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A supply chain model with service level constraints and strategies under uncertainty

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Transportation discounts;
Distribution-free approach (DFA);
Service level constraints (SLC)

Abstract In the current socio-economic situation, the daily demand for essential goods in the business sector is always changing owing to various unavoidable reasons. As a result, choosing the right method for profitable business has become quite tricky. This study introduces different business strategies based on constant and fuzzy demands. There are two types of constraints considered in this model to avoid the backorder cost. However, combining the service-level constraints with the constant and fuzzy demand, this study compares the total costs, and finally, the best strategy is established. Moreover, investing a small amount, this model improves the quality of the products and reduces the vendor's setup cost. Depending on the number of transported products, this model follows the transportation discount policy for hassle-free delivery of the products with a minimum delivery rate. The Kuhn-Tucker optimization technique is employed, and global optimality is verified numerically, analytically using the Hessian matrix. This model's robustness is discussed through a comparative study, numerical examples, sensitivity analysis, graphical representation, and managerial insights. Finally, some concluding remarks along with future extensions are discussed.

1. Introduction

Any production process yields perfect and defective products, although the number of defective products may not be high. Defective productions and fluctuation of demand can disturb the supply-chain business process. Keeping in mind, a combination of three different strategies-constant demand, fuzzy demand, and service level constraints (SLCs)-makes a revolution for a supply chain management (SCM) model. For a distribution-free model, further investment for improved quality and a reduction in setup cost can improve the business process with a minimum total cost. This study addresses this issue using the fuzzy method and compares the results among the four different issues. However, asymmetric approaches for reducing and eliminating waste, rework, and losses in the production process are involved in the quality improvement policy. Further, the seller and buyer incur the cost required to set up the equipment for processing different goods among the supply chain members. This study demonstrates a strategy

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to improve the quality of the products and decrease the setup cost.

By screening the delivery products received from a vendor, the buyer distinguishes them as perfect and defective products. The buyer further incurs a holding and transportation cost for the defective products returns to the vendor for a replacement. This model numerically demonstrates an increase in the number of transported products with a decrease in the transportation cost. However, the customers cannot afford to wait for the desired products. They choose the alternative from the other corner. For such cases hence, the buyer invests in decreasing the time between the order delivery and delivery supplied, i.e., reduce the lead time crashing cost.

Finally, it can be said briefly that, fuzzy concept considers the uncertain matter. In the fluctuating demand situation, the fuzzy concept can play a vital role in the smooth-running business process. More research from many profitable business directions overcame many more obstacles through fuzzy demand [1]. On the other side, imperfect production is a big push to supply chain management. Controllable lead time and service level constraints can overcome this situation [2]. However, transportation cost and inspection policy can overcome the unavoidable situation [3]. From the buyer side, inspection errors and the return of defective items for replacement are common for profitable cases [4]. Moreover, from the vendor side, quality improvement and setup cost reduction is another challenging matter [5–7]. A fuzzy supply chain model developed here with service level constraints and different strategies to determine the best method of a smooth-running business process.

The rest of this paper is organized as follows. Section 2 describes the related literature review of the keywords. Section 3 presents the definition of the problem, relevant notations, and associated assumptions. Mathematical modeling is described in Section 4, and Section 5 provides the methodology to determine the solution. Numerical examples are described in Section 6, and Section 7 presents a sensitivity analysis of the input parameters. Section 8 provides managerial insights into this study, and Section 9 presents the conclusions. Finally, the Appendix and relevant references are provided.

2. Related literature review

The existing research related to this field, contribution, and research gap are elaborately described in this section and an author contribution table. The following subsections are provided to illustrate the related research and gap in this direction. To make a supply chain more robust and flexible, the following research history is very much essential.

2.1. Supply chain management

A supply chain is a system of organization, people, activities, information, and resources involved in moving a product or service from supplier to customer. Every manufacturing system needs a perfect way to supply its products to retailers as well as customers. The concept vendor, buyer, supplier, and retailer are the most widely used concept in the supply chain. In the present socio-economic situation, the supply chain is the most restorative procedure for supplying the customers' necessary products. Its speed depends on the members. A vendor-buyer two-echelon supply chain model was first developed by Hill [8], who considered it a generalized policy. Cárdenas-Barrón [9] optimized inventory decisions for running a smooth business in supply-chain management in different stages with a multi-type customer. Cárdenas-Barrón [10] described a model based on different inventory model variations using algebra and analytical geometry. Tenget al. [11] presented an integrated vendor-buyer model considering the economic lot quantity to obtain a closed-type optimal solution. Kusukawa and Alozawa et al. [12] focused on a green supply chain based on achieving quality recycled parts' optimal operation. Using a contract policy, this green supply chain developed into a recycling activity. SCM and manufacturing models help the industries to generate low-cost models that ensure business productivity and success. The focus is to make valuable decisions regarding system design used in manufacturing systems and supply chains. Further, Sarkar et al. [13] developed a supply chain model based on a different strategy. Recently, Sarkar et al. [14] reported a cooperative advertising collaboration policy on a supply chain. [15] invented a supply chain incorporating unequal lot size under variable transportation. Further, their model introduces unpredictable conditions within the supply chain. Mishra et al. [16] conducted a study based on previous studies on carbon emission. They formulated a production inventory model under shortages. Bhuniya et al. [17] discussed a smart production process under an SCM with energy consumption-dependent demand. In this way, research on SCM is going on, but supply chain model with constant and fuzzy demand for the case of distribution-free approach so far, no researcher has considered.

2.2. Distribution free approach

The time gap between order receives and successful delivery is said to be a lead time. The expected demand during the lead time is known as lead time demand. In many cases, it is challenging to collect information about lead time demand. Without lead time demand distribution, it is not possible to calculate the exact shortage amount. To collect the necessary information regarding the lead time demand distribution, managers need to pay much money. In this situation, the min-max distribution-free approach is beneficial to solve the model. In these cases, lead time demand does not follow any distribution except the mean and standard deviation. This concept is known as the distribution-free approach. Scarf [18] invented a DFA. A newsvendor problem was used for his min-max solution. Using a specified distribution function, his model solved the pre-stated example. Gallego and Moon [19] extended the previous model through a different ordering method and cost. Moon [20,47] used a different DFA process to solve the previous inventory model. Wu et al. [21] modified their research through an extraordinary exponential cost with variable lead time-dependent demand with a computational algorithm for an optimal policy. Sarkar and Moon [22] proposed a model on defective production by improving the product quality and reducing the setup cost. Sarkar and Mahapatra [1] introduced a fuzzy inventory model in which the demand is a fuzzy demand and considered variable type lead time. They conducted a study considering the DFA concept. Further, Shin et al. [2] published a model based on the DFA policy with lead time and another technique SLC.

Majumder et al. [23] studied a supply chain model with a distribution-free approach. They made a comparison between a distribution and distribution-free approach. Kutlu [24] proposed a distribution-free stochastic frontier instrumental variable method with time-varying efficiency. Cui and Zhong [25] presented a distribution-free test of independence based on the mean-variance index. Chan et al. [26] discussed a single period assortment optimization problem with a multinomial logit model of consumer choice and static substitution. Many researches in a different direction based on a distribution-free approach has developed. However, a distribution-free approach for supply chain management with fuzzy demand is still not considered now.

2.3. Imperfect production

In any production process, defective production is an issue for profits. More production of defective products increases the loss and decreases the popularity. Many studies reuse defective products instead of selling defective products as less-quality goods at cheap rates. Chan et al. [27] extended the research conducted by Salameh and Jaber [28]. They proposed three different ideas for the defective items-either defective products sell at lower prices or are discarded or reused. Jaber et al. [29] proposed a model where defective products are returned for rework or replaced with a local supplier's right products. They reported that it is beneficial to either buy or repair the fraction of defective products rather than the replacement unit cost threshold value. Yu et al. [30] extended the previous model by considering the defective products as a new product using reworking. Many researchers develop their models by detecting defective items and return them for a new replacement. Sarkar and Saren [31] studied an imperfect production-based model in which warranty policy and inspection errors control the market demand. Cheng-Kang et al. [32] presented an imperfect production model, where warranty and rework have a massive impact on selling price-dependent demand. Cheikhrouhou et al. [4] proposed a model for imperfect production by involving an inspection cost. Their model consists of two possible cases: first, defective lots are received from the market through the retailer's strategy and own expenses and send back to the supplier. Secondly, using supplier investment defective products from the market and retailer returns in the next lot, such as in the food industry. Tayyab et al. [33] proposed a model of defective items. Rework under fuzzy demand plays an important role in facing the maximum profit. Khanna et al. [34] recently developed a defective production model in which the warranty and a maintenance policy were used, and a vendor-buyer strategy was considered. Recently, Ahmed et al. [35] discussed a model of Reworking for Imperfect Quality Items with the Integration of Multi-Period Delay-in-Payment and Partial Backordering in Global Supply Chains. Imperfect production for the fuzzy supply chain management with service level constraints so far no research has developed.

2.4. Service level

In the current socio-economic situation, it is essential to survive in business competition. However, it can be found that there are several competitions among many manufacturing companies. Companies continue to strive to make their products better than others on the market. In this case, the service level of products performs a significant role in promoting and selling their products. Customers are always more sensitive to buy any item if they can get more service facilities. Products with more service level increases demand of the product and vice versa. Hence, service level constraints significantly affect controlling the market demand and companies' reputation and advertisement. Gift policy, safety stock, warranty policy, rework facility, delivery facility are part of the service level constraints. Thus, the demand for huge products ensures more profit for manufacturing companies while reducing the total cost. Chen and Krass [36] proposed a model where stockout cost was used rather than the minimal service level constraint. Their model developed an optimal ordering strategy, which was obtained using minimal service level constraints. Lee et al. [37] introduced objective functions for service level constraints in their model instead of stockout cost. Their research used the backorder rate. They also assumed SLC and introduced a normal distribution for lead time. Jha and Shanker [38] presented an integrated model with participating vendors and buyers where service level constraints were considered instead of shortage costs. They also considered normally distributed stochastic demand. Taleizadeh et al. [39] developed a model based on multi-products where the system's capacity is limited in terms of production and service rate constraint. Sarkar et al. [40] considered a distribution-free model by incorporating two concepts to produce improved quality products and reduce setup cost to minimize the total cost. Their model also had a provision of using the service level constraint policy. Shin et al. [2] reviewed a model with service level constraints by considering distribution and distribution-free approaches. Gruson et al. [41] proposed service levels for the deterministic lot-sizing problem. Escalona et al. [42] presented the effect of two service-level measures on the design of a critical-level policy for fast-moving items. Sett et al. [43] discussed a supply chain model with a single vendor and sing buyer participating members with upgraded service. They developed the strategy to increase service in the presence of an unreliable vendor and an online-to-offline (O2O) channeling system. However, no researcher has researched service level constraints with fuzzy demand and distribution-free approach.

2.5. Quality improvement and setup cost reduction

Nowadays, in the competitive market, improved quality products have a vital role in the production system. Investment for improved quality products and reduced setup cost was introduced by Porteus [44]. He presented the formula for initial setup investment reduction in another paper. His research presents a significant relationship between the quantity and quality of products. Many researchers have made their contributions to this topic. Keller and Noori [45] extended the Porteus model by considering the probabilistic demand function based on lead time. Additionally, the model was modified by introducing the quality improvement cost. Hwang et al. [46] discussed a model where setup cost reduction and quality improvement were achieved through one investment. Their economic production quantity model is considered multi-products. Power functional geometric programs were applied to formulate investment costs, and closed-form solutions were obtained for the problems. Moon [20,47] published a paper in the same direction by improving the quality of multi-products. His economic order quantity model was developed through a one-time initial investment. Hng and Hayya [48] extended a previously published paper by considering budget constraints only. Ouyang et al. [49] presented an integrated SCM model for two different cases, reducing setup investment cost and improved quality. In the first case, lead-time-dependent demand was formulated by a normal distribution. In the second case, lead-time-dependent demand was formulated without considering any distribution. They proved that continuous investment for reducing setup investment cost is better than constant setup investment cost, and improved quality products are better than fixed quality products. Annadurai and Uthayakumar [50] developed an integrated inventory model that also reduced lost sales. A credit period policy was introduced to the seller by Abad and Jaggi [51]. To earn more profit, they also considered unit selling price and end demand as price sensitive. Wee et al. [52] presented a research paper by considering the time length of production and backorder. They considered the same as the decision variables rather than the decision variables lot size and backorder variables for the optimal solution. Pal et al. [53,54] considered a supply chain model and demonstrated an incorporated strategy for improved quality products. Further, their model used a non-linear demand function. Sarkar et al. [55] developed a model with investment for improved quality. Process quality improvement discounts the back order as well as lead time to control the inventory model. In the first case, they used lead time distribution demand. In the second case, they formulated a model without considering distribution demand. Sarkar and Mahapatra [1] developed a model by extending the model proposed by Annadurai and Uthayakumar [50]. They considered fluctuating demand, referred to as fuzzy demand, with investment for improved quality products and reduced setup cost. Majumder et al. [23] considered a supply chain model based on quality improvement and setup cost reduction. Dey et al. [56] presented a paper on a discrete setup cost reduction function in an integrated-type sustainable inventory model. Environmental issue lighted their research with controllable lead time, and this model maximized the profit of the inventory model. Khan et al. [57] presented a model with two participating members considering a transportation discount policy and a reduced manufacturer investment in the setup cost with improved process quality. Their study showed the effect of electrical energy on the supply chain. Sarkar et al. [58] introduced a safety factor in their two-echelon model by increasing the quality and quantity of products and reducing the manufacturer setup cost. Dey et al. [56] improved their previous model by considering the concept of reduced setup time and cost. Further, they used production rate as a variable to illustrate the reliability of the model. Bhuniya et al. [59] developed a model based on consumption energy by decreasing the production system's failure rate. They considered variable demand in a modified version of the model. Guchhait et al. [6] conducted a study on imperfect production. They considered the cost of improving quality and reduced investment in the setup cost. They also included warranty policy and shortages. Their proposed research also considered production system is subject to a random breakdown. Sarkar et al. [7] discussed sustainable supply chain management for improved quality products. However, essential matter carbon emission is also incorporated in their model. Although there is much research on quality improvement, setup cost reduction, transportation discount, the effect of service quality, and distribution-free approach on the improved quality products, research has not been done yet.

2.6. Fuzzy demand

The annual market demand can fluctuate in an actual situation due to several reasons depending on different socio-economic circumstances. To avoid such an uncertain situation, we considered a fuzzy demand rather than constant market demand. Many researchers have already developed models based on this concept. Park et al. [60] first introduced this fuzzy concept to an economic order quantity model. His research model was based on the fuzzy aspect of cost determination. Further, his study reexamined this model using a fuzzy-set-theoretic perspective. Chen et al. [61] extended their previous model using the backorder concept and used the direct derivation method to determine an optimal solution. Yao and Lee [62] developed previous models by introducing a triangular fuzzy number to order quantities; they solved constraints and optimized their model. Ouyang and Yao [63] considered a model to obtain the annual demand confidence interval. They used two types of fuzzy concepts in their model, simple annual fuzzy demand and statistic annual demand fuzzy number. Dutta et al. [64] proposed a new method, known as the garden mean integration representation method, to determine the optimal quantity of orders based on fuzzy random variable demand. Taleizadeh et al. [65] developed a multi-product inventory model using a hybrid intelligent algorithm to solve a non-linear problem of multi-object integers. Taleizadeh et al. [66] presented a model considering three different demands: rough variable demand, stochastic demand, and fuzzy demand. In their model, the incremental discount policy was used with constraints on multiple products and a single period. Sarkar and Moon [22] modified Ouyang et al. [49] model; instead of a fuzzy demand and review period, they considered a constant demand and only used a variable backorder rate. Sarkar and Mahapatra [1] developed a previous model by introducing a distribution-free approach with an average demand distribution. Arqub et al. [67] developed a method for solving fuzzy differential equations based on the reproducing kernel theory under strongly generalized differentiability. Arqub [68] discussed a kernel Hilbert space method to obtain the exact and the numerical solutions of fuzzy Fredholm-Volterra integrodifferential equations. Arqub et al. [69] studied the analytic and approximate solutions of second-order, two-point fuzzy boundary value problems based on the reproducing kernel theory under the assumption of strongly generalized differentiability. Mahapatra et al. [70] extended their previous model by incorporating deteriorating items. Their model developed an economic order quantity system through a fuzzy concept. They used efficient computational algorithms with the objective of a minimum expected total cost for two individual models. Arqub and Al-Smadi [71] proposed a new definition of fuzzy fractional derivative, so-called fuzzy conformable, and discussed fuzzy conformable fractional integral. Recently, Samanta et al. [72] The concept of associated networks is introduced here in a fuzzy environment. Involvement of fuzzy

Table 1 Contribution of the authors.

Author (s)	Quality improvement	Setup cost reduction	Fuzzy demand	Service level constraints	SCM	Transportation discount	Distribution free approach
[1]		√	√				√
[5]		√			√		
[6]	√	√					
[9]					√		
[13]	√	√			√		
[37]				√	√		
[40]	√	√		√			√
[46]	√	√					
[47]							√
[48]	√	√			√		
[51]	√	√					
[53]					√		
[56]	√	√			√		
[57]	√	√		√	√	√	
[63]			√				√
[66]				√			
This Paper	√	√	√	√	√	√	√

demand for the imperfect production model with distribution-free approach and service level constraints under a supply chain management is still not considered in any research model.

In the past, research papers have been advanced with the help of various methods. Arqub and Abo-Hammour [73] presented a genetic algorithm to solve second-order boundary value problems. Their proposed method helps to find out the global nature in terms of the solutions obtained and its ability to solve mathematical and engineering problems. In the other direction, analytical and numerical solution methods for fractional differential equations have been much studied in the literature Akgül [74], Akgül [75], Owolabi et al. [76], Atangana et al. [77], Baleanu et al. [78]. Recently, Kumar et al. [79] discussed a methodology to obtain optimization through a fuzzy linear programming problem in which fuzzy numbers signify the right side parameters. A comparison between previous studies and this study is shown in Table 1.

3. Model purpose, symbolic notation, and assumptions

In this portion purpose of the problem along with symbols and hypotheses are adequately described. At first problem, purpose describes elaborately, then symbols of the mathematical model and at the last portion assuming hypotheses briefly describe.

3.1. Model purpose

The proposed model gives a new direction through a supply chain strategy with two participating members, one vendor, and one buyer. Constant and fuzzy type of demand considers here to compare that which is beneficial for the current market situation. Controllable lead time is considered to increase the customer's confidence and product reputation of availability. However, to avoid the lead time and backorder cost in this study, lead time crashing cost is introduced to decrease the customer waiting time regarding the secondary market availability. The distribution-free approach is considered here to obtain the shortage as well as lead time demand. For controlling the pollution and for reducing the total cost, transportation discounts are proposed here. To avoid the backorder cost, service level constraints are proposed here. Shin et al. [2] is considered a quality improvement, setup cost reduction with distribution-free approach and transportation discount. In the same direction, Guchhait et al. [6] incorporated warranty and shortages. Both of the considered constant types of demand. In the present market situation, product demand varies depending on the session, place, socio-economic situation. In such cases, the fuzzy demand-based supply chain model gives an appropriate model for a smooth business process to fulfill the customer demand. Keeping find, the proposed model considered different cases on behalf of fuzzy demand with service level constraint to identify the best method strategy for the optimization of total SCM cost. This integrated SCM's primary focus is to identify the joint SCM cost by optimizing the verdict variables.

3.2. Notations

The model structure depends on the following variables and parameters.

Verdict variables

Q	order quantity (units)
ψ	probability of imperfect production which may move to out-of-control state
S	setup cost for vendor per setup (\$/setup)
l	length of the lead time (weeks)
n	number of shipments (positive integer)

Input parameters

ξ_i	transportation cost i = 1,2,3,...,n (\$/unit)
σ	standard deviation of the demand per week (units per week)
h_1	retaining cost of good quality products incurred by the vendor(\$ per unit per year)
h_2	retaining cost of defective products incurred by the buyer (\$ per unit per year)
h_3	retaining cost incurred by the vendor (\$ per unit per year)
α	screening rate of the products (units/year)
λ	fraction of annual capital investment cost (\$/year)
g	scaling parameter of the investment for the quality improvement function
G	scaling parameter of the investment for the setup cost reduction function
ψ_0	primary probability of the defective production
S_0	initial cost of setup incurred by the vendor (\$ per setup)
D	average market demand of the products (units/ year)
A	ordering cost incurred by the buyer (\$ per order)
$C(l)$	compensation cost of the lead time per order (\$/order)
φ	fraction of defective products supplied (unit/year)
η	replacing cost of defective products (\$/defective unit)
u	screening cost per unit (\$/unit)
γ	the reciprocal of P (years/unit)
λ_i	the lagrangian coefficient of the kuhn-tucker method (i = 1, 2.)
ϵ	fraction of customer's demand that is satisfied regularly
u_i	minimum duration for ith lead-time component (days), i = 1, 2, ..., n
v_i	normal duration for ith lead-time component (days), i = 1, 2,..., n
m_i	crashing cost per day for ith component of lead-time(\$/day), i = 1, 2,..., n
l_i	length of lead time with components i = 1, 2,..., n (weeks)
P	production rate per unit time (units/year)

3.3. Assumptions

The model is formulated with the following assumptions.

1. A supply chain management model is proposed with the participation of supply chain members-vendor and buyer. The product demand is considered a stochastic and fuzzy demand. Further, defective production to the vendor is considered.
2. The vendor and buyer incurred the P production and screening rate α. The production and screening rate must be greater than the market demand for the supply chain members. When the buyer receives the produced lot, they perform a thorough inspection with the screening rate α, which detects the defective products that are generally produced. The screening of the products is assumed to be free of error.
3. To avoid a stockout situation or reduce the lead time, this model considers SLC based on improved quality and reduction in the setup investment cost. Logarithmic expressions are used for the investment function of the improved-quality products and setup cost reduction technique.
4. A distribution-free approach is considered here. Further, a transportation discount policy is considered that is based on the product quantity. The crashing cost for lead time per cycle is considered. $C(l) = m_i(l_{i-a} - l) + \Sigma_{j=1}^{i-1} m_j(v_j - u_j)$, where $l \in (l_i - l_{i-1})$ [22]
5. The number of perfect products during the inspection is equal to the least market demand, i.e., $Q(1+\psi)(1-\psi) \geqslant DQ(1+\psi)/\alpha$, which implies that $\psi \leqslant 1 - D/\alpha$ (from [28]). The holding cost incurred by the buyer is considered for the perfect and defective products.

4. Model formulation

The proposed model presents an SCM with two participating members with capital investment for improved-quality products and set-up-cost reduction under the SLC. Here, the vendor's production rate is P. Defective products are produced when the system is out of control. The defective product incurs a holding cost for replacing the items. Although this model shows the changes in the fuzzy demand, the buyer further demonstrates a stochastic demand. Fig. 1 represents the framework of the proposed model.

4.1. Vendor's model

The vendor's production is based on stochastic and fuzzy demand. There is a high capital investment for improving the quality of production to decrease the defective products. However, the defective products can be replaced within the lead time. To reduce the lead time, SLCs are considered here, i.e., the time of the product's order and the time taken for the delivery are reduced. The following costs are considered for the vendor.

4.1.1. Vendor's setup cost (VSC)

By investing in this cost, the vendor can set up the equipment to produce a different goods batch. It is the highest cost for a production inventory model and supply chain management. Depending on the developed setup, the production improves faster. Therefore, the total setup investment cost per cycle is

$$VSC = \frac{DS}{nQ} \tag{1}$$

4.1.2. Vendor's quality improvement cost (VQIC)

Defective production is an essential factor for a production system, which gives uncertainty to a supply chain. A production system is reliable when the top products produced are perfect. Here, the vendor invests capital for improving the quality of the products to reduce the production of defective products,

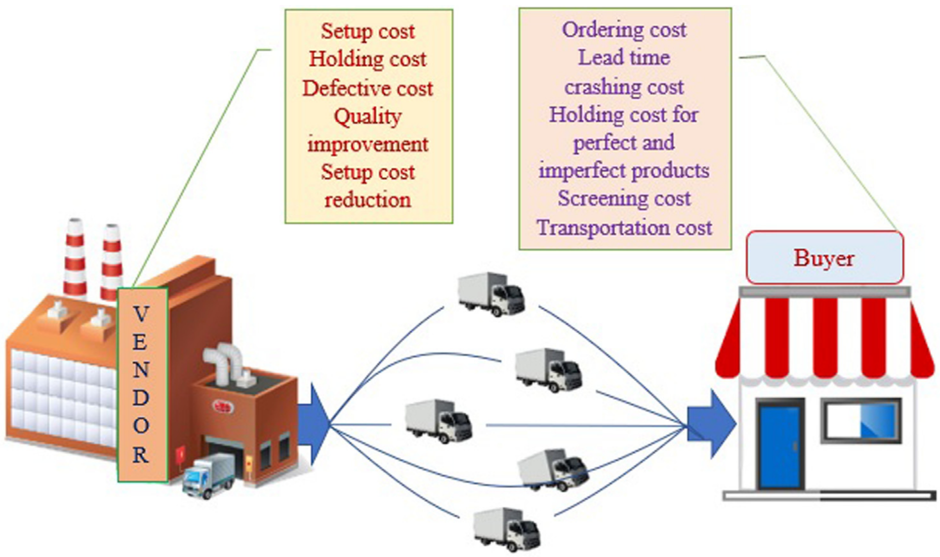

Fig. 1 Proposed Model of Production Management.

i.e., to make the system in-control from out-of-control. Hence, this cost is given as

$$VQIC = \lambda g \log \frac{\psi_0}{\psi} \tag{2}$$

4.1.3. Vendor's setup reduction cost (VSRC)

The production setup is the most critical factor that affects the joint SCM total cost of a supply chain through which a manufacturer builds the setup for production. This model is developed by introducing a small cost that addresses the quality and quantity of the products, and the setup investment cost is reduced. A logarithmic function is used to express this type of cost. Hence, the cost is given as

$$VSRC = \lambda G \log \frac{S_0}{S} \tag{3}$$

4.1.4. Vendor's holding cost (VHC)

The holding cost yields a strong inventory supply chain. All unsold products are stored through this type of investment. This cost is an essential cost of the total supply chain costs parallel to the ordering costs and shortage costs. VHC includes the cost for holding the products, remodifying with an advertisement, and long-time replenishment. Vendors consider the holding cost for holding the products that are produced in their production system until all the products are delivered to the buyer. Hence, the total cost of holding the products is given by

$$VHC = \frac{h_3 Q}{2}[n(1 - D\gamma) - 1 + 2D\gamma] \tag{4}$$

4.1.5. Vendor's defective cost (VDC)

Defective production is a common factor in a production supply chain model. It reduces the quality and quantity of the products. Defective products reduce the total profit and increase the total cost. The vendor invests in replacing the defective products produced when the machine is in an out-of-control situation. The total cost for replacing the defective products is given as

$$VDC = \eta D\psi \tag{5}$$

4.1.6. Vendor total cost (VTC)

Thus, the total cost incurred by a vendor is

$$\begin{aligned} VTC &= (VSC + VQIC + VSRC + VHC + VDC) \\ &= \frac{DS}{nQ} + \lambda g \log \frac{\psi_0}{\psi} + \lambda G \log \frac{S_0}{S} + \frac{h_3 Q}{2}[n(1 - D\gamma) - 1 \\ &\quad + 2D\gamma] + \eta D\psi \end{aligned} \tag{6}$$

4.2. Buyer's model

The buyer receives the products from the vendor by investing in the ordering cost and market transfer. Here, the buyer invests in the holding cost for two different products separately-perfect and defective products. Although the defective products returned to the vendor for replacement, the buyer further invests in the received products' transportation and screening. The buyer considers the following costs for such a model.

4.2.1. Buyer's ordering cost (BOC)

The ordering cost is the most crucial factor of the SCM through which the members are connected. To purchase the products from the vendor, the buyer invests in the ordering cost. There are different ordering methods-a phone call, mail, through a representative. Hence, the total ordering cost that the buyer invests in for his business is given as

To purchase the products from the vendor, the buyer invests in the ordering cost. There are different ordering methods—phone call, mail, through a representative etc. Hence, the total ordering cost that the buyer invests in for his business is given as

$$BOC = \frac{DA}{nQ}$$

4.2.2. Buyer's lead time crashing cost (BLC)

The time gap between receiving the order from a customer and delivering the products is called the lead time. Many researchers considered it to be a negligible amount, but the lead time is a vital factor in the customer demand. This model considered this cost to reduce the time between ordering and delivery of the products. Hence, the crashing cost of lead time is given by

$$BLC = \frac{DC(l)}{Q} \tag{8}$$

4.2.3. Buyer's perfect product holding cost (BPHC)

The buyer receives the products from the vendor and distinguishes them as perfect and defective products. After receiving the products, they may sell them immediately or stock them for a specific time. In this case, the products may become defective. Hence, the buyer invests in two types of costs to hold the products. To maintain the product quality, the buyer invests in the following costs:

$$BPHC = h_1[\psi Q - \frac{\psi(1+\psi)DQ}{2\alpha}] \tag{9}$$

4.2.4. Buyer's imperfect product holding cost (BIPHC)

The defective products incur high losses for the buyer. Currently, most buyers return their products to the vendors, and the vendors accept them for the smooth running of the business. To stock the defective products, the buyer invests in different costs and then returns the products to the vendor for a replacement. In this case, different stock holding costs can identify the perfect products, maintain the lead time, and fulfill the customer demand. Hence, the BIPHC is given by

$$BIPHC = h_2[\frac{Q}{2} + \frac{\sigma^2 l}{4(1-\epsilon)Q} + \frac{\psi(1+\psi)DQ}{2\alpha}] \tag{10}$$

4.2.5. Buyer's screening cost (BSC)

Through the screening cost, the buyer separates the perfect and defective products. The quantity of the perfect products must satisfy the market demand. The buyer invests the capital for screening the products. Here, the buyer can investigate the products' quality to be perfect or imperfect by investing a specific cost. Further, the buyer can transfer the products to different holding places. Hence, the BSC is given as follows:

$$BSC = uD(1+\psi) \tag{11}$$

4.2.6. Buyer's transportation cost (BTPC)

The buyer invests in a transportation cost for transporting the products from the vendor's warehouse. The defective products can be returned to the vendor using the BTPC. To maintain the customer demand, transportation plays a vital role in the vendor and buyer supply-chain system. Furthermore, for the fuzzy demand, the BTPC helps the buyer to transport those products to the customer. Hence, the BTPC is given as follows:

$$BTPC = D\xi_i \tag{12}$$

Here transportation cost reduction techniques consider for reducing the total cost. The transportation cost is not constant, it is directly related to the demand. Here ξ_i is the transportation cost per unit, be such that $\xi_0 > \xi_1 > \xi_2 > \xi_3 > \xi_4 \ldots > \xi_B$. Moreover, ξ_i depends on the transport amount Q, which belongs to $[M_i, M_{i+1})$ and $M_0 = 0$, i.e., Q should lie in some specified range (see [2]).

4.2.7. Buyer total cost (BTC)

Thus, the total cost of buyer per cycle is

$$\begin{aligned} BTC &= (BOC + BLC + BPHC + BIPHC + BSC + BTPC) \\ &= \frac{DA}{nQ} + \frac{DC(l)}{Q} + h_1[\psi Q - \frac{\psi(1+\psi)DQ}{2\alpha}] \\ &\quad + h_2[\tfrac{Q}{2} + \tfrac{\sigma^2 l}{4(1-\epsilon)Q} + \tfrac{\psi(1+\psi)DQ}{2\alpha}] + uD(1+\psi) + D\xi_i \end{aligned} \tag{13}$$

4.3. Total cost of the SCM (TCS)

Hence, the SCM has joint cost that is given by:

$$\begin{aligned} TCS &= VTC + BTC \\ &= \frac{DS}{nQ} + \lambda g \log\frac{\psi_0}{\psi} + \lambda G \log\frac{S_0}{S} + \frac{h_3 Q}{2}[n(1-D\gamma) - 1 + 2D\gamma] \\ &\quad + \eta D\psi + \tfrac{DA}{nQ} + \tfrac{DC(l)}{Q} + h_1[\psi Q - \tfrac{\psi(1+\psi)DQ}{2\alpha}] \\ &\quad + h_2[\tfrac{Q}{2} + \tfrac{\sigma^2 l}{4(1-\epsilon)Q} + \tfrac{\psi(1+\psi)DQ}{2\alpha}] + uD(1+\psi) + D\xi_i \end{aligned} \tag{14}$$

There may arise four cases depending on different assumptions. Case 1 is based on a simple total integrated cost of the supply chain (TCS) with no particular condition. For the smooth production process, the essential criteria for supply chain management are SLCs based on the investment for improved-quality products and set-up-cost reduction. Thus, the SLCs are used as follows: $0 < \psi \leqslant \psi_0$ and $0 < S \leqslant S_0$ which are considered in case 2. Demand D in Eq. (14) is considered stochastic. However, depending on different situations, the demand may change slightly. In this case, the stochastic demand D is considered instead of the fuzzy demand $\tilde{D}$. Thus, case 3 arises. Case 4 consists of the total cost of the SCM depending on the fuzzy demand with SLCs.

4.4. Case: 1. Total cost of the SCM (TCS)

Here, the total cost of the SCM is as aforementioned without a constraint and constant annual demand.

$$\begin{aligned} TCS(Q,\psi,S,l,n) &= \tfrac{DS}{nQ} + \lambda g \log\tfrac{\psi_0}{\psi} + \lambda G \log\tfrac{S_0}{S} + \tfrac{h_3 Q}{2}[n(1-D\gamma) - 1 \\ &\quad + 2D\gamma] + \eta D\psi + \tfrac{DA}{nQ} + \tfrac{DC(l)}{Q} + h_1[\psi Q - \tfrac{\psi(1+\psi)DQ}{2\alpha}] \\ &\quad + h_2[\tfrac{Q}{2} + \tfrac{\sigma^2 l}{4(1-\epsilon)Q} + \tfrac{\psi(1+\psi)DQ}{2\alpha}] + uD(1+\psi) + D\xi_i \end{aligned} \tag{15}$$

4.5. Case: 2. Total integrated cost of the supply chain management including service level constraints (TCSSLC)

Here, SLCs and integrated cost of the supply-chain management are introduced; supply-chain management consists of two SLCs based on improved quality products and set-up-cost reduction. To minimize the total cost, the Kuhn-Tucker condition is incorporated. Thus, the integrated cost of the supply chain with SLCs is

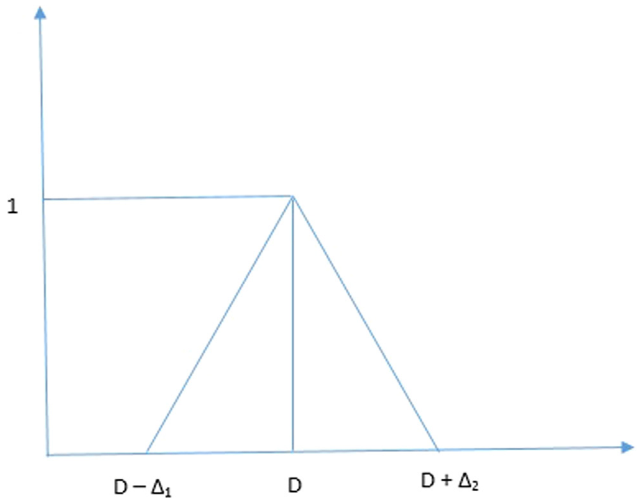

Fig. 2 Triangular fuzzy number $\tilde{D}$.

$$\begin{aligned} TCSSLC(Q,\psi,S,l,n) = {} & \tfrac{DS}{nQ} + \lambda g \log\tfrac{\psi_0}{\psi} + \lambda G \log\tfrac{S_0}{S} + \tfrac{h_3 Q}{2}[n(1-D\gamma) \\ & -1+2D\gamma] + \eta D\psi + \tfrac{DA}{nQ} + \tfrac{DC(l)}{Q} \\ & + h_1[\psi Q - \tfrac{\psi(1+\psi)DQ}{2\alpha}] \\ & + h_2[\tfrac{Q}{2} + \tfrac{\sigma^2 l}{4(1-\epsilon)Q} + \tfrac{\psi(1+\psi)DQ}{2\alpha}] \\ & + uD(1+\psi) + D\xi_i + \lambda_1(\psi-\psi_0) \\ & + \lambda_2(S-S_0) \end{aligned} \tag{16}$$

4.6. Case: 3. Total integrated cost of the supply chain for fuzzy demand (TCSFD)

This case is formulated using a different demand method. Annual constant demand D in Eq. (14) is considered stochastic. Here, considering the current market situation, the annual constant demand fluctuates owing to different situations. Here, the annual constant D is altered by the fluctuate demand $\tilde{D}$. Thus, $\tilde{D}$ = (D-Δ_1, D, D + Δ_2), where $0 < \Delta_1 < D$and $0 < \Delta_2$ (from Fig. 2). The membership contribution is as follows:

$$\begin{aligned} \mu_{\tilde{D}}(x) & = \tfrac{x-D+\Delta_1}{\Delta_1} \;\; if\; D-\Delta_1 \leqslant x \leqslant D \\ & = \tfrac{D+\Delta_2-x}{\Delta_2} \;\; if\; D \leqslant x \leqslant D+\Delta_2 \\ & = 0 \quad otherwise \end{aligned} \tag{17}$$

Then centroid of $\mu_{\tilde{D}}(x)$ is

$$D^* = D + \frac{1}{3}(\Delta_2 - \Delta_1)$$

The following result is considered to obtain the fuzzy demand: Let

$$TCS_{(Q,\psi,S,l,n)}(x) = y(>0) \tag{19}$$

For the fuzzy type of cost, the membership function (from Fig. 3) reduces to: $TCS_{(Q,\psi,S,l,n)}(\tilde{D})$ which gives the formula:

$$\begin{aligned} \mu_{TCS_{(Q,\psi,S,l,n)}(\tilde{D})}(y) & = sup_{x\epsilon TCS^{-1}_{(Q,\psi,S,l,n)}(y)} \mu_{\tilde{D}}(x) \; if\, TCS^{-1}_{(Q,\psi,S,l,n)}(y) \neq \phi \\ & = 0 \;\; if\, TCS^{-1}_{(Q,\psi,S,l,n)}(y) = \phi \end{aligned}$$

From $TCS_{(Q,\psi,S,l,n)}(x) = y$ and Eq. (14) we obtain

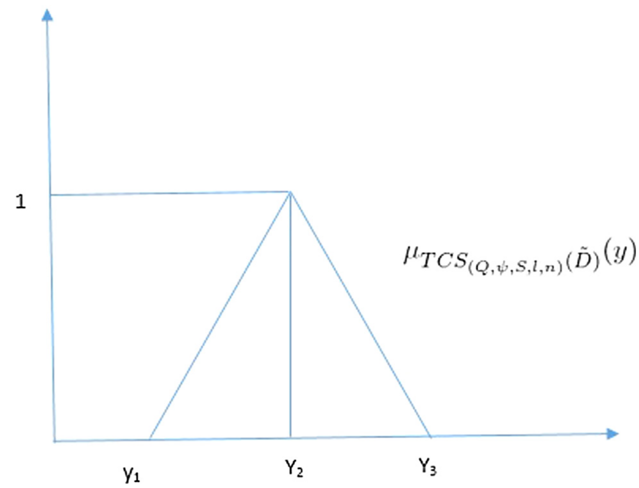

Fig. 3 Triangular fuzzy number $TCS_{(Q,\psi,S,l,n)}(\tilde{D})$ in a normal distribution model.

$$\begin{aligned} y = {} & \tfrac{xS}{nQ} + \lambda g \log\tfrac{\psi_0}{\psi} + \lambda G \log\tfrac{S_0}{S} + \tfrac{h_3 Q}{2}[n(1-x\gamma) - 1 + 2x\gamma] \\ & + \eta x\psi + \tfrac{xA}{nQ} + \tfrac{xC(l)}{Q} + h_1[\psi Q - \tfrac{\psi(1+\psi)xQ}{2\alpha}] \\ & + h_2[\tfrac{Q}{2} + \tfrac{\sigma^2 l}{4(1-\epsilon)Q} + \tfrac{\psi(1+\psi)xQ}{2\alpha}] + ux(1+\psi) + x\xi_i \\ x = {} & \tfrac{y-\phi_1}{\phi_2} \\ & where\; \phi_1 = \lambda g \log\tfrac{\psi_0}{\psi} + \lambda G \log\tfrac{S_0}{S} + h_1\psi Q + h_2\{\tfrac{Q}{2} + \tfrac{\sigma^2 l}{4(1-\epsilon)Q} \\ & -(1-\epsilon)Q\} + \tfrac{h_3 Q}{2}(n-1) \\ & and\; \phi_2 = \tfrac{S}{nQ} + \tfrac{A}{nQ} + \tfrac{C(l)}{Q} - h_1\tfrac{\psi(1+\psi)}{2\alpha} \\ & + h_2\tfrac{\psi(1+\psi)Q}{2\alpha} + u(1+\psi) + h_3\tfrac{Q}{2}\{-n\gamma + 2\gamma\} + n\psi + \xi_i \end{aligned} \tag{20}$$

From Eq. (19) and Eq. (22), the reduced membership function is: $\mu_{TCS_{(Q,\psi,S,l,n)}}(\tilde{D})$ can be written as:

$$\begin{aligned} \mu_{TCS_{(Q,\psi,S,l,n)}(\tilde{D})}(y) & = \tfrac{y-\phi_1}{\phi_2\Delta_1} - \tfrac{D-\Delta_1}{\Delta_1} \;\; if\; y_1 \leqslant y \leqslant y_2 \\ & = \tfrac{D+\Delta_2}{\Delta_2} - \tfrac{y-\phi_1}{\phi_2} \;\; if\; y_2 \leqslant y \leqslant y_3 \\ & where\; y_1 = \phi_1 + (D-\Delta_1)\phi_2 \\ & \quad y_2 = \phi_1 + D\phi_2 \\ & \quad y_3 = \phi_1 + (D+\Delta_2)\phi_2 \end{aligned} \tag{21}$$

The membership function is represented as:$\mu_{TCS_{(Q,\psi,S,l,n)}(\tilde{D})}$ is shown in Fig. 3

The centroid of $\mu_{TCS_{(Q,\psi,S,l,n)}(\tilde{D})}(y)$ is formulated such that:

$$\begin{aligned} \Upsilon(Q,\psi,S,l,n) & = \frac{\int_{-\infty}^{\infty} y\mu_{F(\tilde{D})}(y)dy}{\int_{-\infty}^{\infty} \mu_{F(\tilde{D})}(y)dy} \\ & = \frac{y_1+y_2+y_3}{3} \\ & = \phi_1 + D\phi_2 + \tfrac{(\Delta_2-\Delta_1)\phi_2}{3} \\ & = \lambda g \log\tfrac{\psi_0}{\psi} + \lambda G \log\tfrac{S_0}{S} + h_1\psi Q + h_2\{\tfrac{Q}{2} + \tfrac{\sigma^2 l}{4(1-\epsilon)Q} \\ & \quad -(1-\epsilon)Q\} + \tfrac{h_3 Q}{2}(n-1) + D\{\tfrac{S}{nQ} + \tfrac{A}{nQ} + \tfrac{C(l)}{Q} \\ & \quad -h_1\tfrac{\psi(1+\psi)}{2\alpha} + h_2\tfrac{\psi(1+\psi)Q}{2\alpha} + u(1+\psi) \\ & \quad + h_3\tfrac{Q}{2}\{-n\gamma + 2\gamma\} + n\psi + \xi_i\} + \tfrac{(\Delta_2-\Delta_1)\phi_2}{3} \\ & = TCSFD(Q,\psi,S,l,n) \\ & \quad + \tfrac{(\Delta_2-\Delta_1)}{3}\Big[\tfrac{S}{nQ} + \tfrac{A}{nQ} + \tfrac{C(l)}{Q} - h_1\tfrac{\psi(1+\psi)}{2\alpha} \\ & \quad + h_2\tfrac{\psi(1+\psi)Q}{2\alpha} + u(1+\psi) + h_3\tfrac{Q}{2}\{-n\gamma + 2\gamma\} + n\psi + \xi_i\Big] \end{aligned} \tag{22}$$

4.7. Case: 4. Total integrated cost of the supply chain management for fuzzy demand with service level constraints (TCSFDSLC)

$$\begin{aligned} TCSFDSLC(Q,\psi,S,l,n) = TCS(Q,\psi,S,l,n) + \tfrac{(\Delta_2-\Delta_1)}{3}\Big[\tfrac{S}{nQ} + \tfrac{A}{nQ} \\ + \tfrac{C(l)}{Q} - h_1\tfrac{\psi(1+\psi)}{2\alpha} + h_2\tfrac{\psi(1+\psi)Q}{2\alpha} \\ + u(1+\psi)h_3\tfrac{Q}{2}\{-n\gamma+2\gamma\} + n\psi + \xi_i\Big] \\ + \lambda_1(\psi-\psi_0) + \lambda_2(S-S_0) \end{aligned} \tag{23}$$

5. Solution methodology

Here, to solve the mathematical model, the classical optimization method is considered. The decision variables Q, ψ, S, l, and n are optimized using a discrete optimization technique. As there are multiple decision variables, the Hessian matrix is used to test the solution's globality. At first, the total cost is partially differentiated and equated to zero. Thus, the decision variables obtain the optimum results for case 1 as follows:

$$Q^* = \frac{\sqrt{\frac{DS}{n}+\frac{DA}{n}+DC(l)+\frac{h_2\sigma^2 l}{4(1-\epsilon)}}}{\sqrt{h_1(\psi-\frac{\psi(1+\psi)D}{2\alpha})+h_2(\frac{1}{2}+\frac{\psi(1+\psi)D}{2\alpha})+h_3\frac{1}{2}(n(1-D\gamma)-1+2D\gamma)}}$$

$$\psi^* = -\frac{(\eta+u)D+h_1Q+\frac{DQ(h_2-h_1)}{2\alpha}}{2\frac{DQ}{\alpha}(h_2-h_1)} + \frac{\sqrt{\left((\eta+u)D+h_1Q+\frac{DQ(h_2-h_1)}{2\alpha}\right)^2+4\lambda g\frac{DQ}{\alpha}(h_2-h_1)}}{2\frac{DQ}{\alpha}(h_2-h_1)}$$

$$S^* = \frac{\lambda GnQ}{D}$$

See Appendix A for the calculations of first-order derivatives.

Further, the decision variables obtain the optimum results for case 2 as follows:

$$Q^* = \frac{\sqrt{\frac{DS}{n}+\frac{DA}{n}+DC(l)+\frac{h_2\sigma^2 l}{4(1-\epsilon)}}}{\sqrt{h_1(\psi-\frac{\psi(1+\psi)D}{2\alpha})+h_2(\frac{1}{2}+\frac{\psi(1+\psi)D}{2\alpha})+h_3\frac{1}{2}(n(1-D\gamma)-1+2D\gamma)}}$$

$$\psi^* = -\frac{(\eta+u)D+h_1Q+\frac{DQ(h_2-h_1)}{2\alpha}+\lambda_1}{2\frac{DQ}{\alpha}(h_2-h_1)} + \frac{\sqrt{\left((\eta+u)D+h_1Q+\frac{DQ(h_2-h_1)}{2\alpha}+\lambda_1\right)^2+4\lambda g\frac{DQ}{\alpha}(h_2-h_1)}}{2\frac{DQ}{\alpha}(h_2-h_1)}$$

$$S^* = \frac{\lambda GnQ}{(D+nQ\lambda_2)}$$

$$\lambda_1 = \frac{\lambda g}{\psi} - (\eta+u)D + h_1Q - \frac{(1+2\psi)DQ}{2\alpha}(h_2-h_1)$$

$$\lambda_2 = -\frac{D}{nQ} + \frac{\lambda G}{S}$$

See Appendix B for the calculations of first order derivatives.

Further, the decision variables obtain the optimum results for case 3 as follows:

$$Q^* = \frac{\sqrt{\left(\frac{S(D+m)}{n}+\frac{A(D+m)}{n}+C(l)(D+m)+\frac{h_2\sigma^2 l}{4(1-\epsilon)}\right)}}{\sqrt{\Psi_1+h_3(n(1-(D+m)\gamma)-1+2(D+m)\gamma)}}$$

$$\psi^* = -\frac{(\eta+u)(D+m)+h_1Q+\frac{DQ(h_2-h_1)}{2\alpha}+\frac{m}{2\alpha}(Qh_2-h_1)}{2\left(\frac{DQ}{\alpha}(h_2-h_1)+\frac{m}{\alpha}(Qh_2-h_1)\right)} + \frac{\sqrt{\Psi_2+4\lambda g\left(\frac{DQ}{\alpha}(h_2-h_1)+\frac{m}{\alpha}(Qh_2-h_1)\right)}}{2\left(\frac{DQ}{\alpha}(h_2-h_1)+\frac{m}{\alpha}(Qh_2-h_1)\right)}$$

$$S^* = \frac{\lambda GnQ}{(D+m)}$$

See Appendix C for the calculations of first order derivatives.

Further, the decision variables obtain the optimum results for case 1 as follows:

$$Q^* = \frac{\sqrt{\left(\frac{S(D+m)}{n}+\frac{A(D+m)}{n}+C(l)(D+m)+\frac{h_2\sigma^2 l}{4(1-\epsilon)}\right)}}{\sqrt{\Psi_1+h_3(n(1-(D+m)\gamma)-1+2(D+m)\gamma)}}$$

$$\psi^* = -\frac{(\eta+u)(D+m)+h_1Q+\frac{DQ(h_2-h_1)}{2\alpha}+\frac{m}{2\alpha}(Qh_2-h_1)+\lambda_1}{2\left(\frac{DQ}{\alpha}(h_2-h_1)+\frac{m}{\alpha}(Qh_2-h_1)\right)} + \frac{\sqrt{\Psi_2+4\lambda g\left(\frac{DQ}{\alpha}(h_2-h_1)+\frac{m}{\alpha}(Qh_2-h_1)\right)}}{2\left(\frac{DQ}{\alpha}(h_2-h_1)+\frac{m}{\alpha}(Qh_2-h_1)\right)}$$

$$S^* = \frac{\lambda GnQ}{(D+m+nQ\lambda_2)}$$

$$\lambda_1 = \frac{\lambda g}{\psi} - (\eta+u)(D+m) - h_1Q - \frac{(1+2\psi)DQ}{2\alpha}(h_2-h_1) - \frac{m(Qh_2-h_1)}{\alpha}\left(\psi+\frac{1}{2}\right)$$

$$\lambda_2 = -\frac{(D+m)}{nQ} + \frac{\lambda G}{S}$$

See Appendix D for the calculations of first order derivatives.

Lemma 1. *If l and n are fixed, the Hessian Matrix of TCS(Q, S, n, l, ψ) is always convex for the classic values of verdict variables, then, the TCS(Q, S, n, l, ψ) exhibits the global minimum at that optimum values of decision variables.*

Proof. See Appendix E. □

Lemma 2. *If l and n are fixed, the Hessian Matrix of TCSSLC (Q, S, n, l, ψ) is always convex at the optimum values of decision variables, then, the TCSSLC(Q, S, n, l, ψ) exhibits the global minimum at the optimum values of decision variables.*

Proof. See Appendix F. □

Lemma 3. *If l and n are fixed, the Hessian Matrix of TCSFD (Q, S, n, l, ψ) is always convex at the optimum values of decision variables, then, the TCSFD(Q, S, n, l, ψ) exhibits the global minimum at the optimum values of decision variables.*

Proof. See Appendix G. □

Lemma 4. *If l and n are fixed, the Hessian Matrix of TCSFDSLC(Q, S, n, l, ψ) is always convex at the optimum values of decision variables, then, the TCSFDSLC(Q, S, n, l, ψ) exhibits the global minimum at the optimal values of decision variables.*

Proof. See Appendix H. □

6. Numerical experiment

The numerical example validates the proposed model

6.1. Example

The mathematical model is numerically tested to validate the theoretical solution. All data used are based on the industry visit and certain data are verified by Sarkar and Mahapatra [1]; the following input parameter values are considered here to illustrate the numerical example. D = 500 (units/year); σ = 7 (\$/batch); γ = 1/300 (\$/unit); A = 50 (\$/setup); S_0 = 40 (\$/setup); h_1 = 6 (\$/unit per year); h_2 = 10 (\$/unit per year); h_3 = 4 (\$/unit per year); u = 0.25 (\$/unit); α = 2152 (units/year); η = 20 (\$/defective unit); g = 4000;

Table 2 Transportation discount structure.

Range (Q)	Unit transportation cost ($/unit)	Total cost ($/cycle)
$0 \leqslant Q < 100$	0.20	1955.80
$100 \leqslant Q < 200$	0.15	2084.51
$200 \leqslant Q < 300$	0.10	2519.88
$300 \leqslant Q$	0.05	3854.06

Table 3 Transportation cost structure for the SLC case.

Range (Q)	Unit transportation cost ($/unit)	Total cost ($/cycle)
$0 \leqslant Q < 100$	0.20	1947.09
$100 \leqslant Q < 200$	0.15	2080.66
$200 \leqslant Q < 300$	0.10	2522.65
$300 \leqslant Q$	0.05	3869.74

G = 400; λ = 0.1 ($/year); ψ_0 = 0.08; ϵ = 0.98; C(l) = 28 ($/order); and ξ = 0.20 ($/unit).

6.2. Case 1

The classic results of the verdict variables for the case 1 are such that Q = 86.45 (units); ψ = 0.037; S = 20.75 ($/unit); l = 3 (weeks); n = 3; Total cost = 1955.80 ($/cycle). The globality of this minimum expected total cost is determined using a Hessian matrix. The values of the principal minors are $H_{11} = 0.0929137 > 0, H_{22} = 26536.6 > 0, H_{33} = 3481.76 > 0$. The transportation cost structure is listed in Table 2.

6.3. Case 2

Using an SLC based on the improvement in quality and quantity of products and the reduction in set-up investment cost, optimal solutions are obtained for case 2 with the minimum total cost. The input parameters are given in above example. The optimum numerical values λ_1 and λ_2 are given by (0.13, 0.41). The optimum numerical results of the decision variables for case 2 are Q = 85.84 (units); ψ = 0.04; S = 17.01 ($/unit); l = 3 (weeks); n = 3; Total cost = 1947.09 ($/cycle). The globality of this minimum expected total cost is determined using a Hessian matrix. The values of the principal minors are $H_{11} = 0.0912433 > 0, H_{22} = 25860.40 > 0, H_{33} = 3469.84 > 0$. The transportation cost structure for an SLC listed in Table 3.

6.4. Case 3

In a traditional market, the annual demand is always fixed, but it may fluctuate on the basis of different uncertainty issues of the market situation. Thus, the fuzzy concept arises. The following example is based on the fuzzy demand. The input parameters are same as previous example. The results for case 3 with a considerable fuzzy limit imply $\Delta_1 = 50$ and $\Delta_2 = 80$. The classic results of the verdict variables for case 3 are Q = 85.50 (units); ψ = 0.04; S = 20.94 ($/unit); l = 3 (weeks); n = 3; Total cost = 1949.74 ($/cycle). The globality of this minimum expected total cost is determined using a Hessian matrix. The values of the principal minors are $H_{11} = 0.13825 > 0, H_{22} = 39456.4 > 0, H_{33} = 5280.63 > 0$.

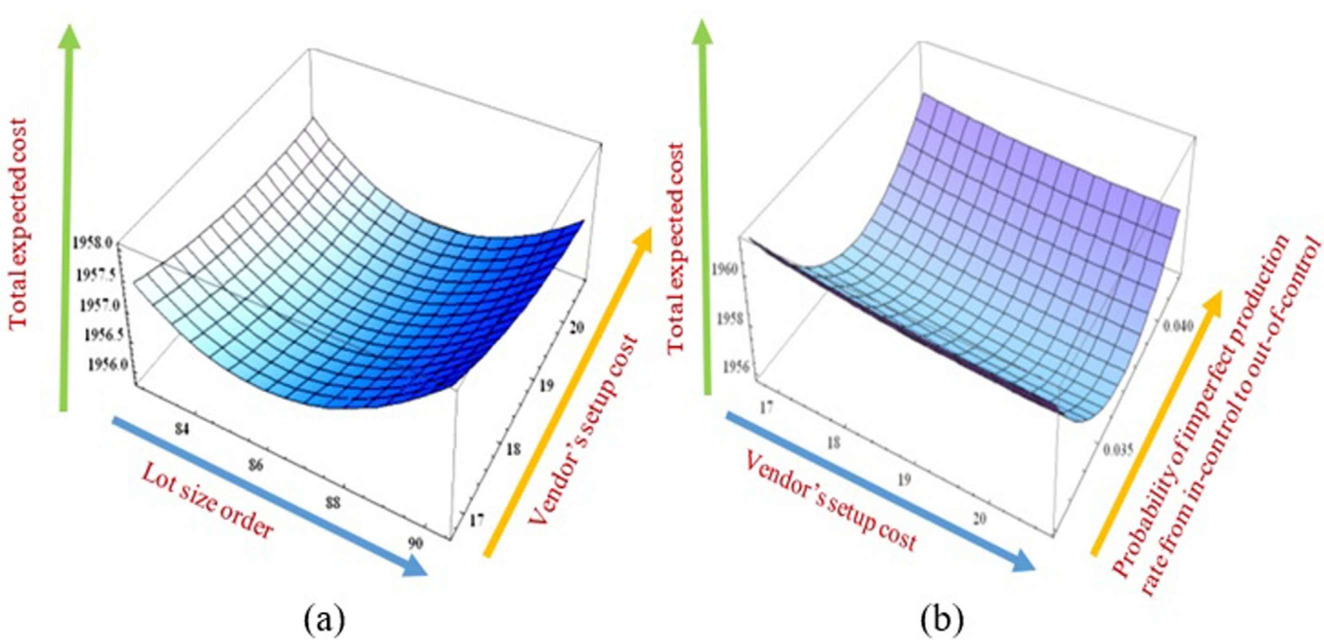

Fig. 4 (a) Total cost versus lot size order quantity and vendor's setup cost. (b) Total cost versus setup investment cost of the vendor and probability of the defective production rate.

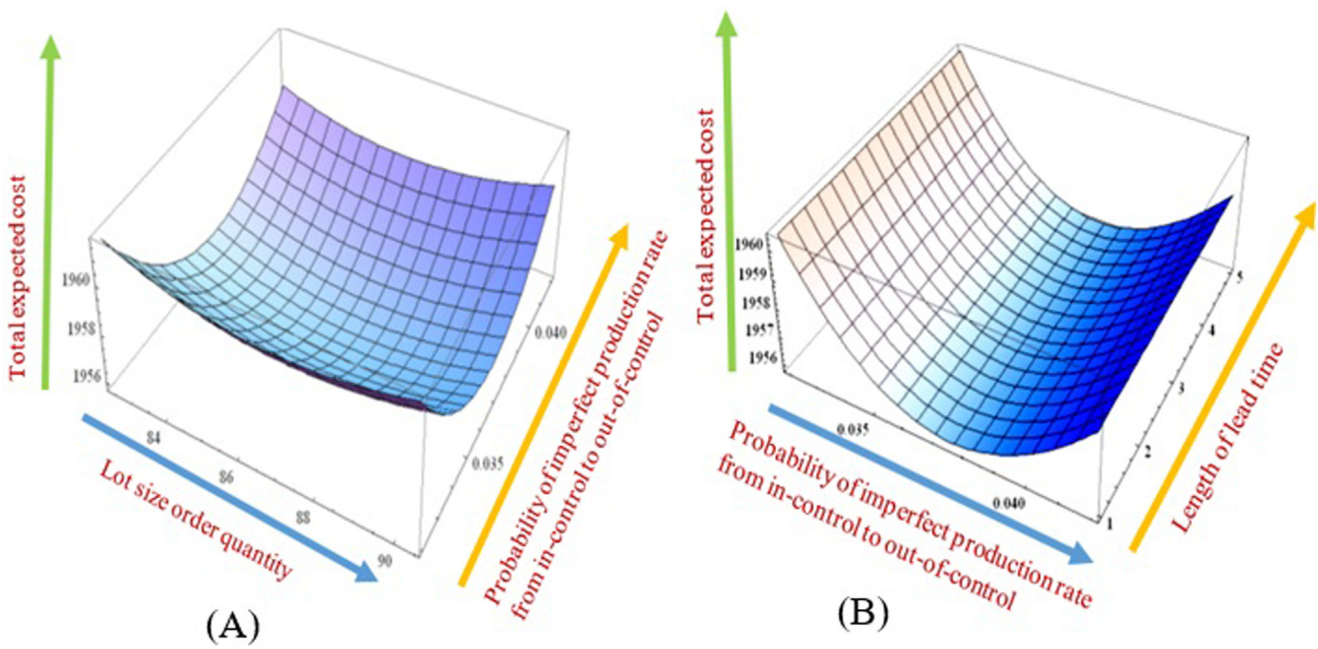

Fig. 5 (A) Total cost versus probability of the defective production rate and lot size order quantity. (B) Total cost versus probability of the defective production rate and length of lead time.

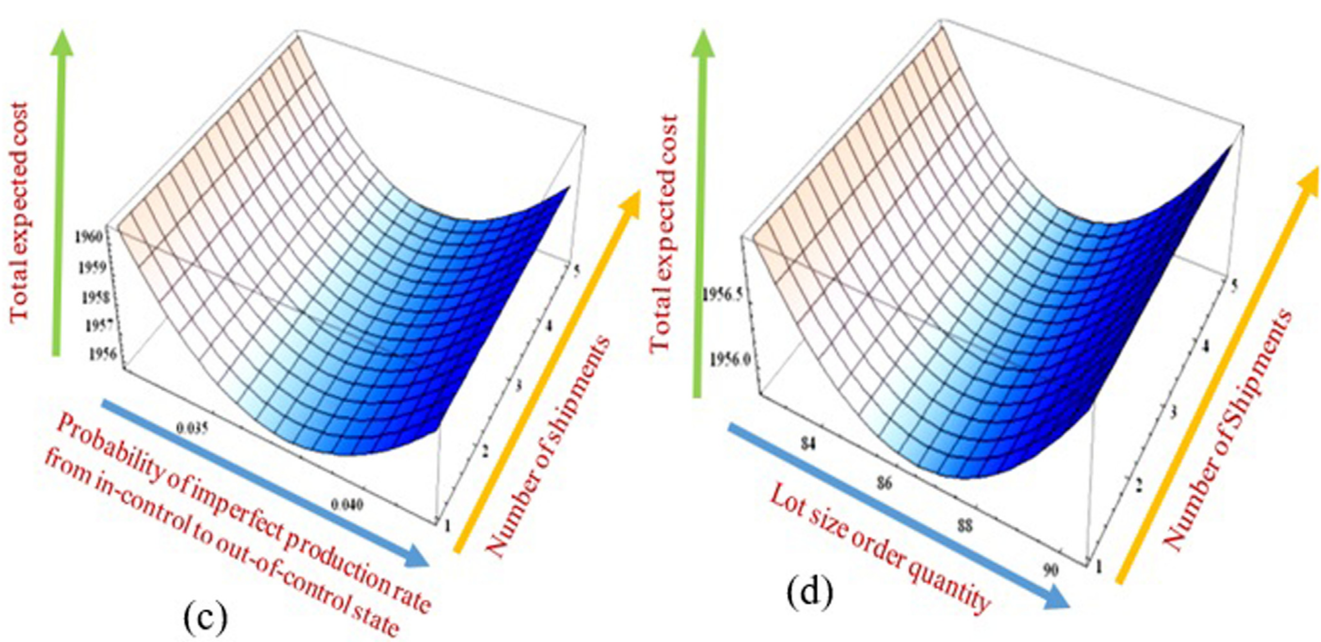

Fig. 6 (c) Total cost versus probability of the defective production rate and shipment numbers. (d) Total cost versus lot size quantity and shipment numbers.

6.5. Case 4

The following example is based on the fuzzy demand with SLCs. The input parameters are same as previous example. The results for case 4 with a considerable fuzzy limit imply $\Delta_1 = 50$ and $\Delta_2 = 80$, and the classic values of λ_1 and λ_2 are given by (0.13, 0.41). The classic results of the verdict variables for case 4 are such that Q = 84.89 (units); ψ = 0.04; S = 17.14 (\$/unit); l = 3 (weeks); n = 3; Total cost = 1941.08 (\$/cycle). The globality of this minimum expected total cost is determined using the Hessian matrix. The values of the principal minors are $H_{11} = 0.136208 > 0$, $H_{22} = 38576.6 > 0, H_{33} = 5280.48 > 0$.

Here all possible combinations of different exceptional cases are considered numerically and analytically. The results' optimality is given analytically in the Lemma section and

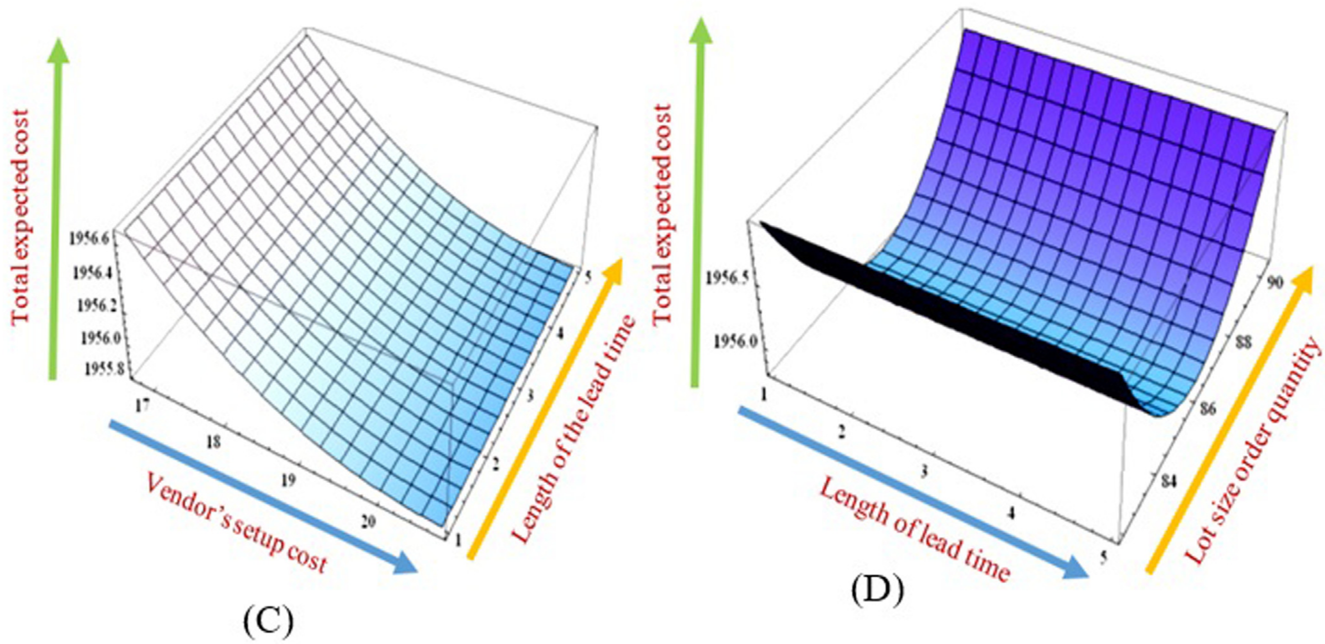

Fig. 7 (C) Total cost versus setup cost of the vendor and lead time length. (D) Total cost versus length of the lead time and lot size order quantity.

Table 4 Comparison table among the total cost of the different cases.

Total cost of the SCM	Case 1	Case 2	Case 3	Case 4
(TCS)	1955.80 ($/cycle)	1947.09 ($/cycle)	1949.74 ($/cycle)	1941.08 ($/cycle)

numerically in the above numerical experiment. Analytically and numerically, in both way the convexity of the result are tested through Hessian matrix as the determinant value of the Hessian matrix has the same sign. The globality of the minimum expected total cost in different cases are also shown in the convex figure Fig. 4–7. However, Table 4 gives the comparison of total cost among different cases. From Table 4, it is clear that the total cost of the SCM is minimum in the case of "Total integrated cost of the supply chain management for fuzzy demand with service level constraints (TCSFDSLC)" than the other different special cases.

7. Sensitivity analysis

Sensitivity analysis for cost and scaling parameters are numerically calculated and major changes demonstrated by these parameters are listed in Table 5 and illustrated Fig. 8.

Table 5 demonstrates the effects of cost parameters and scaling parameters on total cost due to the change such as (-50%, -25%, $+25\%$, $+50\%$). Here, from the following sensitivity table we may conclude that:

1. The most sensitive parameter is the buyer's holding cost for imperfect production. It directly affects the total expected cost. Owing to its small changes, the buyer can stock the products despite their nature. Different uses of the holding cost for perfect and defective products are the acceptable plan in this model. The most sensitive holding cost is the buyer's holding cost for defective production.
2. Due to the small changes in replacing the cost of defective items, the total affected cost is affected more. If the production yields defective products or the buyer returns defective products to the vendor, the supply chain faces more changes in the total cost, detailed in the sensitivity table. It is because the vendor has to replace the perfect products instead of defective products.
3. The sensitivity table indicates that the fractional amount of capital investment for the improved-quality products reduces defective production and is a critical cost parameter, affecting the total cost.
4. The scaling parameters of the investment for the quality-improvement function and set-up-cost reduction further impacts the total cost. The scaling parameters of the investment for improved-quality products is more sensitive than those of the set-up cost-reduction.
5. The effects of lead-time crashing-lot-sale cost significantly affect the total cost. The lead time is co-related to the variations in the crashing cost. As a result, the total cost affected is parallel to the lead time's crashing cost, which is observed in the sensitive table.
6. The transportation cost changes had a significant effect on the joint integrated total cost of the SCM. This result justified the total cost, which directly depends on the

Table 5 Effects of fluctuation of parameters.

Parameters	Changes (in%)	TCS (in%)	Parameters	Changes (in%)	TCS (in%)
	−50	−11.21		−50	−13.05
	−25	−05.05		−25	−05.25
λ	+25	+03.61	η	+25	+04.35
	+50	+06.08		+50	+07.95
	−50	−11.05		−50	−01.00
	−25	−04.64		−25	−00.41
g	+25	+03.33	G	+25	+00.28
	+50	+05.62		+50	+00.50
	−50	−00.43		−50	−17.29
	−25	−00.22		−25	−08.47
h_1	+25	+00.22	h_2	+25	+08.33
	+50	+00.46		+50	+17.65
	−50	−01.51		−50	−01.44
	−25	−00.74		−25	−00.58
h_3	+25	+00.74	S_0	+25	+00.46
	+50	+01.50		+50	+00.84
	−50	−02.56		−50	−03.36
	−25	−01.26		−25	−01.68
A	+25	+01.23	u	+25	+01.68
	+50	+02.54		+50	+03.36
	−50	−04.38		−50	−02.56
	−25	−02.14		−25	−01.28
$C(l)$	+25	+02.06	ξ	+25	+01.28
	+50	+04.03		+50	+02.56

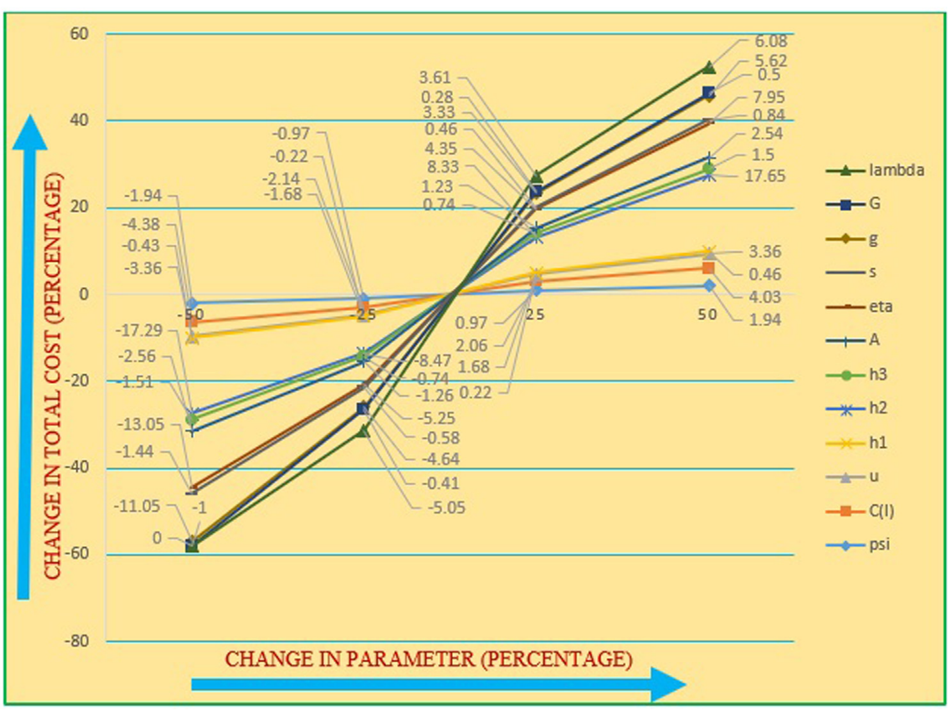

Fig. 8 Effects of changes in parametric values vs total cost.

transportation cost, and one varies according to the other. The transportation cost will decrease with an increase in the number of transported products, i.e., the delivered lot size.

7. Screening cost is an essential investment for imperfect production. The sensitivity table clearly shows that the screening cost parameter is directly related to the joint total cost.

Because Due to increasing of it, the joint total cost will increase and vise versa.

8. The cost of the order of the buyer is another significant effect. Small changes of these cost parameters also have an impact on the joint total cost. Its increasing value increases the joint total cost, and decreases its value decreases the joint total cost.

8. Managerial Insights

The following are the recommendations for improving the industry:

1. It is an imperfect production supply chain model where the total cost is minimized with an optimum different decision variable. The manager should maintain the investment for quality improvement to reduce defective production. It is essential for a reliable supply chain management system. The result helps the manager to reduce the total cost.
2. The manager can avoid any uncertainties regarding customer matters using SLCs. Further, the SCM manager can increase the demand for the products.
3. From the fuzzy concept of this study, the fluctuation in demand can be identified, and in different situations, the production can be controlled depending on the market demand. In this case, the total cost can be reduced.
4. This model considers an investment technique to reduce the setup cost by improving the quality of the products. The manager reduces the setup cost and optimizes the joint supply chain management cost. This research clearly shows variations in the total cost depending on the fuzzy demand and SLC.
5. The manager can identify the high number of transported products to reduce the transportation cost. Although this concept is expected, the research data of this study proves this concept numerically and analytically.

9. Conclusions

This model proved the best way of reducing the total supply chain cost under the fuzzy demand and service level constraints. Here the model highlighted the demand pattern constant and fuzzy type. The study focussed on quality improvement and setup cost reduction to control the total cost and fair market reputation of improved quality products. Transportation discount is another significant contribution of this model. The controllable lead time comes up through the distribution approach. This model's main goal was to obtain the minimum total cost by simultaneously optimizing the decision variables. The Kuhn-Tucker optimization technique with four lemmas was established for the optimal global solution of the model. Mathematica 9.0 was used as a numerical tool for optimal solutions, graphical diagrams, and Numerical Hessian calculation. After explaining different cases, it may conclude that the model's total cost is obtained for the case of fuzzy demand with service level constraints. However, it was proved that as the discard of the defective product can create a loss, the mentioned strategy played an essential role in the supply chain's total cost. Environmental responsibilities are one of the limitation of this model and the discrete investment for reducing setup cost [56]. The buyer's ordering cost can also be reduced through some continuous or discrete investment, as ordering cost can not always be fixed [5]. Moreover, a smart transportation strategy to reduce carbon emission and lead time is another interesting future extension. One can extend the present study by considering rework, warranty, and backlogging [6]. This model can also be extended by considering energy consumption because of environmental responsibilities [17]. Moreover, autonomatated inspection policy to detect defective items, remanufacturing, O2O retailing strategy, some exciting future research direction of the current study.

Appendix A. Assume TCS(Q, S, n, l, ψ)= χ_1, therefore

$$\frac{\partial\chi_1}{\partial Q} = -\frac{1}{Q^2}\left(\frac{DS}{n}+\frac{DA}{n}+DC(l)+\frac{h_2\sigma^2 l}{4(1-\epsilon)}\right)+h_1\left(\psi-\frac{\psi(1+\psi)D}{2\alpha}\right)$$
$$+h_2\left(\frac{1}{2}+\frac{\psi(1+\psi)D}{2\alpha}\right)+h_3(n(1-D\gamma)-1+2D\gamma)$$
$$\frac{\partial\chi_1}{\partial\psi} = -\frac{\lambda g}{\psi}+(\eta+u)D+h_1Q+\frac{(1+2\psi)DQ}{2\alpha}(h_2-h_1)$$
$$\frac{\partial\chi_1}{\partial S} = \frac{D}{nQ}-\frac{\lambda G}{S}$$

Appendix B. Assume TCSSLC(Q, S, n, l, ψ)= χ_2, therefore

$$\frac{\partial\chi_2}{\partial Q} = -\frac{1}{Q^2}\left(\frac{DS}{n}+\frac{DA}{n}+DC(l)+\frac{h_2\sigma^2 l}{4(1-\epsilon)}\right)+h_1\left(\psi-\frac{\psi(1+\psi)D}{2\alpha}\right)$$
$$+h_2\left(\frac{1}{2}+\frac{\psi(1+\psi)D}{2\alpha}\right)+h_3(n(1-D\gamma)-1+2D\gamma)$$
$$\frac{\partial\chi_2}{\partial\psi} = -\frac{\lambda g}{\psi}+(\eta+u)D+h_1Q+\frac{(1+2\psi)DQ}{2\alpha}(h_2-h_1)+\lambda_1$$
$$\frac{\partial\chi_2}{\partial S} = \frac{D}{nQ}-\frac{\lambda G}{S}+\lambda_2$$

Appendix C. Assume TCSFD(Q, S, n, l, ψ)= χ_3, therefore

$$\frac{\partial\chi_3}{\partial Q} = -\frac{1}{Q^2}\left(\frac{S(D+m)}{n}+\frac{A(D+m)}{n}+C(l)(D+m)+\frac{h_2\sigma^2 l}{4(1-\epsilon)}\right)$$
$$+h_1\left(\psi-\frac{\psi(1+\psi)D}{2\alpha}\right)+h_2\left(\frac{1}{2}+\frac{\psi(1+\psi)(D+m)}{2\alpha}\right)$$
$$+h_3(n(1-(D+m)\gamma)-1+2(D+m)\gamma)$$
$$\frac{\partial\chi_3}{\partial\psi} = -\frac{\lambda g}{\psi}+(\eta+u)(D+m)+h_1Q$$
$$+\frac{(1+2\psi)DQ}{2\alpha}(h_2-h_1)+\frac{m(Qh_2-h_1)}{\alpha}\left(\psi+\frac{1}{2}\right)$$
$$\frac{\partial\chi_3}{\partial S} = \frac{(D+m)}{nQ}-\frac{\lambda G}{S}$$
$$where\ m = \frac{(\Delta_2-\Delta_1)}{3}$$
$$\Psi_1 = h_1\left(\psi-\frac{\psi(1+\psi)D}{2\alpha}\right)+h_2\left(\frac{1}{2}+\frac{\psi(1+\psi)(D+m)}{2\alpha}\right)$$
$$\Psi_2 = \left((\eta+u)(D+m)+h_1Q+\frac{DQ(h_2-h_1)}{2\alpha}+\frac{m}{2\alpha}(Qh_2-h_1)\right)^2$$

Appendix D. Assume TCSFDSLC(Q, S, n, l, ψ) = χ_4, therefore

$$\frac{\partial\chi_4}{\partial Q} = -\frac{1}{Q^2}\left(\frac{S(D+m)}{n} + \frac{A(D+m)}{n} + C(l)(D+m) + \frac{h_2\sigma^2 l}{4(1-\epsilon)}\right)$$
$$+h_1\left(\psi - \frac{\psi(1+\psi)D}{2\alpha}\right) + h_2\left(\frac{1}{2} + \frac{\psi(1+\psi)(D+m)}{2\alpha}\right)$$
$$+h_3(n(1-(D+m)\gamma) - 1 + 2(D+m)\gamma)$$
$$\frac{\partial\chi_4}{\partial\psi} = -\frac{\lambda g}{\psi} + (\eta+u)(D+m) + h_1 Q$$
$$+\frac{(1+2\psi)DQ}{2\alpha}(h_2-h_1) + \frac{m(Qh_2-h_1)}{\alpha}(\psi+\tfrac{1}{2}) + \lambda_1$$
$$\frac{\partial\chi_4}{\partial S} = \frac{(D+m)}{nQ} - \frac{\lambda G}{S} + \lambda_2$$
$$where\ m = \frac{(\Delta_2-\Delta_1)}{3}$$

Appendix E. Proof of Lemma 1.

The following calculations are formulated for Hessian Matrix with fixed l, and n:

$$\frac{\partial^2\chi_1}{\partial S^2} = \frac{\lambda G}{S^2}$$
$$\frac{\partial^2\chi_1}{\partial\psi^2} = \frac{\lambda g}{\psi^2} + \frac{DQ}{\alpha}(h_2-h_1)$$
$$\frac{\partial^2\chi_1}{\partial Q^2} = \frac{2}{Q^3}\left(\frac{DS}{n} + \frac{DA}{n} + DC(l) + \frac{h_2\sigma^2 l}{4(1-\epsilon)}\right)$$
$$\frac{\partial^2\chi_1}{\partial S\partial\psi} = \frac{\partial^2\chi_1}{\partial\psi\partial S} = 0$$
$$\frac{\partial^2\chi_1}{\partial S\partial Q} = \frac{\partial^2\chi_1}{\partial Q\partial S} = -\frac{D}{nQ^2}$$
$$\frac{\partial^2\chi_1}{\partial Q\partial\psi} = \frac{\partial^2\chi_1}{\partial\psi\partial Q} = h_1 + \frac{(1+2\psi)}{2\alpha}D(h_2-h_1)$$

The first principal minor is

$$det(H_{11}) = det\left(\frac{\partial^2\chi_1}{\partial S^2}\right) = \frac{\lambda G}{S^2} > 0$$

Here first order principal minor gives positive sign.

$$det(H_{22}) = det\begin{pmatrix} \frac{\partial^2\chi_1}{\partial S^2} & \frac{\partial^2\chi_1}{\partial S\partial\psi} \\ \frac{\partial^2\chi_1}{\partial\psi\partial S} & \frac{\partial^2\chi_1}{\partial\psi^2} \end{pmatrix} = \frac{\lambda G}{S^2}\left(\frac{\lambda g}{\psi^2} + \frac{DQ}{\alpha}(h_2-h_1)\right) > 0$$

The second principal minor is also positive.

The third order principal minor is

$$det(H_{33}) = det\begin{pmatrix} \frac{\partial^2\chi_1}{\partial S^2} & \frac{\partial^2\chi_1}{\partial S\partial\psi} & \frac{\partial^2\chi_1}{\partial S\partial Q} \\ \frac{\partial^2\chi_1}{\partial\psi\partial S} & \frac{\partial^2\chi_1}{\partial\psi^2} & \frac{\partial^2\chi_1}{\partial\psi\partial Q} \\ \frac{\partial^2\chi_1}{\partial Q\partial S} & \frac{\partial^2\chi_1}{\partial Q\partial\psi} & \frac{\partial^2\chi_1}{\partial Q^2} \end{pmatrix}$$
$$= \frac{\partial^2\chi_1}{\partial Q\partial S}det\begin{pmatrix} \frac{\partial^2\chi_1}{\partial S\partial\psi} & \frac{\partial^2\chi_1}{\partial S\partial Q} \\ \frac{\partial^2\chi_1}{\partial\psi^2} & \frac{\partial^2\chi_1}{\partial\psi\partial Q} \end{pmatrix} - \frac{\partial^2\chi_1}{\partial Q\partial\psi}det\begin{pmatrix} \frac{\partial^2\chi_1}{\partial S^2} & \frac{\partial^2\chi_1}{\partial S\partial Q} \\ \frac{\partial^2\chi_1}{\partial\psi\partial S} & \frac{\partial^2\chi_1}{\partial\psi\partial Q} \end{pmatrix}$$
$$+\frac{\partial^2\chi_1}{\partial Q^2}det(H_{22})$$
$$= -\frac{\partial^2\chi_1}{\partial Q\partial S}\frac{\partial^2\chi_1}{\partial S\partial Q}\frac{\partial^2\chi_1}{\partial\psi^2} - \frac{\partial^2\chi_1}{\partial Q\partial\psi}\frac{\partial^2\chi_1}{\partial\psi\partial Q}\frac{\partial^2\chi_1}{\partial S^2} + \frac{\partial^2\chi_1}{\partial Q^2}\frac{\partial^2\chi_1}{\partial S^2}\frac{\partial^2\chi_1}{\partial\psi^2}$$
$$> -\frac{\partial^2\chi_1}{\partial\psi^2}\left(\frac{\partial^2\chi_1}{\partial Q\partial S}\right)^2 - \frac{\partial^2\chi_1}{\partial\psi^2}\left(\frac{\partial^2\chi_1}{\partial Q\partial\psi}\right)^2 + \frac{\partial^2\chi_1}{\partial Q^2}\frac{\partial^2\chi_1}{\partial S^2}\frac{\partial^2\chi_1}{\partial\psi^2}$$
$$> \frac{\partial^2\chi_1}{\partial\psi^2}\left[\frac{\partial^2\chi_1}{\partial Q^2}\frac{\partial^2\chi_1}{\partial S^2} - 2\left(\frac{\partial^2\chi_1}{\partial Q\partial\psi}\right)^2\right] > 0$$

As the term within the third bracket is always positive. Thus, the third principal minor is also positive values. Hence the proof.

Appendix F. Same as Appendix E.

Appendix G. Proof of Lemma 3.

The following calculations are formulated for a Hessian Matrix with fixed l, and n:

$$\frac{\partial^2\chi_3}{\partial S^2} = \frac{\lambda G}{S^2}$$
$$\frac{\partial^2\chi_3}{\partial\psi^2} = \frac{\lambda g}{\psi^2} + \frac{DQ}{\alpha}(h_2-h_1) + \frac{m(Qh_2-h_1)}{\alpha}$$
$$\frac{\partial^2\chi_3}{\partial Q^2} = \frac{2}{Q^3}\left(\frac{(D+m)S}{n} + \frac{(D+m)A}{n} + (D+m)C(l) + \frac{h_2\sigma^2 l}{4(1-\epsilon)}\right)$$
$$\frac{\partial^2\chi_3}{\partial S\partial\psi} = \frac{\partial^2\chi_3}{\partial\psi\partial S} = 0$$
$$\frac{\partial^2\chi_3}{\partial S\partial Q} = \frac{\partial^2\chi_3}{\partial Q\partial S} = -\frac{(D+m)}{nQ^2}$$
$$\frac{\partial^2\chi_3}{\partial Q\partial\psi} = \frac{\partial^2\chi_3}{\partial\psi\partial Q} = h_1 + \frac{(1+2\psi)}{2\alpha}D(h_2-h_1) + \frac{mh_2}{\alpha}(\psi+\tfrac{1}{2})$$

The first principal minor is

$$det(H_{11}) = det\left(\frac{\partial^2\chi_3}{\partial S^2}\right) = \frac{\lambda G}{S^2} > 0$$

Here, the first order principal minor is positive.

$$det(H_{22}) = det\begin{pmatrix} \frac{\partial^2\chi_3}{\partial S^2} & \frac{\partial^2\chi_3}{\partial S\partial\psi} \\ \frac{\partial^2\chi_3}{\partial\psi\partial S} & \frac{\partial^2\chi_3}{\partial\psi^2} \end{pmatrix} = \frac{\lambda G}{S^2}\left(\frac{\lambda g}{\psi^2} + \frac{DQ}{\alpha}(h_2-h_1) + \frac{m(Qh_2-h_1)}{\alpha}\right) > 0$$

The aforementioned principal minor of second order is positive.

$$det(H_{33}) = det\begin{pmatrix} \frac{\partial^2\chi_3}{\partial S^2} & \frac{\partial^2\chi_3}{\partial S\partial\psi} & \frac{\partial^2\chi_3}{\partial S\partial Q} \\ \frac{\partial^2\chi_3}{\partial\psi\partial S} & \frac{\partial^2\chi_3}{\partial\psi^2} & \frac{\partial^2\chi_3}{\partial\psi\partial Q} \\ \frac{\partial^2\chi_3}{\partial Q\partial S} & \frac{\partial^2\chi_3}{\partial Q\partial\psi} & \frac{\partial^2\chi_3}{\partial Q^2} \end{pmatrix}$$
$$= \frac{\partial^2\chi_3}{\partial Q\partial S}det\begin{pmatrix} \frac{\partial^2\chi_3}{\partial S\partial\psi} & \frac{\partial^2\chi_3}{\partial S\partial Q} \\ \frac{\partial^2\chi_3}{\partial\psi^2} & \frac{\partial^2\chi_3}{\partial\psi\partial Q} \end{pmatrix} - \frac{\partial^2\chi_3}{\partial Q\partial\psi}det\begin{pmatrix} \frac{\partial^2\chi_3}{\partial S^2} & \frac{\partial^2\chi_3}{\partial S\partial Q} \\ \frac{\partial^2\chi_3}{\partial\psi\partial S} & \frac{\partial^2\chi_3}{\partial\psi\partial Q} \end{pmatrix}$$
$$+\frac{\partial^2\chi_3}{\partial Q^2}det(H_{22})$$
$$= -\frac{\partial^2\chi_3}{\partial Q\partial S}\frac{\partial^2\chi_3}{\partial S\partial Q}\frac{\partial^2\chi_3}{\partial\psi^2} - \frac{\partial^2\chi_3}{\partial Q\partial\psi}\frac{\partial^2\chi_3}{\partial\psi\partial Q}\frac{\partial^2\chi_3}{\partial S^2} + \frac{\partial^2\chi_3}{\partial Q^2}\frac{\partial^2\chi_3}{\partial S^2}\frac{\partial^2\chi_3}{\partial\psi^2}$$
$$> -\frac{\partial^2\chi_3}{\partial\psi^2}\left(\frac{\partial^2\chi_3}{\partial Q\partial S}\right)^2 - \frac{\partial^2\chi_3}{\partial\psi^2}\left(\frac{\partial^2\chi_3}{\partial Q\partial\psi}\right)^2 + \frac{\partial^2\chi_3}{\partial Q^2}\frac{\partial^2\chi_3}{\partial S^2}\frac{\partial^2\chi_3}{\partial\psi^2}$$
$$> \frac{\partial^2\chi_3}{\partial\psi^2}\left[\frac{\partial^2\chi_3}{\partial Q^2}\frac{\partial^2\chi_3}{\partial S^2} - 2\left(\frac{\partial^2\chi_3}{\partial Q\partial\psi}\right)^2\right] > 0$$

As the term within the third bracket is always positive. Thus, the principal minor of third order is positive. Hence, it is proved.

Appendix H. Same as Appendix G.

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