number of suppliers. On the other hand, when the probability of local disruption is low, and the confidence level is high, i.e., $\theta = 0.95 and 0.99$, the decision-maker prefers to apply the supplier diversification strategy by selecting more suppliers to mitigate the impact of disruption risks (see Table 3 and Fig. 6a). As it is shown in Fig. 6a, all ten suppliers are selected when the decision-maker is very risk-averse and has the highest confidence level. Although, when the confidence level decreases to the next lower level, i.e., $\theta = 0.95$, the number of selected suppliers decreases to 7. Therefore, these results demonstrate that the suggested strategy by (Chopra & Sodhi, 2014), i.e., locating suppliers in different geographical regions, is an efficient way to reduce the disruption risk for risk-averse decision-makers. In addition, we note that the largest orders are placed on the domestic suppliers that are unreliable but offer more

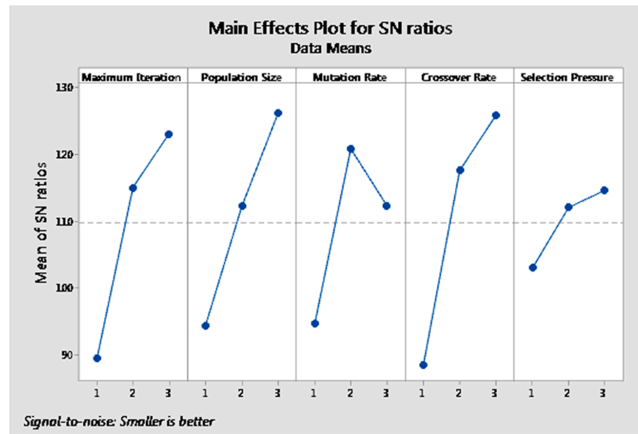

Fig. 8. Average S/N ratio levels for GA's parameters.

Table 5
Comparison of GA, PSO and GAMS commercial solvers in small and medium-sized instances.

Problem size$\|J\| \times \|I\| \times \|S\|$	GAMS	Genetic Algorithm		Particle swarm optimization algorithm	
		Objective function value (cost)	Gap	Objective function value (cost)	Gap
$2 \times 2 \times 4$	−1810930	−1810930	0%	−1810930	0%
$3 \times 2 \times 4$	−2850170	−2850170	0%	−2850170	0%
$2 \times 3 \times 8$	−2126760	−2118494	0.3%	−2126760	0%
$2 \times 4 \times 16$	−2207523	−2207256	0.01%	−2207523	0%
$2 \times 6 \times 64$	−2207523	−2196952	0.4%	−2207520	0%

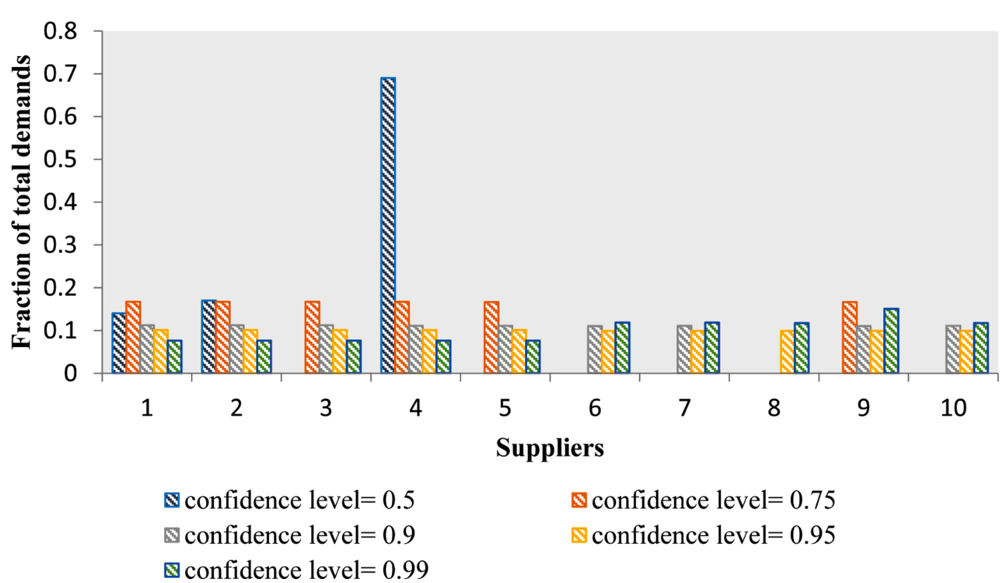

Fig. 7. Average S/N ratio levels for GA's parameters.

Table 6
Comparison of GA and PSO algorithms for large-sized instances.

Problem size $\|J\| \times \|I\| \times \|S\|$	Genetic Algorithm		Particle swarm optimization algorithm	
	Objective function value (cost)	Number of function evaluation	Objective function value (cost)	Number of function evaluation
$2 \times 8 \times 256$	−2311876	520,200	−2328370	400,200
$2 \times 10 \times 1024$	−2328350	1,040,400	−2328378	800,400
$3 \times 6 \times 64$	−3434240	110,100	−3434260	100,100
$3 \times 8 \times 256$	−3611144	520,200	−3623034	400,200
$3 \times 10 \times 1024$	−3595970	1,040,400	−3622775	800,400
$5 \times 3 \times 8$	−5598874	110,100	−5620307	100,100
$5 \times 6 \times 64$	−5623297	440,200	−5678173	400,200
$5 \times 10 \times 1024$	−6489943	1,040,400	−6748010	800,400

competitive prices.

In addition, Table 3 indicates that for $\alpha_i \in [0.06, 0.15]$, domestic suppliers are selected when the confidence level is low. For example, for $\theta = 0.75$, five domestic suppliers and a foreign supplier are selected. Although, when the confidence level increases, the number of selected foreign suppliers also increases. Similar to $\alpha_i \in [0, 0.06]$, the probability of local disruption is the main factor for selecting the suppliers, but unlike $\alpha_i \in [0, 0.06]$, the fraction of allocated demand to each selected supplier is identical. Therefore, in general, Fig. 6 demonstrates the effect of local disruption risk on the optimal risk-averse supply portfolios. Also, the results show that the reliability of suppliers is a key selection element in two levels of local disruption probability for the risk-averse model. Finally, a comparison between both models indicates the expected cost of the risk-neutral model is smaller than the risk-averse model. In the risk-averse model, reducing the disruption risk is important as much as reducing the cost, and the model finds the optimal solution to the trade-off between these two elements.

6. Performance of the developed algorithms

It is shown that metaheuristic algorithms can be successfully used to solve complex problems with a high number of possible solutions. In this section, we present several different parameters to evaluate the performance of the GA method. Also, to evaluate the developed algorithms' performance, a comparison between GA, PSO, and GAMS commercial solver (DICOPT algorithm) is made.

6.1. Parameters input for the GA algorithm

We investigate the performance of the different parameters of developed GA in this section as adjusting parameters of *meta*-heuristic algorithms affect their performance significantly. We use the Taguchi method (Taguchi, 1986) to design the experiments. When the number of factors increases significantly in the full factorial design, the number of level combinations also increases very fast, which is not effective in practice. Therefore, In the Taguchi method, the orthogonal array is used to organize the parameters affecting the process and the different levels such that study a large number of decision variables with a small number of experiments (Taguchi, 1986). This method examines pairs of combinations instead of test all possible combinations like the factorial design. Therefore, the orthogonal arrays method saves time and resource with a minimum amount of experimentation. The Taguchi arrays can be drawn out manually in the small size. Based on the number of parameters (factors) and the number of levels (states), the orthogonal arrays are calculated for large size. The orthogonal arrays in different levels and parameters can be found online or in the Minitab software as a file (Design of Experiments via Taguchi Methods - Orthogonal Arrays., 2020)

Based on the literature and trial and error experiment, we consider five parameters in the three different levels (see Table 4). Thus, we apply the L27 (3ˆ5) scheme as an appropriate orthogonal array according to the Taguchi method.

In the Taguchi method, the signal-to-noise (S/N) ratio is used to determine the best level of each factor as follows:

$$S/Nratio = -10log_{10}\left(\frac{\sum (Objectivefunction)^2}{n}\right) \tag{23}$$

Due to the stochastic nature of GA and also in order to obtain the optimal value of the objective function, each model is run ten times. Then, the best solution of all runs is selected as the value of the objective function. In addition, a common performance measure (i.e., relative percentage deviation (RPD)) is employed to evaluate the algorithm. This measure demonstrates that how much an algorithm differs from the best-obtained solution on average. RPD is calculated as follows:

$$RPD = \frac{Alg_{sol} - Min_{sol}}{Min_{sol}} \times 100, \tag{24}$$

where Alg_{sol} represents the best-obtained solution for ten runs, $Min_{sol}$is the minimum obtained solution. After changing the objective values to RDPs, the mean RPD is computed in each trial. Based on instructions of the Taguchi parameter design, these mean RPDs are transformed into S/N ratio using Eq. (48). The mean S/N ratio for GA's parameters is presented in Fig. 8.

Based on Fig. 7, the best level of the parameters can be determined as follows:

Maximum Iteration = 500, Population Size = 50, Mutation Rate = 0.8, Crossover Rate = 0.3, Selection Pressure = 15.

Note that values of Maximum Iteration and Population Size are dependent on the scale of the problem. 4 shows the parameters for the small-sized increases. We need to increase the value of Maximum Iteration and Population Size for medium and large-sized instances.

6.2. Comparison of the performance of PSO, GA, and GAMS commercial solver

In this section, we compare the obtained solutions of the GA, PSO, and GAMS commercial solver (DICOPT algorithm) in Tables 5 and 6 to evaluate the performance of the developed algorithms. We have run small, medium, and large-sized instances. As we can see from Table 5, the GAMS solver can only solve small and medium-sized instances. In addition, note that the DCIOPT algorithm (GAMS solver) does not guarantee to find the global optimum as the proposed models are non-convex programming models. Finally, Table 6 show the comparison between GA and PSO algorithms for large-sized instances in which the GAMS commercial solver is not able to find the solution.

In general, Compared to GA, PSO is easy to implement and has no evolution operators such as crossover and mutation. Tables 5 and 6 indicate that generally, the PSO algorithm performs better than the GA algorithm. In addition, Table 5 shows that the results of the PSO algorithm are exactly equal to the optimal solution in the GAMS software (if it is available). In large-sized instances, the PSO algorithm reaches a better objective function in the lower number of function evaluations compared to the GA algorithm.

7. Theoretical and practical implications

In this section, we provide several implications to the theory and practice in the supply chain. As mentioned in the literature review, the supplier selection problem can be divided into two main categories:1) Quantitative models and 2) Descriptive models. (Sawik, 2014) addressed that most studies conducted on quantitative supplier selection inventory models ignore disruption risk and multi-sourcing aspects. Even though the costs of the supply chain can be reduced by single sourcing, but due to the presence of capacity constraints, disruption risks, etc., supply chain performance can be compromised. Therefore, we present a joint supplier selection and order allocation problem that explicitly is taken into consideration multi-suppliers with regard to inventory decisions, capacity constraints, and disruption risks. (Sawik, 2014) showed when supplier selection and order allocation are made simultaneously, these decisions can be considered as a short- to medium-term planning horizon. Therefore the managers can apply our model to strategic partnerships (supplier selection) and inventory policy based on the quantitative criteria for medium-term decisions.

In the supplier selection literature, there are two approaches to modeling the supplier selection and order allocation problem: 1) The supplier selection problem and order allocation problem are formulated as an integrated model. 2) The modeling of supplier selection and order allocation problem is divided into two main phases. In the first phase, the suppliers are usually evaluated using a multi-criteria tool, and in the second phase, orders allocate to selected suppliers using mathematical programming to take into account the system constraints. For instance, please refer to (Ghodsypour & O'Brien, 1998; Hong et al., 2005; Kaur & Singh, 2021; Mendoza & Ventura, 2010). Coordination of supplier selection phase and order quantities allocation leads to a significantly improvement in the performance of a multi-echelon supply chain under disruption risks. We propose a joint supplier selection and order allocation model under disruption risks that it causes coordination between of supplier selection phase and the phase of order quantities allocation.

(Chopra & Sodhi, 2014) show that even though the initial investment in the development of capabilities to mitigate disruption risks may not appear cost-effective at first, managers can improve the supply chain by investing in developing these capabilities. We used the strategy of locating suppliers in different geographical regions to mitigate disruption risks. Much of Toyota production at plants across Japan was suspended by this company after the Japanese earthquake and tsunami in 2011. Therefore, the world was faced with a shortage of parts. In the wake of this event, the strategy of locating suppliers in different geographical regions has been introduced as a practical resilience driver for many automotive manufacturing firms such as Toyota and Nissan (Hosseini et al., 2019). Our results support the benefits of the strategy of locating suppliers in different geographical regions to increase supply chain capabilities, especially when the decision-maker is highly risk-averse. Also, sensitivity analyses have been conducted on the parameters of disruption probability and confidence levels to create insight into the behavior of the model and the numerical example. The result shows that our models are sensitive to the probability of disruption and therefore, the reliability of the supplier is a key selection element.

Since supplier selection models are sometimes complicated and have nonlinear objective cost functions (Alejo-Reyes et al., 2019, 2020; Mendoza & Ventura, 2013), the number of possible solutions can increase exponentially. Therefore, the exact solutions methods, e.g., the L-Shaped method, can not solve these problems (Pariazar & Sir, 2018). Also, in large instances, the solution process may time-consuming by commercial software, and it becomes impossible to evaluate the cost for all of them. The literature shows that metaheuristic algorithms present promising alternatives to reduce the computational burden associated with solving, such as Genetic algorithms, ant colony optimization, particle swarm, differential evolution, etc. (Alejo-Reyes et al., 2020; Bianchi et al., 2009; Pariazar & Sir, 2018). For this reason, we use GA and PSO algorithms to solve our proposed model. For this reason, we use GA and PSO algorithms to solve our proposed model. The findings of this study indicate that the PSO algorithm is more efficient than GA. This result is in line with what (Zhu et al., 2011) discussed the high computational efficiency for the non-linear constrained portfolio optimization problems.

8. Conclusion

In this paper, we develop a mixed-integer nonlinear programming model to integrate supplier selection and order allocation problem in a centralized multi-product supply chain under disruption risks. Although most of the supplier selection models in the literature have studied only buyer cost minimization in their models, the centralized supply chain in this paper focuses on the simultaneous optimization of the buyer and the supplier. The buyer–supplier relationship leads to coordination or collaboration with upstream partners to ensure an efficient supply of materials along the supply chain. Also, we investigate how decision-maker attributes affect the outcome of a supplier selection decision and buyer performance. Therefore, according to the taxonomy of (Kundu et al., 2015), our problem is placed in the category of the upstream problem. In addition, by applying the CVaR concept as a risk assessment measure, we provide the decision-maker with a simple tool to control the risk and coordinate the flow of the products from suppliers to the buyer. This method allows the decision-maker to shape the cost distribution by selecting the optimal supply portfolio and optimal allocation of the customers' orders. We show that the developed model is non-convex, and as a result, the problem is complex to solve with commercial solution solvers. Therefore, we develop at first a particle swarm optimization algorithm as the solution method and then, we evaluate its efficiency by comparing with a proposed Genetic algorithm and the commercial GAMS solver. We also apply the Taguchi design method to adjust the best parameters for the proposed Genetic algorithm.

The comparisons between the two risk-averse and risk-neutral models indicated when the shortage cost of parts dominates the purchasing cost; both models are sensitive to the probability of disruption and the reliability of the supplier is a key selection element. In the risk-neutral model, the decision-maker selects mostly the suppliers among reliable foreign suppliers and then allocates the demands based on the production cost of each supplier and their capacities. A specific supplier is selected based on the supplier's non-disruption probability than based on the other factors such as purchasing cost or capacity of the supplier. In the risk-neutral model, increasing disruption probability has no impact on the key factors of supplier selection and order allocation. In the risk-averse model, with the increase in the decision maker's risk-averseness, the decision-maker applies the supplier diversity policy. The suppliers with smaller disruption risks are selected, and the orders for all parts are simultaneously placed on the low price, less reliable domestic suppliers. Therefore, the computational experiments indicate that for both the risk-neutral and the risk-averse models, supplier reliability is a key selection parameter, even for a cost-based objective function. With increasing local disruption probability, this strategy occurs at a higher confidence level. In contrast, the risk-neutral model focuses on the expected cost only and limitedly uses a diversity strategy to mitigate disruption risks. Our finding in line with what (Hosseini et al., 2019; Kamalahmadi & Parast, 2017) discussed the positive impacts of geographical segregation of suppliers on risk management. Our results confirm the diversity strategy in the supply chain by allocating less demand to a major number of suppliers and multiple-sourcing from two regions, which can help companies reduce the effect of local and regional disruption risk.

The performed analysis in this work can be expanded in different aspects. In this study, it is assumed that there exists a central decision-maker to achieve coordination among the supply chain-related decisions. For further studies in the future, it is recommended to consider a decentralized supply chain with competition between the buyer and suppliers. We also considered only two levels of centralization, but more levels of the supply chain could be introduced into the analysis. This leads to coordination with downstream partners to influence demand in a beneficial procedure. Finally, considering both supply and demand risks simultaneously can be an interesting subject for further research.

CRediT authorship contribution statement

Elham Esmaeili-Najafabadi: Conceptualization, Data curation, Formal analysis, Methodology, Validation, Writing - original draft. **Nader Azad:** Methodology, Validation, Writing - review & editing. **Mohammad Saber Fallah Nezhad:** Conceptualization, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Proof of Lemma 1. Base on the definition of convex functions, it is sufficient to prove that

$$\pi^{sc}\left(\mu Q_j^{(1)}+(1-\mu)Q_j^{(2)}\right)\leq\mu\pi^{sc}\left(Q_j^{(1)}\right)+(1-\mu)\pi^{sc}\left(Q_j^{(2)}\right) \tag{A.1}$$

By replacing Eq. (8) into Eq. (A.1), we obtain that

$$\begin{aligned}&\sum_{j\in J}\sum_{i\in I}\frac{D_j}{\left(\mu Q_j^{(1)}+(1-\mu)Q_j^{(2)}\right)}A_{ij}X_i+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\frac{\left(\mu Q_j^{(1)}+(1-\mu)Q_j^{(2)}\right)}{2}Y_{ij}^2\left(h_i^b+h_i^v\right)\\&+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\frac{D_j^2S_{ij}Y_{ij}}{\left(\mu Q_j^{(1)}+(1-\mu)Q_j^{(2)}\right)Cap_i}+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_sD_jC_{ij}Y_{ij}+\sum_{j\in J}\sum_{s\in S}\beta_sD_jB_jU_j^s\\&-\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_sD_jP_jY_{ij}\leq\sum_{j\in J}\sum_{i\in I}\frac{\mu D_j}{Q_j^{(1)}}A_{ij}X_i+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\frac{\mu Q_j^{(1)}}{2}Y_{ij}^2\left(h_i^b+h_i^v\right)+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\frac{\mu D_j^2S_{ij}Y_{ij}}{Q_j^{(1)}Cap_i}\\&+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\mu\beta_sD_jC_{ij}Y_{ij}+\sum_{j\in J}\sum_{s\in S}\mu\beta_sD_jB_jU_j^s-\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\mu\beta_sD_jP_jY_{ij}+\sum_{j\in J}\sum_{i\in I}\frac{(1-\mu)D_j}{Q_j^{(2)}}A_{ij}X_i\\&+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\frac{(1-\mu)Q_j^{(2)}}{2}Y_{ij}^2\left(h_i^b+h_i^v\right)+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\frac{(1-\mu)D_j^2S_{ij}Y_{ij}}{Q_j^{(2)}Cap_i}\\&+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}(1-\mu)\beta_sD_jC_{ij}Y_{ij}+\sum_{j\in J}\sum_{s\in S}(1-\mu)\beta_sD_jB_jU_j^s-\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}(1-\mu)\beta_sD_jP_jY_{ij}.\end{aligned} \tag{A.2}$$

After simplifying, the Eq. (A.2) can be rewritten as

$$\begin{aligned}&\sum_{j\in J}\sum_{i\in I}\frac{D_j}{\left(\mu Q_j^{(1)}+(1-\mu)Q_j^{(2)}\right)}A_{ij}X_i+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\frac{D_j^2S_{ij}Y_{ij}}{\left(\mu Q_j^{(1)}+(1-\mu)Q_j^{(2)}\right)Cap_i}\\&\leq\sum_{j\in J}\sum_{i\in I}\left(\frac{\mu}{Q_j^{(1)}}+\frac{(1-\mu)}{Q_j^{(2)}}\right)D_jA_{ij}X_i+\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\left(\frac{\mu}{Q_j^{(1)}}+\frac{(1-\mu)}{Q_j^{(2)}}\right)\frac{D_j^2S_{ij}Y_{ij}}{Cap_i}.\end{aligned} \tag{A.3}$$

The inequality (A.3) can be broken into two inequality as follows

$$\sum_{j\in J}\sum_{i\in I}\frac{D_j}{\left(\mu Q_j^{(1)}+(1-\mu)Q_j^{(2)}\right)}A_{ij}X_i\leq\sum_{j\in J}\sum_{i\in I}\left(\frac{\mu}{Q_j^{(1)}}+\frac{(1-\mu)}{Q_j^{(2)}}\right)D_jA_{ij}X_i, \tag{A.4}$$

and

$$\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\frac{D_j^2S_{ij}Y_{ij}}{\left(\mu Q_j^{(1)}+(1-\mu)Q_j^{(2)}\right)Cap_i}\leq\sum_{j\in J}\sum_{s\in S}\sum_{i\in I_s}\beta_s\left(\frac{\mu}{Q_j^{(1)}}+\frac{(1-\mu)}{Q_j^{(2)}}\right)\frac{D_j^2S_{ij}Y_{ij}}{Cap_i}. \tag{A.5}$$

Thus, it suffices to prove inequalities (A.4) and (A.5) simultaneously. Due to the similarity of the proof of two inequality, we show only the proof for Eq. (A.4).

According to Lemma 2, the following inequality hold

$$\sum_{i\in I}\frac{D_j}{\left(\mu Q_j^{(1)}+(1-\mu)Q_j^{(2)}\right)}A_{ij}X_i\leq\sum_{i\in I}\left(\frac{\mu}{Q_j^{(1)}}+\frac{(1-\mu)}{Q_j^{(2)}}\right)D_jA_{ij}X_i, \tag{A.6}$$

Thus,

$$\sum_{j\in J}\sum_{i\in I}\frac{D_j}{\left(\mu Q_j^{(1)}+(1-\mu)Q_j^{(2)}\right)}A_{ij}X_i\leq\sum_{j\in J}\sum_{i\in I}\left(\frac{\mu}{Q_j^{(1)}}+\frac{(1-\mu)}{Q_j^{(2)}}\right)D_jA_{ij}X_i \tag{A.7}$$

Similarly, Eq. (A.5) is proved, and the proof is complete. ■

Appendix B

***Proof of Lemma 2*:** The left-hand side (LHS) can be restated

$$\frac{1}{\mu Q_j^{(1)} + (1-\mu)Q_j^{(2)}} = \frac{1}{\mu\left(Q_j^{(1)} - Q_j^{(2)}\right) + Q_j^{(2)}}, \tag{B.1}$$

And the right-hand side (RHS) can be rewritten as follows:

$$\left(\frac{\mu}{Q_j^{(1)}} + \frac{(1-\mu)}{Q_j^{(2)}}\right) = \frac{\mu Q_j^{(2)} + (1-\mu)Q_j^{(1)}}{Q_j^{(1)}Q_j^{(2)}} = \frac{\mu\left(Q_j^{(2)} - Q_j^{(1)}\right) + Q_j^{(1)}}{Q_j^{(1)}Q_j^{(2)}}. \tag{B.2}$$

Therefore, it suffices to prove that

$$\frac{1}{\mu\left(Q_j^{(1)} - Q_j^{(2)}\right) + Q_j^{(2)}} \le \frac{\mu\left(Q_j^{(2)} - Q_j^{(1)}\right) + Q_j^{(1)}}{Q_j^{(1)}Q_j^{(2)}}. \tag{B.3}$$

By cross multiplying and simplifying the RHS, we hold

$$\begin{aligned}
&Q_j^{(1)}Q_j^{(2)} \le \left(\mu\left(Q_j^{(2)} - Q_j^{(1)}\right) + Q_j^{(1)}\right)\left(\mu\left(Q_j^{(1)} - Q_j^{(2)}\right) + Q_j^{(2)}\right)\\
&= -\mu^2\left(Q_j^{(2)} - Q_j^{(1)}\right)^2 + \mu Q_j^{(2)}\left(Q_j^{(2)} - Q_j^{(1)}\right) + \mu Q_j^{(1)}\left(Q_j^{(1)} - Q_j^{(2)}\right) + Q_j^{(1)}Q_j^{(2)}\\
&= -\mu^2\left(Q_j^{(2)} - Q_j^{(1)}\right)^2 + \mu\left(Q_j^{(2)} - Q_j^{(1)}\right)\left(Q_j^{(1)} - Q_j^{(2)}\right) + Q_j^{(1)}Q_j^{(2)}\\
&= -\mu^2\left(Q_j^{(2)} - Q_j^{(1)}\right)^2 + \mu\left(Q_j^{(2)} - Q_j^{(1)}\right)^2 + Q_j^{(1)}Q_j^{(2)}\\
&= \left(Q_j^{(2)} - Q_j^{(1)}\right)^2(\mu - \mu^2) + Q_j^{(1)}Q_j^{(2)} = \mu(1-\mu)\left(Q_j^{(2)} - Q_j^{(1)}\right)^2 + Q_j^{(1)}Q_j^{(2)}.
\end{aligned} \tag{B.4}$$

Due to $\mu \in [0,\ 1]$, the value of $\mu(1-\mu)$ is always lager than or equal to zero. As $\left(Q_j^{(2)} - Q_j^{(1)}\right)^2 \ge 0$ always holds. Therefore, $Q_j^{(1)}Q_j^{(2)} \le \mu(1-\mu)\left(Q_j^{(2)} - Q_j^{(1)}\right)^2 + Q_j^{(1)}Q_j^{(2)}$ holds, and the proof is complete. ■

Appendix C

Proof of Theorem 1. *For simplicity, the matrix form of Expression* (11) *is defined as follows*:

$$f(X.Y.U) = \sqrt{2D(\boldsymbol{a}^T\boldsymbol{x} + D\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s} \oslash \boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{*}\boldsymbol{Y}_s)\boldsymbol{h})} + D\boldsymbol{\beta}^T\boldsymbol{Y}_s\boldsymbol{c} + DB\boldsymbol{\beta}^T\boldsymbol{u} - DP\boldsymbol{\beta}^T\boldsymbol{Y}_s. \tag{C.1}$$

where$\boldsymbol{Y}_s = \boldsymbol{I}_s\boldsymbol{Y}$. $\boldsymbol{I}_s$ is the matrices of different scenarios for each supplier. To prove the non-convexity of the Function (C.1), we use proof by contradiction. Therefore assume that the Function (C.1) is convex. It will suffice to merely show that the Hessian matrix for the Expression (C.1) is a positive semi-definite. We provide the Hessian matrix by calculating the derivatives of function (C.1) as follows:

$$\frac{\partial f}{\partial \boldsymbol{x}} = \frac{D\boldsymbol{a}(\boldsymbol{h}^T(\boldsymbol{Y}_s^T \odot \boldsymbol{Y}_s^T)\boldsymbol{\beta})}{\sqrt{2D(\boldsymbol{a}^T\boldsymbol{x} + \boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s} \oslash \mathbf{cap}))(\boldsymbol{\beta}^{\mathbf{T}}(\mathbf{Y_s} \odot \mathbf{Y_s})\mathbf{h})}}. \tag{C.2}$$

Note that the derivatives have been calculated using the methods introduced in (Petersen & Pedersen, 2012). we calculate $\frac{\partial f}{\partial \boldsymbol{Y}_s}$ using the Hadamard product properties (Petersen & Pedersen, 2012) as follows:

$$diag(\boldsymbol{\beta}) = \mathbf{M}, diag(\boldsymbol{h}) = \boldsymbol{H},$$

$$\boldsymbol{\beta}^T(\boldsymbol{Y}_s{*}\boldsymbol{Y}_s)\boldsymbol{h} = \boldsymbol{tr}\left(\mathrm{M}^T\boldsymbol{Y}_s\boldsymbol{H}\boldsymbol{Y}_s{}^T\right) = \boldsymbol{tr}\left(\mathbf{M}\boldsymbol{Y}_s\boldsymbol{H}\boldsymbol{Y}_s{}^T\right) \tag{C.3}$$

$$\frac{\partial \boldsymbol{tr}\left(\mathrm{M}\boldsymbol{Y}_s\boldsymbol{H}\boldsymbol{Y}_s{}^T\right)}{\partial \boldsymbol{Y}_s} = \boldsymbol{M}^T\boldsymbol{I}^T\boldsymbol{Y}_s\boldsymbol{H}^T + \boldsymbol{IM}\boldsymbol{Y}_s\boldsymbol{H} = \boldsymbol{M}^T\boldsymbol{Y}_s\boldsymbol{H}^T + \boldsymbol{M}\boldsymbol{Y}_s\boldsymbol{H} = 2\boldsymbol{M}\boldsymbol{Y}_s\boldsymbol{H} \tag{C.4}$$

where $diag(\boldsymbol{x})$ is the diagonal matrix of vector $\mathbf{x}$, $\boldsymbol{tr}(X)$ is the trace of matrix $\mathbf{X}$, and $\boldsymbol{I}$ is the identity matrix (unit matrix) with a size of $S \times S$. According to Eqs. (C.3) and (A.4), the derivative of $\frac{\partial f}{\partial \boldsymbol{Y}_s}$ is calculated as follow:

$$\frac{\partial f}{\partial \boldsymbol{Y}_s} = \frac{D(\boldsymbol{\beta}(\boldsymbol{s} \oslash \boldsymbol{cap})^T)(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{*}\boldsymbol{Y}_s)\boldsymbol{h})}{\sqrt{2D(\boldsymbol{a}^T\boldsymbol{x} + \boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s} \oslash \boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{*}\boldsymbol{Y}_s)\boldsymbol{h})}} + \frac{4D(\boldsymbol{a}^T\boldsymbol{x} + \boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s} \oslash \boldsymbol{cap}))(\mathrm{M}\boldsymbol{Y}_s\boldsymbol{H})}{\sqrt{2D(\boldsymbol{a}^T\boldsymbol{x} + \boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s} \oslash \boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{*}\boldsymbol{Y}_s)\boldsymbol{h})}} + D\boldsymbol{\beta}\boldsymbol{c}^T - DP\boldsymbol{\beta}$$

$$= \frac{D(\beta(s \oslash cap)^T)(\beta^T(Y_s{*}Y_s)h)}{\sqrt{2D(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)}} + \frac{4D(a^Tx+\beta^TY_s(s\oslash cap))((\beta h^T){*}Y_s)}{\sqrt{2D(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)}} + D\beta c^T - DP\beta. \tag{C.5}$$

Similarly, the other required derivatives are calculated as follows:

$$\frac{\partial f}{\partial u} = DB\beta, \tag{C.6}$$

$$\frac{\partial^2 f}{\partial x^2} = -\frac{Da(h^T(Y_s^T{*}Y_s^T)\beta)(\beta^T(Y_s{*}Y_s)h)a^T}{\sqrt{8D(a^Tx+\beta^TY_s(s\oslash cap))^3(\beta^T(Y_s{*}Y_s)h)^3}} < 0. \tag{C.7}$$

To calculate $\frac{\partial^2 f}{\partial {Y_s}^2}$, the derivative of the expression $(\beta h^T){*}Y_s$ is calculated by applying the theorems provided in (Bentler & Lee, 1978):

$$\frac{\partial((\beta h^T){*}Y_s)}{\partial Y_s} = \frac{\partial(\beta h^T)}{\partial Y_s}D_{Y_s} + \frac{\partial Y_s}{\partial Y_s}D_{\overline{\beta h^T}} = D_{\overline{\beta h^T}} \tag{C.8}$$

Where matrix $D_{\overline{\beta h^T}}$ is a diagonal matrix, the entries of which on the main diameter are $d_{ii} = \beta h_i^T$. Also, βh_i^T is equal to the i^{th} entry of the vector $vec(\beta h^T)$, Thus, $\frac{\partial^2 f}{\partial {Y_s}^2}$is calculated as follows:

$$\frac{\partial^2 f}{\partial {Y_s}^2} = \frac{[6D(MY_sH)((s\oslash cap)\beta^T) + 4D(a^Tx+\beta^TY_s(s\oslash cap))D_{\overline{\beta h^T}}]}{2D(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)} \times \frac{[\sqrt{2D(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)}]}{2D(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)}$$

$$- \frac{\left[\frac{D(\beta(s\oslash cap)^T)(\beta^T(Y_s{*}Y_s)h)+4D(a^Tx+\beta^TY_s(s\oslash cap))(MY_sH)}{\sqrt{2D(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)}}\right]}{2D(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)} \times \frac{[D(\beta(s\oslash cap)^T)(\beta^T(Y_s{*}Y_s)h)+4D(a^Tx+\beta^TY_s(s\oslash cap))(MY_sH)]}{2D(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)}$$

$$= \frac{[6D(MY_sH)((s\oslash cap)\beta^T) + 4D(a^Tx+\beta^TY_s(s\oslash cap))D_{\overline{\beta h^T}}]}{\sqrt{8D(a^Tx+\beta^TY_s(s\oslash cap))^3(\beta^T(Y_s{*}Y_s)h)^3}} \times \frac{[2D(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)]}{8D(a^Tx+\beta^TY_s(s\oslash cap))^3(\beta^T(Y_s{*}Y_s)h)^3}$$

$$- \frac{[D(\beta(s\oslash cap)^T)(\beta^T(Y_s{*}Y_s)h)+4D(a^Tx+\beta^TY_s(s\oslash cap))(MY_sH)]^2}{\sqrt{8D(a^Tx+\beta^TY_s(s\oslash cap))^3(\beta^T(Y_s{*}Y_s)h)^3}}. \tag{C.9}$$

The denominator in Eq. (C.9) is positive. Therefore, $\frac{\partial^2 f}{\partial {Y_s}^2}$ is positive if the numerator in Eq. (C.9) is positive that is as follows:

$$12D^2(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)(MY_sH)((s\oslash cap)\beta^T)$$

$$+8D^2(a^Tx+\beta^TY_s(s\oslash cap))^2(\beta^T(Y_s{*}Y_s)h)D_{\overline{\beta h^T}}$$

$$-D^2(\beta^T(Y_s{*}Y_s)h)^2(\beta(s\oslash cap)^T(s\oslash cap)\beta^T)$$

$$-4D^2(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)(MY_sH)((s\oslash cap)\beta^T)$$

$$-4D^2(\beta^T(Y_s{*}Y_s)h)(a^Tx+\beta^TY_s(s\oslash cap))(MY_sH)((s\oslash cap)\beta^T)$$

$$-16D^2(a^Tx+\beta^TY_s(s\oslash cap))^2(MY_sH)(HY_s^TM)$$

$$=8D^2(a^Tx+\beta^TY_s(s\oslash cap))^2(\beta^T(Y_s{*}Y_s)h)D_{\overline{\beta h^T}}$$

$$+4D^2(a^Tx+\beta^TY_s(s\oslash cap))(\beta^T(Y_s{*}Y_s)h)(MY_sH)((s\oslash cap)\beta^T)$$

$$-D^2(\beta^T(Y_s{*}Y_s)h)^2(\beta(s\oslash cap)^T(s\oslash cap)\beta^T)$$

$$-16D^2(a^Tx+\beta^TY_s(s\oslash cap))^2(MY_sH)(HY_s^TM)$$

$$=8D^2(a^Tx+\beta^TY_s(s\oslash cap))^2(\beta^T(Y_s{*}Y_s)h)D_{\overline{\beta h^T}}$$

$$+(D(\beta(s\oslash cap)^T)(\beta^T(Y_s{*}Y_s)h) - 4D(a^Tx+\beta^TY_s(s\oslash cap))(MY_sH))^2 > 0 \tag{C.10}$$

Because Eq. (C.10) is positive, $\frac{\partial^2 f}{\partial {Y_s}^2}$ is also positive, and the objective function (A.1) is convex in the direction of Y_s.

To calculate $\frac{\partial f}{\partial x \partial Y_s}$, we need to calculate the derivative of $a^Tx((\beta h^T){*}Y_s)$; however, since $a^Tx((\beta h^T){*}Y_s)$ is a matrix, the derivative is not defined (Petersen & Pedersen, 2012), and the following equations are used to calculate it:

$$\boldsymbol{a}^T\boldsymbol{x}\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)=\begin{bmatrix} a_1 & a_2 & \cdots & a_n\end{bmatrix}\begin{bmatrix} x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix}\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)=a_1x_1\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)+a_2x_2\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)+\cdots+a_nx_n\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right) \tag{C.11}$$

Thus, we have:

$$\begin{aligned}
\frac{\partial f}{\partial \boldsymbol{x}\partial \boldsymbol{Y}_s}&=\frac{\frac{-D^2\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)\left(\boldsymbol{\beta}(\boldsymbol{s}\oslash\boldsymbol{cap})^T\right)\boldsymbol{a}\left(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h}\right)}{\sqrt{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}}}{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}\\
&+\frac{4D\boldsymbol{a}\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)\sqrt{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}}{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}\\
&-\frac{\frac{4D^2\boldsymbol{a}\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)\left(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap})\right)\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)}{\sqrt{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}}}{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}\\
&=\frac{-D\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)(\boldsymbol{\beta}(\boldsymbol{s}\oslash\boldsymbol{cap})^T)\boldsymbol{a}(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}}\\
&+\frac{8D\boldsymbol{a}\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}}\\
&-\frac{4D\boldsymbol{a}\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}}\\
&=\frac{-D\boldsymbol{a}\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)(\boldsymbol{\beta}(\boldsymbol{s}\oslash\boldsymbol{cap})^T)(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}}\\
&+\frac{4D\boldsymbol{a}\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))\left(2(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})-\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)\right)}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}}\\
&=\frac{-D\boldsymbol{a}\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)(\boldsymbol{\beta}(\boldsymbol{s}\oslash\boldsymbol{cap})^T)(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}}\\
&+\frac{4D\boldsymbol{a}\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}}-\frac{4D\boldsymbol{a}\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}} \qquad \text{(C.12)}\\
&=\frac{D\boldsymbol{a}(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})\left(-\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)(\boldsymbol{\beta}(\boldsymbol{s}\oslash\boldsymbol{cap})^T)+4\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))\right)}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}}
\end{aligned}$$

Note that in the above equations, $\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)$ and $(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})$ are scalars with equal values, and they can be used interchangeably in matrix multiplications. In addition, the derivative of $(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})$ can be used instead of the derivative of $(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta})$ to calculate $\frac{\partial f}{\partial \boldsymbol{Y}_s\partial \boldsymbol{x}}$ due to their equal values. Therefore, we have:

$$\begin{aligned}
\frac{\partial f}{\partial \boldsymbol{Y}_s\partial \boldsymbol{x}}&=\frac{2D\boldsymbol{a}\boldsymbol{M}\boldsymbol{Y}_s\boldsymbol{H}\sqrt{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}}{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}\\
&-\frac{\frac{D^2(\boldsymbol{\beta}(\boldsymbol{s}\oslash\boldsymbol{cap})^T)(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})\boldsymbol{a}(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta})}{\sqrt{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}}}{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}\\
&-\frac{\frac{4D^2(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)\boldsymbol{a}(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta})}{\sqrt{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}}}{2D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}\\
&=\frac{8D\boldsymbol{a}\left(\left(\boldsymbol{\beta}\boldsymbol{h}^T\right)^*\boldsymbol{Y}_s\right)(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^{**}\boldsymbol{Y}_s)\boldsymbol{h})}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}}\\
&\frac{-D\left(\boldsymbol{h}^T\left(\boldsymbol{Y}_s{}^{T*}\boldsymbol{Y}_s{}^T\right)\boldsymbol{\beta}\right)(\boldsymbol{\beta}(\boldsymbol{s}\oslash\boldsymbol{cap})^T)\boldsymbol{a}(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})}{\sqrt{8D(\boldsymbol{a}^T\boldsymbol{x}+\boldsymbol{\beta}^T\boldsymbol{Y}_s(\boldsymbol{s}\oslash\boldsymbol{cap}))^3(\boldsymbol{\beta}^T(\boldsymbol{Y}_s{}^*\boldsymbol{Y}_s)\boldsymbol{h})^3}}
\end{aligned}$$

$$-\frac{4Da\left(\boldsymbol{h}^{T}\left(\boldsymbol{Y}_{s}{}^{T}*\boldsymbol{Y}_{s}{}^{T}\right)\boldsymbol{\beta}\right)\left(\boldsymbol{a}^{T}\boldsymbol{x}+\boldsymbol{\beta}^{T}\boldsymbol{Y}_{s}(\boldsymbol{s}\oslash\boldsymbol{cap})\right)\left(\left(\boldsymbol{\beta}\boldsymbol{h}^{T}\right)*\boldsymbol{Y}_{s}\right)}{\sqrt{8D\left(\boldsymbol{a}^{T}\boldsymbol{x}+\boldsymbol{\beta}^{T}\boldsymbol{Y}_{s}(\boldsymbol{s}\oslash\boldsymbol{cap})\right)^{3}\left(\boldsymbol{\beta}^{T}\left(\boldsymbol{Y}_{s}*\boldsymbol{Y}_{s}\right)\boldsymbol{h}\right)^{3}}} \tag{C.13}$$

Finally, the Hessian matrix can be obtained as follows:

$$H_0=\begin{bmatrix}\frac{\partial^2 f}{\partial \boldsymbol{x}^2} & \frac{\partial^2 f}{\partial \boldsymbol{x}\partial \boldsymbol{Y}_s} & 0\\ \frac{\partial^2 f}{\partial \boldsymbol{Y}_s\partial \boldsymbol{x}} & \frac{\partial^2 f}{\partial \boldsymbol{Y}_s{}^2} & 0\\ 0 & 0 & 0\end{bmatrix}$$

As it is shown the $\frac{\partial^2 f}{\partial x^2}$ and is $\frac{\partial^2 f}{\partial Y_s{}^2}$negative and positive, respectively, and the values of $\frac{\partial^2 f}{\partial x\partial Y_s}$and $\frac{\partial^2 f}{\partial Y_s\partial x}$ are equal. According to the definition of a positive semi-definite matrix (Bhatia, 2007; Petersen & Pedersen, 2012), the Hessian matrix is positive semi-definite if and only if all of its eigenvalues are non-negative. Therefore, the eigenvalues of the Hessian matrix is calculated as follows:

$$|H_0-\lambda I|=0\Rightarrow\begin{vmatrix}\frac{\partial^2 f}{\partial \boldsymbol{x}^2}-\lambda & \frac{\partial^2 f}{\partial \boldsymbol{x}\partial \boldsymbol{Y}_s} & 0\\ \frac{\partial^2 f}{\partial \boldsymbol{Y}_s\partial \boldsymbol{x}} & \frac{\partial^2 f}{\partial \boldsymbol{Y}_s{}^2}-\lambda & 0\\ 0 & 0 & -\lambda\end{vmatrix}=\left(\frac{\partial^2 f}{\partial \boldsymbol{x}^2}-\lambda\right)\left(-\lambda\left(\frac{\partial^2 f}{\partial \boldsymbol{Y}_s{}^2}-\lambda\right)-0\right)-\frac{\partial^2 f}{\partial \boldsymbol{x}\partial \boldsymbol{Y}_s}\left(-\lambda\frac{\partial^2 f}{\partial \boldsymbol{Y}_s\partial \boldsymbol{x}}-0\right)$$

$$=-\lambda\left(\frac{\partial^2 f}{\partial \boldsymbol{x}^2}-\lambda\right)\left(\frac{\partial^2 f}{\partial \boldsymbol{Y}_s{}^2}-\lambda\right)+\lambda\left(\frac{\partial^2 f}{\partial \boldsymbol{x}\partial \boldsymbol{Y}_s}\right)^2=0\Rightarrow\begin{cases}\lambda_1=0\\ \left(\frac{\partial^2 f}{\partial \boldsymbol{x}^2}-\lambda\right)\left(\frac{\partial^2 f}{\partial \boldsymbol{Y}_s{}^2}-\lambda\right)=\left(\frac{\partial^2 f}{\partial \boldsymbol{x}\partial \boldsymbol{Y}_s}\right)^2\end{cases}$$

Since $\left(\frac{\partial^2 f}{\partial x\partial Y_s}\right)^2$ is positive, $\left(\frac{\partial^2 f}{\partial x^2}-\lambda\right)\left(\frac{\partial^2 f}{\partial Y_s{}^2}-\lambda\right)$ will be positive. Therefore, the inequalities are held as follows:

$$\begin{cases}1)\left(\frac{\partial^2 f}{\partial \boldsymbol{x}^2}-\lambda\right)\le 0 and\left(\frac{\partial^2 f}{\partial \boldsymbol{Y}_s{}^2}-\lambda\right)\le 0\\ \qquad or\\ 2)\left(\frac{\partial^2 f}{\partial \boldsymbol{x}^2}-\lambda\right)\ge 0 and\left(\frac{\partial^2 f}{\partial \boldsymbol{Y}_s{}^2}-\lambda\right)\ge 0\end{cases} \tag{C.14}$$

From the Equation (C.14) and case 2, it is obvious that $\frac{\partial^2 f}{\partial x^2}\ge\lambda$ and $\frac{\partial^2 f}{\partial Y_s{}^2}\ge\lambda$. Due to $\frac{\partial^2 f}{\partial x^2}$ is negative, $\lambda<0$. In case 1, since $\frac{\partial^2 f}{\partial x^2}\le\lambda$, $\frac{\partial^2 f}{\partial Y_s{}^2}\le\lambda$ and $\frac{\partial^2 f}{\partial Y_s{}^2}$ is positive, λ can be to greater and equal zero. Therefore, the condition of the positive semi-definite matrix definition does not always hold. This is a contradiction. Thus, the function (C.1) is non-convex. The proof is now completed

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