Quantum Pythagorean Fuzzy Evidence Theory: A Negation of Quantum Mass Function View

Xiaozhuan Gao, Lipeng Pan, and Yong Deng

***Abstract*—Dempster–Shafer (D-S) evidence theory is an effective methodology to handle unknown and imprecise information because it can assign probability into the power set. However, the process of obtaining information is a complex task, which can consider the rational, conscious, objective evaluation of utility with behavioral effects. Besides, in most cases, information can be obtained from different angles at the same time. The quantum model of mass function (QM) uses amplitude and phase angle to easily express those properties of information that can extend D-S evidence theory to the unit circle in a complex plane. Moreover, everything in nature will have its opposite, which is a kind of universality. The Bayes theorem is essentially the process of negation. However, in most cases, decisions can be made by only fully considering the known information without considering the other side of the information. Hence, considering the negation of information is a question to be investigated deeply, which can analyze information from the other point. This article proposes negation of QM by using the subtraction of vectors in the unit circle, which can degenerate into negation proposed by Yager in standard probability theory and negation proposed by Yin *et al.* in D-S evidence theory. Negation can provide us more information to consider the problem from both positive and negative aspects. In this article, negation can be understood information, which does not belong to event A, that is to say, negation can be regarded as nonmembership by using the fuzzy terms. Based on the above discussion, this article proposes the quantum pythagorean fuzzy evidence theory (QPFET), which is the novel work to consider QPFET from the point of negation. Besides, there are some numerical examples to explain the proposed method. In order to explore the applications of QPFET, this article discusses the possibility of the VI š ekriterijumsko Kompromisno Rangiranje method under QPFET to handle multicriteria decision-making that enables us to capture 2-D data, considering not only amplitude but also phase angle.**

***Index Terms*—Dempster–Shafer (D-S) evidence theory, negation, pythagorean fuzzy sets (PFSs), quantum mass function, quantum pythagorean fuzzy evidence theory (QPFET), VIšekriterijumsko Kompromisno Rangiranje (VIKOR).**

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I. Introduction

KNOWLEDGE representation is an interesting topic that can describe the information we obtained, which has been used in many fields, such as economics [1], [2], physical [3], [4], artificial intelligence [5]–[7], and so on [8]–[10]. However, uncertainty information representation is one of the most difficult challenges to knowledge representation. To address this issue, there are many methodologies, such as probability theory [11], fuzzy sets [12], Dempster–Shafer (D-S) evidence theory [13], [14], and so on. D-S evidence theory can better handle unknown and imprecise information because it can assign probability into the power set [13]–[15]. Therefore, D-S evidence theory has bigger ability to express uncertainty [16] and has been used extensively in many fields, such as evidential reasoning [17], information fusion [18], [19], classification [20], [21], and so on [22]–[25].

Although D-S evidence theory can describe more complex information by considering objective lack of complete knowledge or subjective preferences and biases, some experiments have proved that data can fluctuate at a given execution time, which is irreversible [26]. Besides, the real physical world is disposable and irreversible. Quantum mechanics provides a new view to explore and express information, which is more general than the standard (Kolmogorov) probability theory [27], [28]. This article focuses on quantum probabilities, which are nonadditive probabilities and have been used in many fields, such as decision-making [29], machine learning [30], and so on [31], [32]. Quantum probability has bigger uncertainty and can expand probability into the 2-D space by considering the amplitude and the phase angle; besides, it can obey the power set. Hence, how to understand D-S evidence theory under quantum theory is a question of value inquiry. He and Jiang [33] proposed an evidential Markov model and an evidential dynamical model to predict interference in decision-making. In the full lattice, quantum probabilities violate the additivity property [34], [35]. Vourdas set the connection between D-S evidence theory and quantum theory from the view of lattice [28], [36]. Xiao [37] presented a complex evidence theory, which has been used in many fields [26]. Gao and Deng [38] studied the quantum model of mass function (QM). The QM expanded D-S evidence theory into quantum theory, which has largest spatial and temporal representation capability. Quantum probability can better consider the events from different locations, and at the same time, in this case, the different locations can be understood as the different angles, which can be reflected by the phase angle [39]. For the weather with high uncertainty, it can often be predicted to have a probability of rain of 0.8; then, there is a probability of 0.2 that can include all possible weather conditions. Using QM can better express uncertain information from different angles. Besides, in most cases, the behavior of people has the inherent dual property. The process of decision-making is a complex dual-nature task, which can consider the rational, conscious, objective evaluation of utility with behavioral effects, such as irrational emotions, subconscious feelings, and subjective biases [40]. In the process of real decision, those properties can be understood as the 2-D information. The QM can use the amplitude and the phase angle to express the 2-D information. Hence, it can be seen that the QM is an effectiveness tool to express uncertain information.

In most cases, the decisions can be made by only fully considering the known information without considering the other side of the information. As we all know, everything in nature will have its opposite, which is a kind of universality. Hence, considering the other side of information is a question to be deeply investigated. On the other hand, the QM can satisfy nonadditivity, which has some connections with logic. In logic, negation plays an essential role [41]. Besides, negation is also an important mathematical tool [42]. Negation can help us better understand the obtained information; for example, it very hard to prove $a + b > c$; however, it is easily to prove $a + b < c$. From this point, it can be seen that negation is an essential tool in mathematics. Besides, the Bayes theorem is essentially the process of negation [43]; it can be seen that negation is also important in artificial intelligence. Recently, much work about the negation of information to study the meaning of negation has been done [44]–[47]. Yager [48] proposed the negation of probability theory, which can increase the entropy. Yin *et al.* [49] proposed the negation of mass function, which can be used to consider conflict. Torres-Blanc *et al.* [50] proposed new and strong negations based on type-2 fuzzy sets. Srivastava and Maheshwari [51] discussed some properties of negation. Srivastava and Kaur [52] used the Shannon entropy function and the Kullback–Leibler divergence to determine the uncertainty related to the negation. Kang *et al.* [53] proposed the negation of discrete Z-numbers. Hence, studying the negation of QM is a problem of value study, which is also an open issue.

Based on the above discussion, this article proposed a new negation method based on the QM. First, the QM is similar to a vector in a 2-D space, so the QM can be regarded as vector $\overrightarrow{QM}$. Besides, the amplitude of QM is always smaller than 1. In probability theory, using $1 - p(A)$ represents the negation of A. Hence, in this article, the negation of QM can be presented by using $\overrightarrow{1} - \overrightarrow{QM}$, which is similar to the subtraction between two vectors. Besides, in probability theory, probability represents the probability of something happening, that is to say, the membership degree by using a fuzzy term. Then, the negation of probability can be understood as the impossible probability of something, namely, the nonmembership degree by using a fuzzy item. Hence, the negation sets the connection between membership and nonmembership degrees. Membership degree is an essential tool in the fuzzy set, which has been used in many fields [54]–[56]. Besides, in most cases, the fuzzy sets can be set by people, which can include the larger subjective factors. Hence, how to generate the fuzzy sets is also an open issue. The view of this article can decrease the subjective factors by considering information from different views. The pythagorean fuzzy set (PFS) satisfies the law that the sum of squares is 1, which has bigger information representation space and uncertainty [57], [58]. Based on the above discussion, this article proposed quantum pythagorean fuzzy evidence theory (QPFET) based on the proposed negation, which can consider both the amplitude and the phase angle. Besides, the fuzzy set can be well applied to multicriteria decision making (MCDM) [59]. Hence, to explore the application of QPFET, this article proposed QPFET-VIKOR based on VI š ekriterijumsko Kompromisno Rangiranje (VIKOR), meaning multicriteria optimization and compromise solution [60]). In this article, quantum probability can be expressed by using the Euler formula, not imaginary probability. In essence, they can be equivalent to each other. However, the Euler formula can better reflect the change of phase angle, which is more intuitive. Besides, the amplitude of Euler formula can be easily obtained by the classical probability. In the application of this article, the QPFET-VIKOR can consider the amplitude and the phase angle separately. Hence, using the Euler formula to present the quantum probability is more concise and convenient.

The primary contributions of this article are summarized as follows.

1) This article considered the negation based on QM from the view of vector. Specifically, this article considered the negation in 2-D space, which can help us consider problems from the positive and negative views. Besides, the proposed negation is compatible with Yager's negation and Yin *et al.*'s [49] negation.
2) Based on the negation of QM, this article proposed QPFET. This is the novel work to consider a fuzzy set from the negation, which can help us obtain more information.
3) A decision-making algorithm is proposed based on QPFET and VIKOR, which can consider not only amplitude (probability), but also phase angle. Finally, the effectiveness of the proposed method is verified through experiments.

The rest of this article is structured as follows. In Section II, some preliminaries of QM, PFSs, and Yager's negation are introduced. In Section III, the negation of QM and the QPFET-based negation are presented. Besides, some numerical examples are used to analyze the proposed methods. In Section IV, we presented the QPFET-VIKOR and used the proposed method into the specific application. Finally, Section V concludes this article.

II. Preliminaries

This section introduces some preliminaries of QM, Yager's negation, and Yin *et al.*'s [49] negation.

A. Quantum Mass Function

Definition II.1 (Quantum frame of discernment) [38]: Let Θ be the set of mutually exclusive and collectively exhaustive events $|A_i>$, namely

$$|\Theta>= \{|A_1>, |A_2>, \ldots, |A_n>\}. \tag{1}$$

The power set of $|\Theta>$ composed of 2^N elements is indicated by $2^{|\Theta>}$, as follows:

$$2^{|\Theta>} = \{\phi, \{|A_1>\}, \{|A_2>\}, \ldots, \{|A_1, A_2>\}, \ldots, |\Theta>\}. \tag{2}$$

Definition II.2 (Quantum mass function): In quantum frame of discernment, the QM $\mathbb{M}$ is defined as follows [38]:

$$\mathbb{M}(|A>) = \psi e^{j\theta} \tag{3}$$

which is a mapping of $\mathbb{Q}$ from 0 to 1 and satisfies

$$\mathbb{M}(\phi) = 0 \tag{4}$$

$$\sum_{|A>\subseteq|\Theta>} |\mathbb{M}(|A>)| = 1 \tag{5}$$

where $|\mathbb{M}(|A>)| = \psi^2$ is the amplitude of the Euler formula and represents the probability of event $|A>$, $0 \le \psi^2 \le 1$. θ_i represents the phase angle of event $|A>$.

The QM is also called quantum basic probability assignment, where $|\psi_1|^2$ represents the belief degree to $|A>$, namely, evidence supports the proposition or hypothesis $|A>$. θ represents the phase angle of $|A>$, and the range of θ is $[0, 360°]$ [38].

B. Pythagorean Fuzzy Sets

Definition II.3 (PFSs): X is a nonempty set; the PFS A in X is defined as follows [57], [58]:

$$A = \{\langle x, A_Y(x), A_N(x)\rangle \mid x \in X\} \tag{6}$$

where $0 \le A_Y(x) \le 1$ and $0 \le A_N(x) \le 1$, which represent membership and nonmembership degrees, respectively, Besides, $A_Y(x)^2 + A_N(x)^2 \le 1$ should be satisfied, which can also be rewritten as $r = A_Y(x)^2 + A_N(x)^2$. According to r, the hesitancy degree in the PFS can be written as follows [57]:

$$A_H(x) = \sqrt{1 - r}. \tag{7}$$

Definition II.4 (Yager's negation): Consider a probability distribution

$$p = \left\{p_1, p_2, \ldots, p_n; 0 \leqslant p_i \leqslant 1; \sum_{i=1}^{n} p_i = 1\right\} \tag{8}$$

defined on the set $X = (x_1, x_2, \ldots, x_n)$. The negation of the probability distribution $\bar{p}$ is defined as follows [48]:

$$\bar{p} = \left\{\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_n; 0 \leqslant \bar{p}_i \leqslant 1; \sum_{i=1}^{n} \bar{p}_i = 1\right\} \tag{9}$$

where

$$\bar{p}_i = \frac{1 - p_i}{n - 1} = \frac{1 - p_i}{\sum_{i=1}^{n}(1 - p_i)} = \frac{\sum_{j=1, i\neq j}^{n} p_j}{n - 1} \tag{10}$$

where n is the number of probabilities and p_i is the original probability.

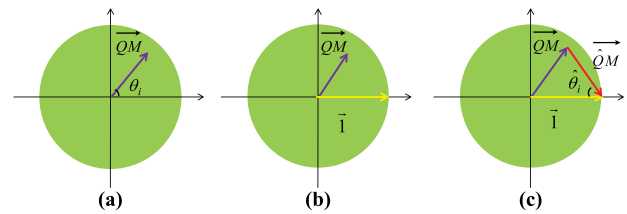

Fig. 1. Explanation of negation.

Definition II.5 (Yin's negation): Consider a mass function

$$M = \left\{m_{e_1}, m_{e_2}, \ldots, m_{e_{2^n}}; 0 \leqslant m_{e_i} \leqslant 1; \sum_{i=1}^{2^n} m_{e_i} = 1\right\}. \tag{11}$$

The negation of probability distribution $\bar{M}$ is defined as follows [49]:

$$\bar{M} = \left\{\bar{m}_{e_1}, \bar{m}_{e_2}, \ldots, \bar{m}_{e_{2^n}}; 0 \leqslant \bar{m}_{e_i} \leqslant 1; \sum_{i=1}^{2^n} \bar{m}_{e_i} = 1\right\} \tag{12}$$

where

$$\bar{m}_{e_i} = \frac{1 - m_{e_i}}{n - 1} = \frac{1 - m_{e_i}}{\sum_{i=1}^{n}(1 - m_{e_i})} = \frac{\sum_{j=1, i\neq j}^{n} m_j}{n - 1} \tag{13}$$

where n is the number of mass functions and m_{e_i} is the original mass function.

III. Quantum Pythagorean Fuzzy Evidence Theory

In this section, the negation of QM is proposed, which can be computed by using subtraction between vectors. Based on the proposed negation, the QPFET can be presented, which can consider the amplitude and the phase angle. Finally, some numerical examples are used to further analyze the proposed method.

A. Negation of Quantum Mass Function

Negation can provide us a new insight to make decisions by making full use of known information. Hence, exploring the negation of QM can expand the application of QM.

This section proposes the negation of QM. QM can expand the D-S evidence theory into 2-D space by considering the amplitude and the phase angle, as shown in Fig. 1(a). Besides, the proposed negation can be considered from the view of vector, which can be shown in Fig. 1.

Fig. 1 gives us an intuitive explanation about the negation of QM. In Fig. 1(a), $\overrightarrow{QM}$ represents the QM $\mathbb{M}(|A_i>) = \psi_i e^{j\theta_i}$, whose amplitude is ψ_i^2 and phase angle is θ_i. In Fig. 1(b), $\vec{1}$ represents the unit vector and its phase angle is 0. In Fig. 1(c), $\overrightarrow{\hat{QM}}$ represents the negation of QM, which can be computed by $\vec{1} - \overrightarrow{QM}$, whose phase angle is $\hat{\theta}_i$. Next, specific steps of negation are introduced in detail as follows.

Step 1: Calculate the complementary of QM

Use $1-\mathbb{M}(|A_i>)$ to represent the complementary of $\mathbb{M}(|A_i>)$, as follows:

$$\hat{\mathbb{M}}(|A_i>) = 1-\mathbb{M}(|A_i>) = 1-\psi_i e^{j\theta_i}. \tag{14}$$

Step 2: Calculate the amplitude and the phase angle of $Q\hat{M}$

From Fig. 1(c), it can be seen that $\overrightarrow{QM}$, $\overrightarrow{Q\hat{M}}$, and vector $\overrightarrow{1}$ can form a triangle; hence, it is easy and reasonable by using a trigonometric function to obtain amplitude and phase angle of $Q\hat{M}$, which can be shown as follows.

(i) Calculate the amplitude

$$\cos\theta_i = \frac{1^2+|\mathbb{M}(|A_i>)|^2-|\hat{\mathbb{M}}(|A_i>)|^2}{2*1*|\mathbb{M}(|A_i>)|} \tag{15}$$

$$\Downarrow$$

$$|\hat{\mathbb{M}}(|A_i>)| = \sqrt{1+|\mathbb{M}(|A_i>)|^2-2*1*|\mathbb{M}(|A_i>)|} \tag{16}$$

$$\Downarrow$$

$$\widehat{\psi}_i = \sqrt{|\hat{\mathbb{M}}(|A_i>)|} \tag{17}$$

where $|\mathbb{M}(|A_i>)|$ represents the amplitude of $\mathbb{M}(|A_i>)$. $|\hat{\mathbb{M}}(|A_i>)|$ represents the amplitude of $\hat{\mathbb{M}}(|A_i>)$

(ii) Calculate the phase angle

$$\cos\widehat{\theta}_i = \frac{1+|\hat{\mathbb{M}}(|A_i>)|^2-|\mathbb{M}(|A_i>)|^2}{2*1*|\hat{\mathbb{M}}(|A_i>)|^2}. \tag{18}$$

According to (15)–(18), the negation of QM can be obtained as follows:

$$\hat{\mathbb{M}}(|A_i>) = \widehat{\psi}_i e^{j\widehat{\theta}_i}. \tag{19}$$

Step 3: Normalization. Because the sum of amplitudes is not equal to 1, it should normalize.

(i) Calculate the sum σ of negation of QM as follows:

$$\sigma = \sum \widehat{\psi}_i^{\,2} \tag{20}$$

where $\widehat{\psi}_i$ represents the amplitude of $\hat{\mathbb{M}}(|A_i>)$.

(ii) Note that the sum σ might not equal to 1; hence, the negation results of normalization are as follows:

$$|\hat{\mathbb{M}}(|A_i>)| = \frac{\widehat{\psi}_i^{\,2}}{\sigma}. \tag{21}$$

In fact, negation can be obtained by using the complement of each focal element. Besides, the negation in QM can be changed with the change in the phase angle. When the QM degenerates the probability theory, $Q\hat{M}$ is similar to the negation of Yager. When the QM degenerates the mass function, $Q\hat{M}$ and Yin *et al.*'s [49] negation are consistent.

Proof 1: When the QM can degenerate the classical probability, $\mathbb{M}(|A_i>) = \psi_i e^{j\theta_i}$ can be rewritten as $\mathbb{M}(|A_i>) = \psi_i e^{j0}$, namely, $P(A_i) = \psi_i^2$. By using (14)–(21), the negation of $P(A_i) = \psi_i^2$ can be computed as follows:

$$\hat{P}(A_i) = 1-|\mathbb{M}(|A_i>)| = 1-\psi_i^2.$$

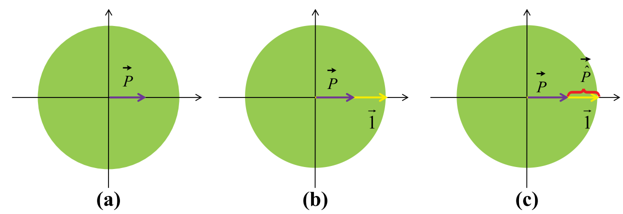

Fig. 2. Explanation of proof.

The final results after normalization are

$$\hat{P}(A_i) = \frac{1-\psi_i^2}{\sum_{i=1}^{i=n} 1-\psi_i^2} = \frac{1-\psi_i^2}{n-\sum\psi_i^2} = \frac{1-\psi_i^2}{n-1}.$$

The more specific explanations are shown in Fig. 2. In Fig. 2, $\vec{P}$ represents the classical probability, and its phase angle is 0. $\vec{\hat{P}}$ represents the negation of $\vec{P}$; it can be seen that the phase angle of $\vec{\hat{P}}$ is 0, and the amplitude of $\vec{\hat{P}}$ is $1-\psi_i^2$.

Besides, when the QM degenerates the classical mass function, the proof is similar to Proof 1.

B. Examples of Negation

In this section, some numerical examples are used to explain the proposed negation method.

Example III.1: Assume the QFD $|\Theta>=\{|a>,|b>,|c>\}$; the QMs are as follows:

$$\mathbb{M}(|a>) = \sqrt{0.2}, \mathbb{M}(|b>,|c>) = \sqrt{0.7}, \mathbb{M}(|c>) = \sqrt{0.1}.$$

QMs in this case are similar to the mass function in D-S evidence theory. After the negation, the QMs are as follows:

$$\hat{\mathbb{M}}(|a>)=\sqrt{0.40}, \hat{\mathbb{M}}(|b>,|c>)=\sqrt{0.15}, \hat{\mathbb{M}}(|c>)=\sqrt{0.45}.$$

It can be seen that the proposed method can degenerate the Yin *et al.*'s [49] negation.

Example III.2: Assume the QFD $|\Theta>=\{|a>,|b>,|c>\}$; the QMs are as follows:

$$\mathbb{M}(|a>) = \sqrt{0.2}e^{j90^\circ}, \mathbb{M}(|b>,|c>) = \sqrt{0.7}e^{j45^\circ}$$
$$\mathbb{M}(|c>) = \sqrt{0.1}e^{j120^\circ}.$$

After the negation, the QMs are as follows:

$$\hat{\mathbb{M}}(|a>) = \sqrt{0.3668}e^{j11.304^\circ}$$
$$\hat{\mathbb{M}}(|b>,|c>) = \sqrt{0.2543}e^{j44.424^\circ}$$
$$\hat{\mathbb{M}}(|c>) = \sqrt{0.3789}e^{j4.715^\circ}.$$

Comparing Examples III.1 and III.2, it can be seen that the phase angle plays an essential role in negation of QM, which can influence the results of negation.

Example III.3: Assume the QFD $|\Theta>=\{|a>,|b>,|c>\}$; the QMs are as follows:

$$\mathbb{M}(|a>) = \sqrt{0.2}e^{j\cdot 2\pi\cdot a}, \mathbb{M}(|b>,|c>) = \sqrt{0.8}e^{j\cdot 2\pi\cdot b}.$$

This example is used to discuss that the negation changes with the change in the phase angle, as shown Figs. 3 and 5, where a

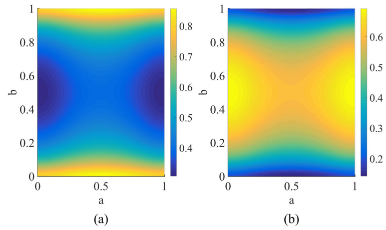

Fig. 3. (a) Amplitude change of $|a>$ after negation in Example III.3. (b) Amplitude change of $|b,c>$ after negation in Example III.3.

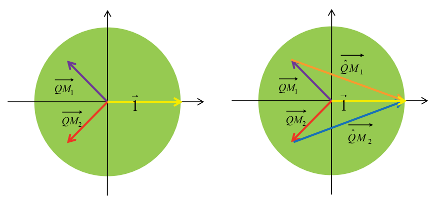

Fig. 4. Explanation of axis of symmetry.

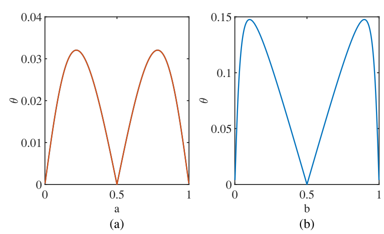

Fig. 5. (a) Phase angle change of $|a>$ after negation in Example III.3. (b) Phase angle change of $|b,c>$ after negation in Example III.3.

and b can change from 0 and 1, namely, the range of phase angle is $\theta_i \in [0, 360°]$.

First, the amplitude changes of $|a>$ and $|b,c>$ are symmetrical by analyzing Fig. 3, whose axis of symmetry is the phase angle of 180°. The more intuitive explanation is given in Fig. 4.

Next, the change in the phase angle can be further discussed. From Fig. 5, it can be seen that the phase angle after negation can be 0 under $\theta_i = 0°, 180°, 360°$. Besides, comparing Fig. 5(a) and 5(b), it can be seen that bigger amplitude has bigger phase angle.

Example III.4: Assume the QFD $|\Theta>=\{|a>, |b>, |c>\}$; the QMs are as follows:

$$\mathbb{M}(|a>) = \sqrt{a}e^{j90°}, \mathbb{M}(|b>, |c>) = \sqrt{b}e^{j45°}$$
$$\mathbb{M}(|c>) = \sqrt{1-a-b}e^{j120°}$$

where a represents the amplitude of $|a>$, b represents the amplitude of $|b,c>$, and $1-a-b$ represents the amplitude of $|c>$,

Example III.4 mainly discusses how the final results after negation changes with amplitudes. In this case, the values of a, b, and c are not equal to 0. Fig. 6 shows the changes in the amplitudes of $|a>$, $|b,c>$, and $|c>$ after negation. From Fig. 6(a), it can be seen that the amplitude increases with the increase in a while b remains the same. When a gets closer to 1 and b gets closer to 0, $|a>$ and $|b,c>$ have the biggest amplitudes, and $|c>$ has the smallest amplitude. Besides, when a gets closer to 0 and b gets closer to 0, $|a>$ and $|b,c>$ have the smallest amplitudes, and $|c>$ has the biggest amplitude.

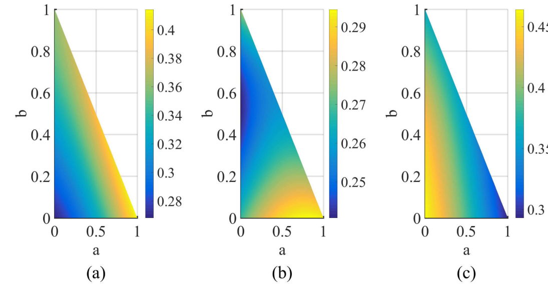

Fig. 6. (a) Amplitude change of $|a>$ after negation in Example III.4. (b) Amplitude change of $|b,c>$ after negation in Example III.4. (c) Amplitude change of $|c>$ after negation in Example III.4.

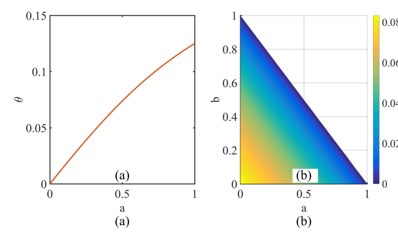

Fig. 7. (a) Phase angle of $|a>$ after negation in Example III.4. (b) Phase angle of $|c>$ after negation in Example III.4.

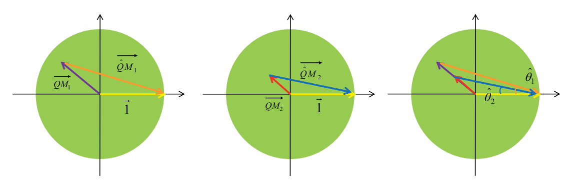

Fig. 8. Influence of amplitude on phase angle.

Next, the changes in the phase angle of $|a>$ and $|b,c>$ after negation can be shown in Fig. 7. Fig. 7(a) shows the phase angle of $|a>$ after negation; it can be seen that the phase angle can increase with the increase in a. Fig. 8 gives an intuitive explanation from the view of vector. It can be seen that bigger amplitude can have bigger phase angle after negation when the phase angle remains the same. Besides, the phase angle of $|b,c>$ has the same change with $|a>$. Similarly, when a and b have smallest values, the phase angle of $|c>$ has biggest values.

C. Quantum Pythagorean Fuzzy Evidence Theory

In this section, the QPFET can be introduced in detail.

Negation can be obtained by the proposed method, which can be understood as quantum probability, which does not belong to A, namely, negation can be regarded as the nonmembership of A.

Next, the specific steps of QPFET can be introduced as follows.

Step 1: Calculate the negation of $\mathbb{QM}$.

In this step, the negation can be obtained by using (14)–(21), i.e., $\hat{\mathbb{M}} = \hat{\psi}_i e^{j\cdot 2\pi\cdot\hat{\theta}_i}$.

Step 2: Construct QPFET.

This article considers membership as the probability that the target is A, and nonmembership as the probability that the target is not A. In this step, $\mathbb{M}$ is considered as the quantum membership degree, and $\hat{\mathbb{M}}$ is considered as the quantum nonmembership degree. Hence, we can construct the QPFET as follows.

Definition III.1: Supposing that QFD is $|\Theta>=\{|A_1>, |A_2>, \dots\}$, which is the nonempty set, the QPFET is defined as follows:

$$|\Theta>=\{<(A, \mathbb{M}, \hat{\mathbb{M}})>|A \in 2^{|\Theta>}\} \tag{22}$$

where $\mathbb{M}$ represents the quantum membership and $\hat{\mathbb{M}}$ represents the quantum nonmembership, namely, the negation of $\mathbb{M}$.

Step 3: Obtain quantum uncertainty or quantum hesitancy degree.

(I) Calculate the sum of squares of $|\mathbb{M}|$ and $|\hat{\mathbb{M}}|$, as follows:

$$\rho = |\mathbb{M}|^2 + |\hat{\mathbb{M}}|^2 \tag{23}$$

where $|\mathbb{M}|$ represents the amplitude of $\mathbb{M}(A)$ and $|\hat{\mathbb{M}}|$ represents the amplitude of $\hat{\mathbb{M}}(A)$.

(II) Obtain the amplitude of quantum uncertainty $|\iota|$. In this step, if ρ is larger than 1, then $|\iota|$ should be regarded as 0; otherwise, $|\iota|$ should be computed by using the following equation:

$$|\iota| = (1-\rho)^{0.5}. \tag{24}$$

(III) Calculate the phase angle of quantum uncertainty ε_i. In this step, all phase angles should be in the form $2\pi \cdot \theta_i$:

$$\zeta_i = \theta_i^2 + \hat{\theta}_i^2 \tag{25}$$

$$\varepsilon_i = (1-\zeta_i)^{0.5} \tag{26}$$

where θ_i represents the phase angle of $\mathbb{M}(A)$ and $\hat{\theta}_i$ represents the phase angle of $\hat{\mathbb{M}}(A)$. The quantum uncertainty is $\iota e^{j\cdot 2\pi\cdot\varepsilon_i}$

This view is inspired by [60]. Next, numerical examples are used to explain the proposed method.

D. Examples of QPFET

Example III.2 is used to further explain the procedures of QPFET.

Taking the event a as an example, the quantum membership is $\sqrt{0.2}e^{j2\pi\cdot 0.25}$, and the quantum nonmembership $\sqrt{0.3668}e^{j2\pi\cdot 0.0314}$ can be obtained by using the proposed negation method. Next, the process of quantum uncertainty can be introduced.

The amplitude of quantum uncertainty $|\iota|$ can be computed as

$$|\iota| = \sqrt{1-(0.2^2+0.3668^2)} = 0.9086.$$

Next, the phase angle of quantum uncertainty ε can be obtained as

$$\varepsilon = \sqrt{1-(0.25^2+0.0314^2)} = 0.9677.$$

Hence, $\mathbb{PM}(a)$ can be obtained as follows:

$$\mathbb{PM}(a) = [\sqrt{0.2}e^{j2\pi\cdot 0.25}, \sqrt{0.91}e^{j2\pi\cdot 0.97}, \sqrt{0.37}e^{j2\pi\cdot 0.03}].$$

Other $\mathbb{PM}s$ are as follows:

$$\mathbb{PM}(b,c) = [\sqrt{0.7}e^{j2\pi\cdot 0.13}, \sqrt{0.67}e^{j2\pi\cdot 0.98}, \sqrt{0.25}e^{j2\pi\cdot 0.12}]$$

$$\mathbb{PM}(c) = [\sqrt{0.1}e^{j2\pi\cdot 0.33}, \sqrt{0.92}e^{j2\pi\cdot 0.94}, \sqrt{0.38}e^{j2\pi\cdot 0.01}].$$

It can be seen that negation can build a bridge in QPFET and QM, which can help us obtain more information to make decision.

IV. QPFET-VIKOR Method

MCDM provides a novel method to evaluate, assess, and select alternatives, which originated from operational research. In most cases, information can be obtained by experts to handle the MCDM problem, which can have a certain subjective attitude. Using QM to express the information can better describe the two dimensions. Moreover, the QM can describe information well from different angles by using phase angles, which is more in line with human thinking. Besides, the proposed QPFET can well describe the fuzziness of information. Hence, it can be seen that QPFET is the effective tool to handle MCDM problems. There are many methodologies for dealing with MCDM. VIKOR was proposed by Opricovic to handle with precise and crisp data, and it has been extended to account for wider informational settings ever since [60], [61]. VIKOR can rank a set of alternatives in scenarios with conflicting criteria, which can get a compromise with priority in handling the MCDM problem [61]. Hence, this article mainly consider the possibility of the VIKOR method under QPFET that can help us obtain more information by analyzing the known data.

The proposed QPFET-VIKOR can be introduced in detail as follows.

A. Problem Statement

There is the set of decision makers (also named experts) $E = \{e_1, e_2, \dots, e_n\}$, who can choose the best candidate from the set of candidates (which is also named alternatives) $\Theta = \{A_1, A_1, \dots, A_m\}$. Besides, there are some criteria (which are also named attributes) $\mathcal{C} = \{c_1, c_2, \dots, c_k\}$. The expert can assign a probability to candidate A_i by considering the criteria. Next, this article gives a method to choose the best performance candidate by using the proposed QPFET-VIKOR.

B. Proposed QPFET-VIKOR

In QPFET-VIKOR, the known information can use the proposed QPFET to generate the QPFET. QPFET-VIKOR can make full use of the known information by considering the negation, which can be used to handle MCDM. The specific steps of QPFET-VIKOR are as follows.

Step 1: Obtain the weights of experts. All experts may not be equally important. Hence, considering the weights of experts can help us get the more precise decision. The importance of expert can be obtained by using linguistic terms. The specific steps are explained as follows.

(i) Obtain the information of experts. There are some linguistic terms to describe the importance of experts. For example, the importance of e_1 can be understood as *very important* with the probability of 0.8; then, the probability of unknown (which can include all the possibilities) is 0.2. Then, the QM of experts can be obtained.

(ii) Generate QPEFT. In this step, we used the proposed method of QPFET to obtain $\mathbb{PM} = [\mathbb{M}, \mathbb{O}, \hat{\mathbb{M}}]$ of experts, where $\mathbb{M} = m \cdot e^{j2\pi \cdot \alpha}$ represents the membership of events, $\hat{\mathbb{M}} = \hat{m} \cdot e^{j2\pi \cdot \gamma}$ represents the nonmembership of events, and $\mathbb{O} = o \cdot e^{j2\pi \cdot \beta}$ represents the uncertainty of events.

(iii) Calculate the weights of experts. By using the following equation, we can obtain the weight of experts:

$$\gamma_i = \frac{\sum_{j=1}^{y} \kappa_{ij} [|\mathbb{M}_{ij}|^2 + |\hat{\mathbb{M}}_{ij}|^2 \frac{|\mathbb{M}_{ij}|^2}{|\mathbb{M}_{ij}^2| + |\mathbb{O}_{ij}^2|} + \alpha_{ij}^2 + \gamma_{ij}^2 \frac{\alpha_{ij}^2}{\alpha_{ij}^2 + \beta_{ij}^2}]}{\sum_{i=1}^{x} \sum_{j=1}^{y} \kappa_{ij} [|\mathbb{M}_{ij}|^2 + |\hat{\mathbb{M}}_{ij}|^2 \frac{|\mathbb{M}_{ij}|^2}{|\mathbb{M}_{ij}^2| + |\mathbb{O}_{ij}^2|} + \alpha_{ij}^2 + \gamma_{ij}^2 \frac{\alpha_{ij}^2}{\alpha_{ij}^2 + \beta_{ij}^2}]} \quad (27)$$

where i represents the number of experts, j represents the amount of QPFET, and κ represents the importance of the linguistic term. In addition, the sum of γ_i should be 1.

Step 2: Construct a fused QPFET decision matrix. In MCDM problems, different experts can give different perspectives, so they have to fuse the individual information to make a reasonable decision. In this step, the fused decision matrix can be introduced.

(I) Obtain $\mathbb{M}_{mn}^i$ given by experts: Every expert can give some information of each criterion for the individual candidate. $\mathbb{M}_{mn}^i$ can be obtained by using Step 1(i), where m represents the number of candidates, n represents the number of criteria, and i represents the number of experts.

(II) Fuse $\mathbb{M}_{mn}^i$ to generate new $\mathbb{M}_{mn}$: All the information provided by experts is fused to generate more reasonable and precise information, which can be computed as follows:

$$\mathbb{M}_{mn} = \sum_{i=1} \gamma_i \mathbb{M}_{mn}^i \quad (28)$$

where γ_i represents the weight of the expert, and $\mathbb{M}_{mn}^i$ represents the information about the candidate and the criterion given by expert e_i.

(III) Use QM to obtain QPFET. Use $\mathbb{M}_{mn}$ to obtain QPFET $\mathbb{PM}_{mn}$, which is similar to Step 1(ii).

Step 3: Evaluation weights of criteria. The importance of individual criterion can be obtained by linguistic terms given by experts.

(I) Obtain $\mathbb{M}$ of criteria from experts. Expert e_i can assign a linguistic term to the criterion c_n in the form of probability, so $\mathbb{M}_n^i$ can be obtained.

(II) Fuse $\mathbb{M}_n^i$ to generate new $\mathbb{M}_n$. The fusion equation can be computed as follows:

$$\mathbb{M}_n^i = \sum_{i=1} \gamma_i \mathbb{M}_n^i. \quad (29)$$

(III) Use the fused $\mathbb{M}_n$ to generate the QPFET.

(IV) Calculate the weight of criteria (ω_n) by using (27).

Step 4: Construct a score matrix. $\mathbb{PM}_{mn}$ of every candidate's criterion has been obtained in Step 2. The score function can transform $\mathbb{PM}_{mn}$ to crisp values, which can be computed as

$$\mathbb{G}_{mn} = \sum_{i=1} \kappa^i (|\mathbb{M}_{mn}^i|^2 - |\hat{\mathbb{M}}_{mn}^i|^2 + (\alpha_{mn}^i)^2 - (\beta_{mn}^i)^2) \quad (30)$$

where κ represents the importance of linguistic term, $\mathbb{M}_{mn}^i$ and $\hat{\mathbb{M}}_{mn}^i$ represent the membership and nonmembership of linguistic term i, respectively, m represents the criterion, n represents the candidate, and $\mathbb{G}_{mn}$ represents the score degree of $\mathbb{PM}_{mn}$.

Hence, the score matrix is as follows:

$$\mathbb{G} = \begin{bmatrix} \mathbb{G}_{11} & \mathbb{G}_{12} & \cdots & \mathbb{G}_{1n} \\ \mathbb{G}_{21} & \mathbb{G}_{22} & \cdots & \mathbb{G}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{G}_{m1} & \mathbb{G}_{m2} & \cdots & \mathbb{G}_{mn} \end{bmatrix}. \quad (31)$$

Step 5: Identify the best and worst values. The best and worst values of every criterion are identified regarding the nature of that criterion, which can be obtained by the score matrix. Besides, $\mathfrak{b}$ and $\mathfrak{c}$ represent the collection of benefit-type and cost-type criteria, respectively.

The best value $\mathbb{G}_m^+$ and the worst value $\mathbb{G}_m^-$ of criterion c_m are obtained by using the following equations:

$$\begin{cases} \mathbb{G}_m^+ = \max \mathbb{G}_{mn}, & \text{if } c_i \in \mathfrak{b} \\ \mathbb{G}_m^+ = \min \mathbb{G}_{mn}, & \text{if } c_i \in \mathfrak{c} \end{cases} \quad (32)$$

$$\begin{cases} \mathbb{G}_m^- = \min \mathbb{G}_{mn}, & \text{if } c_i \in \mathfrak{b} \\ \mathbb{G}_m^- = \max \mathbb{G}_{mn}, & \text{if } c_i \in \mathfrak{c} \end{cases}. \quad (33)$$

Step 6: Determine $\mathcal{S}$, $\mathcal{R}$, and $\mathcal{Q}$. In this step, the group utility measure $\mathcal{S}_n$ and regret measure $\mathcal{R}_n$ relative to alternative A_i can be determined as follows:

$$\mathcal{S}_n = \sum_{m=1} \omega_n \left(\frac{\mathbb{G}_m^+ - \mathbb{G}_{mn}}{\mathbb{G}_m^+ - \mathbb{G}_m^-} \right) \quad (34)$$

$$\mathcal{R}_n = \max_m \omega_n \left(\frac{\mathbb{G}_m^+ - \mathbb{G}_{mn}}{\mathbb{G}_m^+ - \mathbb{G}_m^+} \right) \quad (35)$$

where ω_n represents the weight of individual criterion. According to $\mathcal{S}_n$ and $\mathcal{R}_n$, the ranking measure $\mathcal{Q}_n$ can be computed by

$$\mathcal{Q}_n = \upsilon \left(\frac{\mathcal{S}_n - \mathcal{S}^-}{\mathcal{S}^+ - \mathcal{S}^-} \right) + (1 - \upsilon) \left(\frac{\mathcal{R}_n - \mathcal{R}^-}{\mathcal{R}^+ - \mathcal{R}^-} \right) \quad (36)$$

where $\mathcal{S}_n^- = \min_n \mathcal{S}_n$, $\mathcal{S}_n^+ = \max_n \mathcal{S}_n$, $\mathcal{R}_n^- = \min_n \mathcal{R}_n$, and $\mathcal{R}_n^+ = \min_n \mathcal{R}_n$. Besides, the parameter υ represents the weight for the strategy of the maximum group utility, which should belong to [0,1]. In this method, υ is considered as 0.5, which provides the maximum group utility along with the minimum individual regret.

Step 7: Rank the candidates. $\mathcal{S}$, $\mathcal{R}$, and $\mathcal{Q}$ ordering of candidates can be obtained, which can help us choose the best candidate with the smaller value of $\mathcal{S}$, $\mathcal{R}$, and $\mathcal{Q}$.

The algorithm of QPFET-VIKOR is shown in Algorithm 1.

Algorithm 1: Algorithm of QPFET-VIKOR.

Input: The data about the importance of expert; The data of candidates given by experts; The data of criterion given by experts.
Output The ranking order of candidates.

function GENERATING QM
  for $i = 1$ to x **do**
    $\mathbb{QM}_i(e_j) = a_j$;
    $\mathbb{QM}_i(\Theta) = 1 - \sum a_j$;
  end for
end function
(1) Obtaining the weights of experts
for $i = 1$ to x **do**
  function Generating QM
end for
for $i = 1$ to x **do**
  for $j = 1$ to y **do**
    Generating the QPFET by using (14)–(21) and (23)–(26);
  end for
end for
for $i = 1$ to x **do**
  for $j = 1$ to y **do**
    Getting the weights of experts by using (27);
  end for
end for
(2) Construct fused QPFET decision matrix
for $i = 1$ to m **do**
  for $y = 1$ to n **do**
    Function Generating QM
  end for
end for
for $i = 1$ to x **do**
  Getting the new $\mathbb{QM}_{ij}$ after fusion by using (28);
end for
for $i = 1$ to m **do**
  for $j = 1$ to n **do**
    Generating the QPFET decision matrix by using (14)–(21) and (23)–(26);
  end for
end for
(3) Evaluation weights of criteria
for $i = 1$ to x **do**
  for $j = 1$ to n **do**
    Function Generating QM
  end for
end for
for $i = 1$ to x **do**
  Getting the new $\mathbb{QM}_i$ after fusion by using (28);
end for
for $i = 1$ to m **do**
  Generating the QPFET by using (14)–(21) and (23)–(26);
end for
for $i = 1$ to m **do**
  Getting the weights of criterion by using (27);
end for
(4) Construction of score matrix
for$i = 1$ to m **do**
  for $j = 1$ to n **do**
    Getting the score matrix by using (30);
  end for
end for
(5) Identification of best and worst values
for $j = 1$ to n **do**
  Getting best and worst values of score matrix by using (32) and (33);
end for
(6) Determine S, R, Q
for $i = 1$ to n **do**
  Getting S, R, Q by using (34)–(36);
end for
(7) Ranking of candidates
for $i = 1$ to n **do**
  Candidates are ranked from small to large;
end for

C. Application of QPFET-VIKOR

In this section, the proposed QPFET-VIKOR is used to choose the best renewable energy project. A brief introduction is as follows. There are five candidates: bio fuels (BF), hydroelectric (HD), biomass (BM), wind power (WP), and solar thermo-electric (ST). There are four criteria for individual candidate: power (PW), operating hours (OP), tons of CO_2 avoided (CO_2), and investment ratio (IV). For more details on this application, see [60] and [62].

Some linguistic terms are introduced in Table I, which can be referred to [60]. For example, the expert considers that the candidate is good. However, it is not sure, and the expert only has the probability of good of 0.8 by analyzing some properties. Hence, it is more convenient to describe the obtained information.

The steps of QPFET-VIKOR to choose the best candidate are described as follows. In this application, all results reserve two decimal fractions for simplicity.

Step 1: Obtain the weights of experts.

(I) Obtain the information of experts:

Some information about experts is given in Table II. Taking expert e_1 as an example, the importance of e_1 is VI, I with the probability of 0.9. Hence, the probability of unknown Θ is 0.1. $\mathbb{M}_1$ can be obtained as follows:

$$\mathbb{M}_1(VI, I) = \sqrt{0.9 \cdot \frac{1+0.8}{2}} e^{j\cdot 2\pi \cdot \frac{1+0.8}{2}} = \sqrt{0.81} e^{j\cdot 2\pi\cdot 0.9}$$

$$\mathbb{M}_1(\Theta) = \sqrt{0.1 \cdot \frac{3}{5}} e^{j\cdot 2\pi\cdot \frac{3}{5}} = \sqrt{0.06} e^{j\cdot 2\pi \cdot 0.6}.$$

In this case, 0.9 represents the probability of VI, I, and $\frac{1+0.8}{2}$ represents the average of VI and I. Θ can include all possibilities (VI, I, M, UI, and VUI).

The results of normalization are as follows:

$$\mathbb{M}_1(VI, I) = \sqrt{0.931} e^{j\cdot 2\pi\cdot 0.9}, \mathbb{M}_1(\Theta) = \sqrt{0.069} e^{j\cdot 2\pi\cdot 0.6}.$$

TABLE I
LINGUISTIC TERMS

Linguistic terms	Linguistic terms	Importance	$\mathbb{Q}$
VG	VI	1	$e^{j\cdot 2\cdot\pi\cdot 1}$
G	I	0.8	$\sqrt{0.8}e^{j\cdot 2\cdot\pi\cdot 0.8}$
M	M	0.6	$\sqrt{0.6}e^{j\cdot 2\cdot\pi\cdot 0.6}$
UG	UI	0.4	$\sqrt{0.4}e^{j\cdot 2\cdot\pi\cdot 0.4}$
VUG	VUI	0.2	$\sqrt{0.2}e^{j\cdot 2\cdot\pi\cdot 0.2}$

By using the above methods, QMs of experts can be obtained, as shown in the second column of Table III.

(II) Generate QPFET. Use the proposed QPFET to generate $\mathbb{PM}$, as shown in the third column of Table III.

(III) Calculate the weights of experts. The weights of experts are also different; fusing the information given by experts is helpful to make decision. In this step, the amplitude and the phase angle can be considered simultaneously; the final results are shown in Table III. Analyzing Table III, it can be seen that $\gamma_1 > \gamma_3 > \gamma_2$, which is in line with our expectations. Taking weight γ_1 of e_1 as an example, the specific steps are as follows:

$$\gamma_1 = \frac{0.9\cdot(0.93^2 + 0.36^2\cdot\frac{0.93^2}{0.93^2+0.07^2})}{1.8687+1.1364+1.5017} + \frac{0.9\cdot(0.9^2 + 0.18^2\cdot\frac{0.9^2}{0.9^2+0.40^2})}{1.8687+1.1364+1.5017} + \frac{0.6\cdot(0.07^2 + 0.64^2\cdot\frac{0.07^2}{0.07^2+0.77^2})}{1.8687+1.1364+1.5017} + \frac{0.6\cdot(0.60^2 + 0.01^2\cdot\frac{0.60^2}{0.60^2+0.80^2})}{1.8687+1.1364+1.5017} = 0.41$$

where 0.9 represents the importance of VI and I and can be computed by $\frac{1+0.8}{2}$, and 0.6 represents the importance of unknown. In this step, the amplitude and the phase angle can be aggregated because using the specific values to represent the weights of experts can help us better analyze the problems. Besides, the number of experts can be set according to [60], [63], and [64].

Step 2: Construct fused QPFET decision matrix. The information of each criterion for all candidates provided by experts e_i is shown in Table IV.

Take BF and BW as an example, which is VB with the probability of 0.8, and the probability of all possibilities is 0.2 given by expert e_1.

(I) Using Step 1(I), the QMs of candidates can be obtained. Table V represents the QM given by experts e_1, e_2, and e_3. The first column represents the experts e_i, the second column represents the candidates, and the rest of the columns represent criteria. Taking the PW and BF given e_1 in Table V as an example, the QM of VB is $\sqrt{0.57}e^{j\cdot 2\pi\cdot 0.2}$, and the QM of Θ is $\sqrt{0.43}e^{j\cdot 2\pi\cdot 0.6}$.

(II) Fuse all information by using the weights of experts to get the new QM, as shown Table VI. Taking PW of BF as an example, the specific steps are shown as follows:

TABLE II
IMPORTANCE OF EXPERTS

Expert	Linguistic terms	Probability
e_1	VI,I	0.9
e_2	I,M	0.7
e_3	I	0.8

$$\mathbb{M}(VB) = \sum_{i=1}^{3}\gamma_i\cdot\mathbb{M}_i(VB), \mathbb{M}(\Theta) = \sum_{i=1}^{3}\gamma_i \cdot\mathbb{M}_i(\Theta)$$

where γ_i represents the weight of expert, and $\mathbb{M}_i$ represents the information given by different experts. The results of fusion are $\mathbb{M}(VB) = \sqrt{0.61}e^{j\cdot 2\pi\cdot 0.2}$ and $\mathbb{M}(\Theta) = \sqrt{0.39}e^{j\cdot 2\pi\cdot 0.6}$.

(III) Use the new QM to get QPFET. Use the new QM to generate the QPFET by the proposed method, as shown in Table VII.

Step 3: Evaluation weights of criteria. There is some information about criteria obtained from experts e_1, e_2, and e_3, which is shown in Table VIII.

(I) Obtain $\mathbb{M}$ of criterion from experts. $\mathbb{M}$ of criterion from experts can be obtained by using Step 1(I), as shown in Table IX.

(II) Fuse $\mathbb{M}_n^i$ to generate the new $\mathbb{M}_n$. Next, use (29) to obtain the fused QMs, which are shown in the second column of Table X.

(III) Use the fused $\mathbb{M}_n$ to generate the QPFET. $\mathbb{PM}$ is shown in the third column of Table X.

(IV) Calculate the weight of criteria ω_n by using (27), as shown in Table X.

Step 4: Construct the score matrix. The score matrix of candidates can be shown in Table XI.

Take $\mathbb{PM}$ of PW and BF as an example to explain the computation of score function by using (28)

$$\mathbb{G}_{PW}^{BF} = 0.2\cdot(0.61^2 - 0.43^2 + 0.2^2 - 0.10^2) + 0.6\cdot(0.39^2 - 0.57^2 + 0.6^2 - 0.03^2) = 0.1552$$

where 0.2 represents the importance of VB, and 0.6 represents the importance of Θ.

Step 5: Identify the best and worst values. Take PW as an example to get the best value $\mathbb{G}_{PW}^{+}$ by using (30)

$$\mathbb{G}_{PW}^{+} = \max_j \mathbb{G}_{PW}^{j} = 2.1558.$$

The worst value $\mathbb{G}_{PW}^{+}$ can be computed by using (31) as follows:

$$\mathbb{G}_{PW}^{-} = \min_j \mathbb{G}_{PW}^{j} = 0.1552.$$

Hence, the best and worst values for the individual criterion can be shown in Table XII.

Step 6: Determine $\mathcal{S}$, $\mathcal{R}$, and $\mathcal{Q}$. Using the above best and worst values, $\mathcal{S}$, $\mathcal{R}$, and $\mathcal{Q}$ can be obtained as shown in Table XIII.

Taking the BF as an example, $\mathcal{S}_{BF}, \mathcal{R}_{BF}$, and $\mathcal{Q}_{BF}$ are computed as follows. The calculation for $\mathcal{Q}$ is performed by

TABLE III
$\mathbb{M}$, $\mathbb{PM}$, AND WEIGHTS OF EXPERTS

	$\mathbb{M}$	$\mathbb{PM}$	Weight
e_1	$\mathbb{M}_1(VI,I)=\sqrt{0.93}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}_1(\Theta)=\sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{PM}_1(VI,I)=[\sqrt{0.93}e^{j\cdot 2\pi\cdot 0.9},\sqrt{0.07}e^{j\cdot 2\pi\cdot 0.40},\sqrt{0.36}e^{j\cdot 2\pi\cdot 0.18}]$ $\mathbb{PM}_1(\Theta)=[\sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.77}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.64}e^{j\cdot 2\pi\cdot 0.01}]$	0.41
e_2	$\mathbb{M}_2(I,M)=\sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}_2(\Theta)=\sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{PM}_2(I,M)=[\sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7},\sqrt{0.43}e^{j\cdot 2\pi\cdot 0.71},\sqrt{0.53}e^{j\cdot 2\pi\cdot 0.08}]$ $\mathbb{PM}_2(\Theta)=[\sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.84}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.47}e^{j\cdot 2\pi\cdot 0.02}]$	0.25
e_3	$\mathbb{M}_3(I)=\sqrt{0.84}e^{j\cdot 2\pi\cdot 0.8}$ $\mathbb{M}_3(\Theta)=\sqrt{0.16}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{PM}_3(I)=[\sqrt{0.84}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.22}e^{j\cdot 2\pi\cdot 0.59},\sqrt{0.49}e^{j\cdot 2\pi\cdot 0.13}]$ $\mathbb{PM}_3(\Theta)=[\sqrt{0.16}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.85}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.51}e^{j\cdot 2\pi\cdot 0.01}]$	0.33

TABLE IV
PERFORMANCE OF CANDIDATES CORRESPONDING TO CRITERIA

Expert	Criteria	Candidate	Linguistic terms	Probability	Criteria	Candidate	Linguistic terms	Probability
The information given by expert $e1$								
e_1	PW	BF	VB	0.8	OP	BF	VG,G	0.9
		HD	M,G	0.7		HD	M,B	0.8
		BM	VG	0.9		BM	VG,G	0.9
		WP	M,G	0.7		WP	M,B	0.8
		ST	VG,G	0.9		ST	M,B	0.7
e_1	CO_2	BF	VG	0.9	IV	BF	M,G	0.9
		HD	M,B	0.8		HD	G, M,B	0.8
		BM	VG,G	0.9		BM	G,M,B	0.8
		WP	G, M,B	0.8		WP	M,G	0.7
		ST	B	0.7		ST	VG,G	0.9
The information given by expert $e2$								
e_2	PW	BF	VB	0.9	OP	BF	VG,G	0.9
		HD	M,G	0.7		HD	VB	0.8
		BM	VG	0.9		BM	VG	0.9
		WP	G,M,B	0.8		WP	B	0.7
		ST	VG	0.9		ST	M,B	0.8
e_2	CO_2	BF	VG	0.9	IV	BF	M,G	0.7
		HD	VB	0.8		HD	M,B	0.8
		BM	VG	0.9		BM	M,G	0.7
		WP	B,G	0.7		WP	M,G	0.9
		ST	M,B	0.9		ST	VG	0.9
The information given by expert $e3$								
e_3	PW	BF	VB	0.8	OP	BF	VG	0.9
		HD	M,G	0.7		HD	M,B	0.9
		BM	VG	0.9		BM	VG	0.8
		WP	G,M,B	0.8		WP	B	0.7
		ST	VG,G	0.9		ST	M,B	0.7
e_3	CO_2	BF	VG	0.9	CO_2	BF	M,G	0.9
		HD	M,B	0.8		HD	M,B	0.8
		BM	VG	0.9		BM	M,G	0.7
		WP	VG,G	0.9		WP	M,G	0.7
		ST	M,B	0.8		ST	VG	0.9

taking $\upsilon = 0.5$:

$$\mathcal{S}_{BF} = 0.32 \cdot \frac{2.1558 - 0.1552}{2.1558 - 0.1552} + \cdots$$
$$+ 0.09 \cdot \frac{2.1203 - 0.7519}{2.1203 - 0.5593}$$
$$= 0.3200 + 0.1163 + 0.0749 + 0.0789 = 0.5901$$
$$\mathcal{R}_{BF} = \max\{0.3200, 0.1163, 0.0749, 0.0789\} = 0.3300$$
$$\mathcal{Q}_{BF} = 0.5 \cdot \frac{0.5856 - 0.1626}{0.8878 - 0.1626}$$
$$+ 0.5 \cdot \frac{0.3300 - 0.0970}{0.3800 - 0.0970} = 0.7033.$$

Step 7: Rank the candidates. The ranking order of candidates according to $\mathcal{S}, \mathcal{R}, \mathcal{Q}$ ascending order can be obtained as follows.

Analyzing Table XIV, it can be seen that BM has better performance than other candidates.

In QPFET-VIKOR, the uncertainty can be fully considered under the frame of quantum. The QM can express the hesitant state of mind, which is consistent with human thinking. QPFET can better consider the fuzziness of information based on the proposed negation. Besides, the phase angle of quantum probability can easily describe the information from different angles at the same time. The amplitude and the phase angle can be aggregated to specific values, which can help us better analyze the problems. Moreover, the QPFET-VIKOR can consider not only the positive and negative aspects of information, but also the

TABLE V
$\mathbb{M}$ OF EXPERTS

		PW	OP	CO_2	IV
e_1	BF	$\mathbb{M}(VB) = \sqrt{0.57}e^{j\cdot 2\pi\cdot 0.2}$ $\mathbb{M}(\Theta) = \sqrt{0.43}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG,G) = \sqrt{0.93}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(\Theta) = \sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G) = \sqrt{0.91}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.09}e^{j\cdot 2\pi\cdot 0.6}$
	HD	$\mathbb{M}(M,G) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.77}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.23}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(B,M) = \sqrt{0.77}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.23}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(G,M,B) = \sqrt{0.8}e^{j\cdot 2\pi\cdot 0.6}$ $\mathbb{M}(\Theta) = \sqrt{0.2}e^{j\cdot 2\pi\cdot 0.6}$
	BM	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG,G) = \sqrt{0.93}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(\Theta) = \sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG,G) = \sqrt{0.93}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(G,M,B) = \sqrt{0.8}e^{j\cdot 2\pi\cdot 0.6}$ $\mathbb{M}(\Theta) = \sqrt{0.20}e^{j\cdot 2\pi\cdot 0.6}$
	WP	$\mathbb{M}(M,G) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.77}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.23}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(G,M,B) = \sqrt{0.8}e^{j\cdot 2\pi\cdot 0.6}$ $\mathbb{M}(\Theta) = \sqrt{0.20}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$
	ST	$\mathbb{M}(VG,G) = \sqrt{0.93}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(\Theta) = \sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.66}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.33}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(B) = \sqrt{0.61}e^{j\cdot 2\pi\cdot 0.4}$ $\mathbb{M}(\Theta) = \sqrt{0.39}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG,G) = \sqrt{0.93}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(\Theta) = \sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$
e_2	BF	$\mathbb{M}(VB) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.2}$ $\mathbb{M}(\Theta) = \sqrt{0.2727}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG,G) = \sqrt{0.93}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(\Theta) = \sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$
	HD	$\mathbb{M}(M,G) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VB) = \sqrt{0.57}e^{j\cdot 2\pi\cdot 0.2}$ $\mathbb{M}(\Theta) = \sqrt{0.43}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VB) = \sqrt{0.57}e^{j\cdot 2\pi\cdot 0.2}$ $\mathbb{M}(\Theta) = \sqrt{0.43}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.77}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.23}e^{j\cdot 2\pi\cdot 0.6}$
	BM	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$
	WP	$\mathbb{M}(G,M,B) = \sqrt{0.8}e^{j\cdot 2\pi\cdot 0.6}$ $\mathbb{M}(\Theta) = \sqrt{0.20}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(B) = \sqrt{0.61}e^{j\cdot 2\pi\cdot 0.4}$ $\mathbb{M}(\Theta) = \sqrt{0.39}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G) = \sqrt{0.91}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.09}e^{j\cdot 2\pi\cdot 0.6}$
	ST	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{QM}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.77}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.23}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.88}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.12}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$
e_3	BF	$\mathbb{M}(VB) = \sqrt{0.57}e^{j\cdot 2\pi\cdot 0.2}$ $\mathbb{M}(\Theta) = \sqrt{0.43}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.91}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.09}e^{j\cdot 2\pi\cdot 0.6}$
	HD	$\mathbb{M}(M,G) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.88}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.12}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.77}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.23}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.77}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.23}e^{j\cdot 2\pi\cdot 0.6}$
	BM	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.87}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.13}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$
	WP	$\mathbb{M}(G,M,B) = \sqrt{0.8}e^{j\cdot 2\pi\cdot 0.6}$ $\mathbb{M}(\Theta) = \sqrt{0.2}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(B) = \sqrt{0.61}e^{j\cdot 2\pi\cdot 0.4}$ $\mathbb{M}(\Theta) = \sqrt{0.39}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG,G) = \sqrt{0.93}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(\Theta) = \sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G) = \sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta) = \sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$
	ST	$\mathbb{M}(VG,G) = \sqrt{0.93}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(\Theta) = \sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.66}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.34}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B) = \sqrt{0.77}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta) = \sqrt{0.23}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG) = \sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta) = \sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$

different angles of information, which is a flexible and powerful method to handle the MCDM problem.

D. Discussion

In order to further study the performance of the proposed method, this section has a discussion about different methods. Another method cannot consider the information of negation, which is also based on the VIKOR, namely, QM-VIKOR. The specific steps of QM-VIKOR are basically consistent with QPFET-VIKOR; the difference is that the information of negation is not considered.

The final results of QM-VIKOR are as follows. Table XV shows $\mathcal{S}$, $\mathcal{R}$, and $\mathcal{Q}$ of candidates, which is similar to the QPFET-VIKOR. It can be proved that the QPFET-VIKOR is effective and reasonable.

Analyzing Table XV, the ranking order of candidates can be shown in Table XVI.

By analyzing Tables XIV and XV, the final ranking orders are shown in Fig. 9. The finally ranking order of QPFET-VIKOR and QM-VIKOR is similar, which can prove the effectiveness and reasonableness of QPFET-VIKOR.

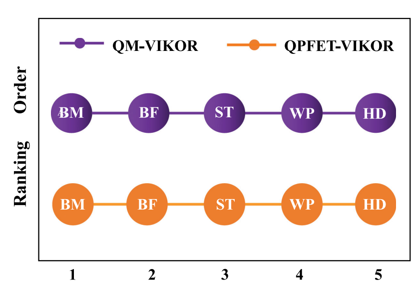

Fig. 9. Comparison between QM-VIKOR and QPFET-VIKOR.

By comparing the results of QM-VIKOR, it can be seen that BM has better performance than other candidates, which is similar to the results of QPFET-VIKOR. Besides, it can be seen that BF is the second candidate in $\mathcal{R}$ and $\mathcal{Q}$ of QM-VIKOR. Hence, we take BF as the second candidate. Similarly, WP is the fourth candidate by analyzing $\mathcal{S}$, $\mathcal{R}$,

TABLE VI
$\mathbb{M}$ OF FUSING ALL EXPERTS

	PW	OP	CO_2	IV
BF	$\mathbb{M}(VB)=\sqrt{0.61}e^{j\cdot 2\pi\cdot 0.2}$ $\mathbb{M}(\Theta)=\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG)=\sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta)=\sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG)=\sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta)=\sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G)=\sqrt{0.87}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta)=\sqrt{0.13}e^{j\cdot 2\pi\cdot 0.6}$
HD	$\mathbb{M}(M,G)=\sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(M,B)=\sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B)=\sqrt{0.61}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(VB)=\sqrt{0.14}e^{j\cdot 2\pi\cdot 0.2}$ $\mathbb{M}(\Theta)=\sqrt{0.24}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B)=\sqrt{0.57}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(VB)=\sqrt{0.14}e^{j\cdot 2\pi\cdot 0.2}$ $\mathbb{M}(\Theta)=\sqrt{0.28}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(G,M,B)=\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.6}$ $\mathbb{M}(M,B)=\sqrt{0.45}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta)=\sqrt{0.22}e^{j\cdot 2\pi\cdot 0.6}$
BM	$\mathbb{M}(VG)=\sqrt{0.94}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta)=\sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG,G)=\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(VG)=\sqrt{0.53}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta)=\sqrt{0.09}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG,G)=\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(VG)=\sqrt{0.55}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta)=\sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G)=\sqrt{0.43}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(G,M,B)=\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.6}$ $\mathbb{M}(\Theta)=\sqrt{0.24}e^{j\cdot 2\pi\cdot 0.6}$
WP	$\mathbb{M}(G,M,B)=\sqrt{0.47}e^{j\cdot 2\pi\cdot 0.6}$ $\mathbb{M}(G,M)=\sqrt{0.30}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta)=\sqrt{0.23}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B)=\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(B)=\sqrt{0.36}e^{j\cdot 2\pi\cdot 0.4}$ $\mathbb{M}(\Theta)=\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(G,M,B)=\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.6}$ $\mathbb{M}(M,G)=\sqrt{0.18}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(VG,G)=\sqrt{0.31}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(\Theta)=\sqrt{0.17}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,G)=\sqrt{0.78}e^{j\cdot 2\pi\cdot 0.7}$ $\mathbb{M}(\Theta)=\sqrt{0.22}e^{j\cdot 2\pi\cdot 0.6}$
ST	$\mathbb{M}(VG)=\sqrt{0.27}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(VG,G)=\sqrt{0.70}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(\Theta)=\sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B)=\sqrt{0.66}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(\Theta)=\sqrt{0.34}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(M,B)=\sqrt{0.48}e^{j\cdot 2\pi\cdot 0.5}$ $\mathbb{M}(B)=\sqrt{0.25}e^{j\cdot 2\pi\cdot 0.4}$ $\mathbb{M}(\Theta)=\sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{M}(VG,G)=\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.9}$ $\mathbb{M}(VG)=\sqrt{0.55}e^{j\cdot 2\pi\cdot 1}$ $\mathbb{M}(\Theta)=\sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6}$

TABLE VII
$\mathbb{PM}$ OF CANDIDATES CORRESPONDING TO CRITERIA

	Criteria (PW)	Criteria (OP)
BF	$\mathbb{PM}(VB)=[\sqrt{0.61}e^{j\cdot 2\pi\cdot 0.2},\sqrt{0.67}e^{j\cdot 2\pi\cdot 0.97},\sqrt{0.43}e^{j\cdot 2\pi\cdot 0.10}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.72}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.57}e^{j\cdot 2\pi\cdot 0.03}]$	$\mathbb{PM}(VG)=[\sqrt{0.94}e^{j\cdot 2\pi\cdot 1},\sqrt{0.34}e^{j\cdot 2\pi\cdot 0},\sqrt{0.06}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.94}e^{j\cdot 2\pi\cdot 0.01}]$
HD	$\mathbb{PM}(M,G)=[\sqrt{0.73}e^{j\cdot 2\pi\cdot 0.7},\sqrt{0.42}e^{j\cdot 2\pi\cdot 0.71},\sqrt{0.53}e^{j\cdot 2\pi\cdot 0.08}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.84}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.47}e^{j\cdot 2\pi\cdot 0.02}]$	$\mathbb{PM}(M,B)=[\sqrt{0.61}e^{j\cdot 2\pi\cdot 0.5},\sqrt{0.67}e^{j\cdot 2\pi\cdot 0.87},\sqrt{0.43}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(VB)=[\sqrt{0.14}e^{j\cdot 2\pi\cdot 0.2},\sqrt{0.96}e^{j\cdot 2\pi\cdot 0.98},\sqrt{0.26}e^{j\cdot 2\pi\cdot 0.02}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.24}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.92}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.02}]$
BM	$\mathbb{PM}(VG)=[\sqrt{0.94}e^{j\cdot 2\pi\cdot 1},\sqrt{0.34}e^{j\cdot 2\pi\cdot 0},\sqrt{0.06}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.94}e^{j\cdot 2\pi\cdot 0}]$	$\mathbb{PM}(VG,G)=[\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.9},\sqrt{0.87}e^{j\cdot 2\pi\cdot 0.43},\sqrt{0.30}e^{j\cdot 2\pi\cdot 0.05}]$ $\mathbb{PM}(VG)=[\sqrt{0.53}e^{j\cdot 2\pi\cdot 1},\sqrt{0.8271}e^{j\cdot 2\pi\cdot 0},\sqrt{0.20}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.24}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.83}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.50}e^{j\cdot 2\pi\cdot 0.02}]$
WP	$\mathbb{PM}(M,G)=[\sqrt{0.30}e^{j\cdot 2\pi\cdot 0.7},\sqrt{0.90}e^{j\cdot 2\pi\cdot 0.71},\sqrt{0.30}e^{j\cdot 2\pi\cdot 0.04}]$ $\mathbb{PM}(G,M,B)=[\sqrt{0.46}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.80}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.38}e^{j\cdot 2\pi\cdot 0.03}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.23}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.92}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.02}]$	$\mathbb{PM}(M,B)=[\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.5},\sqrt{0.89}e^{j\cdot 2\pi\cdot 0.8660},\sqrt{0.34}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(B)=[\sqrt{0.36}e^{j\cdot 2\pi\cdot 0.4},\sqrt{0.87}e^{j\cdot 2\pi\cdot 0.92},\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.03}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.89}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.02}]$
ST	$\mathbb{PM}(VG)=[\sqrt{0.24}e^{j\cdot 2\pi\cdot 1},\sqrt{0.92}e^{j\cdot 2\pi\cdot 0},\sqrt{0.32}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(VG,G)=[\sqrt{0.70}e^{j\cdot 2\pi\cdot 0.9},\sqrt{0.67}e^{j\cdot 2\pi\cdot 0.42},\sqrt{0.25}e^{j\cdot 2\pi\cdot 0.12}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.90}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.44}e^{j\cdot 2\pi\cdot 0.07}]$	$\mathbb{PM}(M,B)=[\sqrt{0.66}e^{j\cdot 2\pi\cdot 0.5},\sqrt{0.50}e^{j\cdot 2\pi\cdot 0.87},\sqrt{0.56}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.34}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.83}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.44}e^{j\cdot 2\pi\cdot 0.02}]$
	Criteria (CO_2)	Criteria (IV)
BF	$\mathbb{PM}(VG)=[\sqrt{0.94}e^{j\cdot 2\pi\cdot 1},\sqrt{0.34}e^{j\cdot 2\pi\cdot 0},\sqrt{0.06}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.06}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.96}e^{j\cdot 2\pi\cdot 0.01}]$	$\mathbb{PM}(M,G)=[\sqrt{0.87}e^{j\cdot 2\pi\cdot 0.7},\sqrt{0}e^{j\cdot 2\pi\cdot 0.71},\sqrt{0.58}e^{j\cdot 2\pi\cdot 0.09}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.13}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.90}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.42}e^{j\cdot 2\pi\cdot 0.01}]$
HD	$\mathbb{PM}(M,B)=[\sqrt{0.57}e^{j\cdot 2\pi\cdot 0.5},\sqrt{0.70}e^{j\cdot 2\pi\cdot 0.87},\sqrt{0.42}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(VB)=[\sqrt{0.14}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.96}e^{j\cdot 2\pi\cdot 0.98},\sqrt{0.26}e^{j\cdot 2\pi\cdot 0.02}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.28}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.90}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.02}]$	$\mathbb{PM}(G,M,B)=[\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.88}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.02}]$ $\mathbb{PM}(M,B)=[\sqrt{0.45}e^{j\cdot 2\pi\cdot 0.5},\sqrt{0.81}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.37}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.22}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.93}e^{j\cdot 2\pi\cdot 0.80},\sqrt{0.30}e^{j\cdot 2\pi\cdot 0.02}]$
BM	$\mathbb{PM}(VG,G)=[\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.9},\sqrt{0.86}e^{j\cdot 2\pi\cdot 0.43},\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.02}]$ $\mathbb{PM}(VG)=[\sqrt{0.55}e^{j\cdot 2\pi\cdot 1},\sqrt{0.81}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.20}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.88}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.47}e^{j\cdot 2\pi\cdot 0.01}]$	$\mathbb{PM}(G,M,B)=[\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.88}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.02}]$ $\mathbb{PM}(M,G)=[\sqrt{0.43}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.83}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.36}e^{j\cdot 2\pi\cdot 0.03}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.24}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.92}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.31}e^{j\cdot 2\pi\cdot 0.02}]$
WP	$\mathbb{PM}(G,M,B)=[\sqrt{0.33}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.89}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.30}e^{j\cdot 2\pi\cdot 0.04}]$ $\mathbb{PM}(M,G)=[\sqrt{0.18}e^{j\cdot 2\pi\cdot 0.7},\sqrt{0.95}e^{j\cdot 2\pi\cdot 0.71},\sqrt{0.25}e^{j\cdot 2\pi\cdot 0.02}]$ $\mathbb{PM}(VG,G)=[\sqrt{0.31}e^{j\cdot 2\pi\cdot 0.9},\sqrt{0.93}e^{j\cdot 2\pi\cdot 0.43},\sqrt{0.18}e^{j\cdot 2\pi\cdot 0.04}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.17}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.95}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.26}e^{j\cdot 2\pi\cdot 0.01}]$	$\mathbb{PM}(M,G)=[\sqrt{0.78}e^{j\cdot 2\pi\cdot 0.7},\sqrt{0.31}e^{j\cdot 2\pi\cdot 0.71},\sqrt{0.55}e^{j\cdot 2\pi\cdot 0.09}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.22}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.86}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.45}e^{j\cdot 2\pi\cdot 0.02}]$
ST	$\mathbb{PM}(M,B)=[\sqrt{0.48}e^{j\cdot 2\pi\cdot 0.5},\sqrt{0.79}e^{j\cdot 2\pi\cdot 0.87},\sqrt{0.38}e^{j\cdot 2\pi\cdot 0}]$ $\mathbb{PM}(B)=[\sqrt{0.25}e^{j\cdot 2\pi\cdot 0.4},\sqrt{0.92}e^{j\cdot 2\pi\cdot 0.9163},\sqrt{0.31}e^{j\cdot 2\pi\cdot 0.02}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.27}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.91}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.31}e^{j\cdot 2\pi\cdot 0.02}]$	$\mathbb{PM}(VG)=[\sqrt{0.55}e^{j\cdot 2\pi\cdot 1},\sqrt{0.81}e^{j\cdot 2\pi\cdot 0},\sqrt{0.20}e^{j\cdot 2\pi\cdot 0.}]$ $\mathbb{PM}(VG,G)=[\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.9},\sqrt{0.86}e^{j\cdot 2\pi\cdot 0.43},\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.05}]$ $\mathbb{PM}(\Theta)=[\sqrt{0.07}e^{j\cdot 2\pi\cdot 0.6},\sqrt{0.88}e^{j\cdot 2\pi\cdot 0.8},\sqrt{0.47}e^{j\cdot 2\pi\cdot 0.01}]$

TABLE VIII
IMPORTANCE OF CRITERIA GIVEN BY EXPERTS

Criteria	e1	e2	e3
PW	VI (0.8)	VI (0.9)	VI (0.9)
OP	VI,I (0.9)	VI (0.8)	VI (0.8)
CO_2	VI,I (0.9)	I,M,UI (0.7)	I,M,UI (0.8)
IV	M,U (0.8)	UI (0.8)	UI (0.9)

and $\mathcal{Q}$. HD is the fifth candidate by comparing $\mathcal{S}$ and $\mathcal{Q}$. Finally, ST can only be regarded as the third candidate, which can different in $\mathcal{S}$, $\mathcal{R}$, and $\mathcal{Q}$. In QPFET-VIKOR, BF is the second candidate by comparing $\mathcal{S}$, $\mathcal{R}$, and $\mathcal{Q}$. ST is the third candidate by analyzing $\mathcal{S}$ and $\mathcal{Q}$. WP can be regarded as the fourth candidate by comparing $\mathcal{S}$, $\mathcal{R}$, and $\mathcal{Q}$. HD is the fifth candidate by analyzing $\mathcal{S}$ and $\mathcal{Q}$. Therefore, it can be seen

TABLE IX
$\mathbb{M}$ OF CRITERIA ACCORDING TO EXPERTS

	$\mathbb{M}$		$\mathbb{M}$
PW	$\mathbb{M}_1(VI)=\sqrt{0.87}e^{J\cdot 2\pi\cdot 1}, \mathbb{M}_1(\Theta)=\sqrt{0.13}e^{J\cdot 2\pi\cdot 0.6}$	OP	$\mathbb{M}_1(VI,I)=\sqrt{0.93}e^{J\cdot 2\pi\cdot 0.9}, \mathbb{M}_1(\Theta)=\sqrt{0.07}e^{J\cdot 2\pi\cdot 0.6}$
	$\mathbb{M}_2(VI)=\sqrt{0.94}e^{J\cdot 2\pi\cdot 1}, \mathbb{M}_2(\Theta)=\sqrt{0.06}e^{J\cdot 2\pi\cdot 0.6}$		$\mathbb{M}_2(VI)=\sqrt{0.87}e^{J\cdot 2\pi\cdot 1}, \mathbb{M}_2(\Theta)=\sqrt{0.13}e^{J\cdot 2\pi\cdot 0.6}$
	$\mathbb{M}_3(VI)=\sqrt{0.87}e^{J\cdot 2\pi\cdot 1}, \mathbb{M}_3(\Theta)=\sqrt{0.13}e^{J\cdot 2\pi\cdot 0.6}$		$\mathbb{M}_3(VI)=\sqrt{0.87}e^{J\cdot 2\pi\cdot 1}, \mathbb{M}_3(\Theta)=\sqrt{0.13}e^{J\cdot 2\pi\cdot 0.6}$
CO_2	$\mathbb{M}_1(VI,I)=\sqrt{0.93}e^{J\cdot 2\pi\cdot 0.9}, \mathbb{M}_1(\Theta)=\sqrt{0.07}e^{J\cdot 2\pi\cdot 0.6}$	IV	$\mathbb{M}_1(M,UI)=\sqrt{0.77}e^{J\cdot 2\pi\cdot 0.5}, \mathbb{M}_1(\Theta)=\sqrt{0.23}e^{J\cdot 2\pi\cdot 0.6}$
	$\mathbb{M}_2(I,M,UI)=\sqrt{0.7}e^{J\cdot 2\pi\cdot 0.6}, \mathbb{M}_2(\Theta)=\sqrt{0.3}e^{J\cdot 2\pi\cdot 0.6}$		$\mathbb{M}_2(UI)=\sqrt{0.72}e^{J\cdot 2\pi\cdot 0.4}, \mathbb{M}_2(\Theta)=\sqrt{0.27}e^{J\cdot 2\pi\cdot 0.6}$
	$\mathbb{M}_3(I,M,UI)=\sqrt{0.8}e^{J\cdot 2\pi\cdot 0.6}, \mathbb{M}_3(\Theta)=\sqrt{0.2}e^{J\cdot 2\pi\cdot 0.6}$		$\mathbb{M}_3(UI)=\sqrt{0.86}e^{J\cdot 2\pi\cdot 0.4}, \mathbb{M}_3(\Theta)=\sqrt{0.14}e^{J\cdot 2\pi\cdot 0.6}$

TABLE X
$\mathbb{M}$, $\mathbb{PM}$, AND WEIGHTS OF CRITERIA

	$\mathbb{M}$	$\mathbb{PM}$	Weight
PW	$\mathbb{M}(VI)=\sqrt{0.89}e^{j\cdot 2\pi\cdot 1}$	$\mathbb{PM}(VI)=[\sqrt{0.89}e^{j\cdot 2\pi\cdot 1}, \sqrt{0.45}e^{j\cdot 2\pi\cdot 0}, \sqrt{0.09}e^{j\cdot 2\pi\cdot 0}]$	0.32
	$\mathbb{M}(\Theta)=\sqrt{0.11}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{PM}(\Theta)=[\sqrt{0.11}e^{j\cdot 2\pi\cdot 0.6}, \sqrt{0.40}e^{j\cdot 2\pi\cdot 0.80}, \sqrt{0.91}e^{j\cdot 2\pi\cdot 0.01}]$	
OP	$\mathbb{M}(VI,I)=\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.9}$	$\mathbb{PM}(VI,I)=[\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.9}, \sqrt{0.87}e^{j\cdot 2\pi\cdot 0.43}, \sqrt{0.31}e^{j\cdot 2\pi\cdot 0.05}]$	0.37
	$\mathbb{M}(VI)=\sqrt{0.51}e^{j\cdot 2\pi\cdot 1}$	$\mathbb{PM}(VI)=[\sqrt{0.51}e^{j\cdot 2\pi\cdot 1}, \sqrt{0.83}e^{j\cdot 2\pi\cdot 0}, \sqrt{0.21}e^{j\cdot 2\pi\cdot 0}]$	
	$\mathbb{M}(\Theta)=\sqrt{0.10}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{PM}(\Theta)=[\sqrt{0.10}e^{j\cdot 2\pi\cdot 0.6}, \sqrt{0.88}e^{j\cdot 2\pi\cdot 0.80}, \sqrt{0.47}e^{j\cdot 2\pi\cdot 0}]$	
CO_2	$\mathbb{M}(VI,I)=\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.9}$	$\mathbb{PM}(VI,I)=[\sqrt{0.39}e^{j\cdot 2\pi\cdot 0.9}, \sqrt{0.90}e^{j\cdot 2\pi\cdot 0.43}, \sqrt{0.22}e^{j\cdot 2\pi\cdot 0.05}]$	0.23
	$\mathbb{M}(VI,M,I)=\sqrt{0.44}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{PM}(VI,M,I)=[\sqrt{0.44}e^{j\cdot 2\pi\cdot 0.6}, \sqrt{0.79}e^{j\cdot 2\pi\cdot 0.80}, \sqrt{0.43}e^{j\cdot 2\pi\cdot 0.03}]$	
	$\mathbb{M}(\Theta)=\sqrt{0.17}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{PM}(\Theta)=[\sqrt{0.17}e^{j\cdot 2\pi\cdot 0.6}, \sqrt{0.92}e^{j\cdot 2\pi\cdot 0.80}, \sqrt{0.35}e^{j\cdot 2\pi\cdot 0.01}]$	
IV	$\mathbb{M}(M,UI)=\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.5}$	$\mathbb{PM}(M,UI)=[\sqrt{0.32}e^{j\cdot 2\pi\cdot 0.5}, \sqrt{0.89}e^{j\cdot 2\pi\cdot 0.87}, \sqrt{0.34}e^{j\cdot 2\pi\cdot 0}]$	0.09
	$\mathbb{PM}(UI)=\sqrt{0.47}e^{j\cdot 2\pi\cdot 0.4}$	$\mathbb{M}(UI)=[\sqrt{0.47}e^{j\cdot 2\pi\cdot 0.4}, \sqrt{0.81}e^{j\cdot 2\pi\cdot 0.92}, \sqrt{0.36}e^{j\cdot 2\pi\cdot 0.03}]$	
	$\mathbb{M}(\Theta)=\sqrt{0.21}e^{j\cdot 2\pi\cdot 0.6}$	$\mathbb{PM}(\Theta)=[\sqrt{0.21}e^{j\cdot 2\pi\cdot 0.6}, \sqrt{0.93}e^{j\cdot 2\pi\cdot 0.80}, \sqrt{0.30}e^{j\cdot 2\pi\cdot 0.02}]$	

TABLE XI
SCORE MATRIX $\mathbb{G}$

	BF	HD	BM	WP	ST
PW	0.1552	0.6417	1.5679	0.7837	2.1558
OP	1.5679	0.4058	2.1239	0.4025	0.3550
CO_2	1.5679	0.5837	2.1222	1.5256	0.4202
IV	0.7519	0.5593	0.6571	0.6770	2.1203

TABLE XII
BEST AND WORST VALUES FOR INDIVIDUAL CRITERION

	PW	OP	CO_2	IV
$\mathbb{G}^+$	2.1558	2.1239	2.1222	2.1203
$\mathbb{G}^-$	0.1552	0.3550	0.4202	0.5593

TABLE XIII
$\mathcal{S}$, $\mathcal{R}$, AND $\mathcal{Q}$ OF CANDIDATES

	BF	HD	BM	WP	ST
$\mathcal{S}$	0.5901	0.8995	0.1784	0.7434	0.6000
$\mathcal{R}$	0.3200	0.3594	0.0940	0.3601	0.3700
$\mathcal{Q}$	0.6949	0.9807	0	0.8738	0.7923

TABLE XIV
RANKING OF CANDIDATES

	Ordering
$\mathcal{S}$	$BM \succ BF \succ ST \succ WP \succ HD$
$\mathcal{R}$	$BM \succ BF \succ HD \succ WP \succ ST$
$\mathcal{Q}$	$BM \succ BF \succ ST \succ WP \succ HD$

TABLE XV
$\mathcal{S}$, $\mathcal{R}$, AND $\mathcal{Q}$ OF CANDIDATES BY USING QM-VIKOR

	BF	HD	BM	WP	ST
$\mathcal{S}$	0.4744	0.9082	0.1299	0.7524	0.5796
$\mathcal{R}$	0.3167	0.3665	0.0759	0.3685	0.2237
$\mathcal{Q}$	0.6328	0.9966	0	0.8999	0.5415

TABLE XVI
RANKING OF CANDIDATES BY USING QM-VIKOR

	Ordering
$\mathcal{S}$	$BM \succ ST \succ BF \succ WP \succ HD$
$\mathcal{R}$	$BM \succ BF \succ HD \succ WP \succ ST$
$\mathcal{Q}$	$BM \succ BF \succ ST \succ WP \succ HD$

that QPFET-VIKOR is a more effective method to handle the MCDM.

V. CONCLUSION

D-S evidence theory can better express uncertainty, which can be caused by objective lack of complete knowledge or subjective preferences and biases. However, in most cases, information can be obtained from different angles at the same time. From the cognitive analysis, the behavior of people has the inherent dual property. Hence, expanding D-S evidence into the quantum framework is essential, which is named QM. The amplitude and the phase angle of the QM can easily express the properties of information and simulate human thinking. Besides, everything in nature will have its negation. For the Bayes theorem, it is the process of negation. Then, for the QM, what is its negation? Based on this question, this article proposed the negation of QM, which can be computed by using subtraction of vectors in the unit circle. The proposed negation is compatible with Yager's negation and Yin *et al.*'s [49] negation when the QM degenerates the probability and mass function, separately. Besides, this article proposed QPFET based on the proposed negation, where the probability of QM can be understood as the membership of event A and the probability after negation can be regarded as the nonmembership of event A. Moreover, some numerical examples were used to explain the proposed method. In order to further explore the QPFET, this article proposed QPFET-VIKOR to handle the MCDM problem. Finally, to prove the effectiveness of the proposed method, this article can give a discussion between QM-VIKOR and QPFET-VIKOR.

The main contributions in QPFET are as follows.

1) It is the novel research to study negation from the view of vectors, which gives the proposed method a more rigorous mathematical proof.
2) We use the proposed negation to generate QPFET, which can build the bridge between the quantum fuzzy set and the QM.
3) In QPFET-VIKOR, the amplitude and the phase angle can be considered separately when aggregating the information.

In summary, the proposed negation method and the QPFET can provide a promising approach to analyze information in the process of solving decision-making problems.

In the future, the physical meaning of negation should be further discussed, which can expand negation to more fields. For the amplitude and the phase angle, how to accurately obtain their values is a problem of value exploration. Besides, the connection between phase angle and uncertainty should be studied, which can help us obtain the more accurate information. How to measure fuzziness of information under quantum discernment is also an open issue. Finally, using quantum theory into more fields has been a problem that needs to be explored for a long time.

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