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The ramification of dynamic investment on the promotion and preservation technology for inventory management through a modified flower pollination algorithm

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ABSTRACT

The dynamic rate of investment in promotion during different price-ranges and the optimum investment in preservation technology are developed for seasonable or fashionable products through their lifespan. An order level inventory model for deteriorating items with promotional price and trapezoidal-type demand rate is used to maximize the profit of a retailer. The deterioration rate is dependent on the preservation technology i.e., more investment in preservation reduces the rate of deterioration. The aim of this study is to obtain the optimum dynamic investment for the promotion. The optimal control theory is employed to obtain the dynamic investment rates. A modified flower pollination Algorithm is applied to find the optimal pricing scheme, preservation technology investment, and replenishment schedule. An illustrated algorithm based numerical experiment is conducted to validate the dynamic behaviour of the investment. Sensitivity analysis has been carried out with respect to the major parameters to prove the novelty of the algorithm and explored the managerial insights of the proposed model. Numerical studies obtain that dynamical investment is really beneficial for the management of stocks.

1. Introduction

For increasing sales, the retail industry always tries for several strategies. Among them the promotional effort is one of the best strategy (Blom et al. (2017)). The aim of sales is always fixed with a certain target. If the target is achieved then the promotion may be offered. Thus, everyone in the retail industry tries hard to obtain the promotion. At the same time, it is very much difficult to decide the investment for this promotion purpose by the management of any industry. Generally, the investment for this promotion is not deterministic, it is dynamic. But utilizing a dynamic investment, it is really hard to make the mathematical modelling. This study continues with the dynamic investment for promotion. However, it is found in the literature that the promotion is an imperative tool which is used for stimulating consumers and wining competitions (Svendsen et al. (2011)). Promotion is targeted to achieve various objectives like boost sales (Srinivasan et al. (2004)); generate attention (Bava et al. (2009)); increase in-store traffic (Spiekermann et al. (2011)); stimulate sales staff (Gao et al. (2016)). As products move through the various stages of the product lifespan, different promotional strategies are employed by the retailer to create anomalies.

At the introduction and growth stages, the foremost objective is to inform the target consumers about its entry, and to increase brand awareness and customer loyalty. At the maturity stage, the retailer stimulates consumers to choose their products over their rivals. Finally, at the decline stage, the retailer generally would like to clear out remaining inventory and generate revenues as much as possible. Therefore, a proper planning for investment for promotion is required to promote each product and it's profit. However, very few researches integrate price, promotion, and replenishment decision. Zhang et al. (2008) analyzed the combine effect of price and promotion for a periodic review inventory model over finite planning horizon. Tsao and Sheen (2008) considered a price- and time-dependent demand function to study the impact of promotion for deteriorating items. Grewal et al. (2011) made some argues that the retailer has considerable opportunities to target customers by applying innovations in pricing and

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promotion. Maihami and Karimi (2014) analyzed an inventory model under price and promotional efforts dependent demand. This is the first study, where the power of dynamic investment in promotion is evaluated under the trapezoidal-type demand. The research gap and the novelty of this study are enlisted below.

1.1. Research gaps in this field of study

From Table 1, one may find that the contribution can be summarized in.

- Though several investigations are available under the trapezoidal-type demand, no one has formulated and investigated the impact of the continuous investment in promotion through the product lifespan.
- These models overlook the impact of preservation technology investment to reduce deterioration rate.
- These models assume predetermine replenishment cycle time.

1.2. Novelty of this study

To fulfil the research gap, the aims of the proposed model are as follows:

- An analytical model is formulated for the optimal investment rates and prices for different stages of the product lifespan.
- The profitability of the investment in preservation technology is verified.
- A suitable heuristic technique is designed for such complicated real-life problem.

2. Literature review

The literature review is developed based on the following area of research.

2.1. Marketing with promotional policy

Pricing scheme and replenishment schedule are two important factors of the inventory management. The availability of items within on-time is the major task of the replenishment schedule. Then, the price stability of the product works good if there is no shortage. The promotional activity for the demand has an influence on the pricing scheme. Dey et al. (2019) investigated the pricing and setup cost reduction policy within an inventory model. They used an additional investment for the reduction of the discrete setup cost of the system. The situation indicates very sensitive when the items are in deteriorating in nature. Noh et al. (2019) discussed about the stochastic replenishment policy. The dynamic pricing policy was studied by Ullah et al. (2019) for a multi-period inventory model. Different types of promotional efforts were discussed by Yu et al. (2020). The preservation technology and the associative investment is the either way to keep the deteriorated kind items fresh for a long time. Das et al. (2020) investigated the pricing scheme with the combination of the preservation within an inventory system. Dye (2013) studied about the preservation for a non-instantaneous deteriorating rate of the items whereas Dye and Yang (2016) discussed about the pricing policy with preservation. Investment for quality improvement (Guchhait et al. (2020)) within the inventory model is another policy of the pricing and long-run business. The promotion has an additional advantage for the steady demand in market. A case study in a milk market of the Germany was investigated by Holzer (2020a). There are several authors who worked on the preservation technology in inventory model and supply chain (Iqbal and Sarkar (2019); Saha et al. (2017)). But there is a big research gap in the literature that preservation with dynamic investment is not considered by anyone till now under the trapezoidal demand pattern.

The demand rate for many products increases gradually during the introduction and finally enters into the decline stage (Wu et al. (2016)). In the introductory and growth stage, it is to be noticed that buyers are less price-insensitive due to lack of knowledge of product's benefits and quality, and the demand increases with respect to the time. An appropriate mixture of promotion and pricing is crucial at this stage, not only for the market price expectations, but also for stimulating the profit trajectory across the lifespan. $\acute{a}\acute{o}$ Cárdenas-Barrón and Sana, 2015 introduced a multi-item supply chain model of delaying period and variable demand with a promotional effort. At the maturity stage, sales growth becomes steady and is approaching the point, where the natural decline of product is started. Defending market share becomes the focal concern at this stage and the retailer has to invest more in promotion to entice customers for buying products. It is strongly commented from a marketing standpoint that the effective promotional strategy can make substantial impact at this stage compared to other stages. Finally, the decline stage occurs. At this stage, demand of the product decreases rapidly with respect to time and retailer commonly improves profit by reducing marketing spending or price-reduction. However, in the absence of pricing and impact, this kind of demand variation of products becomes a function of time through its lifespan. It is termed as trapezoidal-type demand patterns in literature after the pioneer work of Cheng and Wang (2009). Table 1 summarizes the key assumptions of the mathematical models.

The promotional effort has huge impact on the retail industry and customers. It is found from the literature that the coordination between manufacturer and retailers is always beneficial for the business. Nguyen et al. (2020) discussed about the cooperative advertisement policy between retailers and manufacturer. There are several factors, which are working actively in the retailer's market. Afshar-Nadjafi et al. (2016) developed a mathematical model about the time-dependent demand policy. In retail industry, the brand image makes a big impact on the profit of the industry. Grosso et al. (2018) introduced the brand image, the behavior of the staffs, and loyalty have impact on the customer in

Table 1
Research contribution in the direction of this model.

Authors	Planning Horizon	Price consideration	Preservation technology investment	Investment in promotion	Metaheuristic algorithm
Cheng and Wang, 2009	finite	–	–	–	–
Cheng et al., 2011	finite	yes	–	–	–
Lin et al., 2014	finite	–	–	yes	yes
Mishra, 2015	finite	–	yes	–	–
Panda and Saha, 2010	finite	yes	–	–	–
Saha et al., 2011	finite	–	–	yes	–
Uthayakumar and Rameswari, 2012	finite	–	–	–	–
Wu et al., 2016	finite	–	yes	–	–
Zhao, 2014	finite	–	–	–	yes
Present study	finite or infinite	yes	yes	yes	MFPA algorithm

– indicates is not applicable here.

their study to show the major effect on the profit. The customers' demand for the food, beverage, or vegetables i.e., which are deteriorating in nature are usually time-dependent (Holzer (2020b), Lombart et al., 2019)). The vendor managed inventory was explained by Taleizadeh et al. (2020) for the optimal stock replenishment. The demand variation for the fashionable products was discussed by Namin et al. (2017). The fashion products are generally considered as a deteriorating products with respect to the era. A case study about the grocery products was described by Albors-Garrigos (2020). Now, the research context is moved towards the green technology establishment instead of promoting the traditional policies (Kumar and Polonsky (2019)). Through there were several efforts on the promotional efforts by the retailing industry, the trapezoidal-type demand and dynamic investment were not investigated. The proposed research solves this research gap within the literature.

2.2. Preservation technology for trapezoidal-type demand

In addition to trapezoidal-type demand, the related literature with this study mainly includes the following research streams: dynamic investment in preservation technology, investment in promotion, pricing strategy, and a new metaheuristic Algorithm for obtaining the solution of complex inventory control problem. The coordination policy is always fruitful for the supply chain policies for fixed lifespanned products (Sarkar (2016)).

In the retail industry, the perishability of products is a critical factor. In addition to natural deterioration, it may occur as a result of abnormal wear and tear due to maloperations, poor maintenance, display technique, irregular disposal, and harsh environmental conditions. An investment in the preservation technology prevents the deterioration rate significantly, reduces the product loss, and helps retailers to accomplish the profit optimization goal. Specially, due to Covid 19, there is a global pandemic situation. Due to lockdown of routine activities in all aspects, there is s sharp increasing value of deterioration is found. Thus, the investment on preservation of products really important to increase the lifespan of products. Tiwari et al. (2018b) discussed about an optimal pricing policy of deteriorating items in warehouse. The capacity of the warehouse was considered in that model. Tiwari et al. (2018a) analyzed a joint pricing of an inventory model with deteriorating items containing expiry dates. A partial trade-credit policy with partial backlogging was considered in the model. Hsu et al. (2010) introduced a preservation technology strategy with the profitability of a monopolistic retailer. Thereafter, various types of inventory replenishment models for deteriorating items have been discussed recently. Dye and Hsieh (2012) introduced a preservation strategy for the best replenishment policy. They proved that the presentation can reduce the rate determination but they did not prove how much deterioration rate can be controlled that only can be solved through some dynamical study. Chen and Dye (2013) utilized a metaheuristic Algorithm as the particle swam optimization to solve an inventory model with deterioration, which is controlled by using same preservation strategy. Yang et al. (2015) extended this model with an optical dynamic trade-credit policy under the effect of preservation technology. Tsao (2016) developed for the direction of joint location of inventory and the effects of preservation technology on those inventories. They really controlled the rate of deterioration through location-wise preservation technologies. Zhang et al. (2016) introduced a pricing policy within the preservation technology enabled service and investment but they did not consider any dynamic investment, only fixed investment was utilized. This research gap is fulfilled by this proposed research. However, Table 1 demonstrates that the effect of deterioration is considered by several scholars in the absence of the preservation technology investment. The proposed study influences the investment decision in preservation technology under the trapezoidal-type demand.

2.3. Dynamic modelling with metaheuristic approach

The nature inspired metaheuristic algorithms are now becoming popular and efficient tool to obtain the optimal solution of complex real world inventory replenishment and pricing problems. $\acute{a}\acute{o}$ Cárdenas-Barrón et al., 2018 developed an inventory model with discount policy on optimal order. The uncertainty is another real issue which is complex in nature. Sarkar et al. (2020) discussed about the fuzzy method to deal with the promotional policy under uncertain condition. They used signed distance method for fuzzy environment. Unlike gradient descent-type algorithms, metaheuristic algorithms are easy to apply. In recent years, some scholars have successfully applied metaheuristic Algorithm like genetic algorithm (Hong and Kim (2009); Taleizadeh et al. (2013)), particle swarm optimization (Sadeghi et al. (2016); Bhunia and Shaikh (2015)); bat algorithm (Sadeghi et al. (2014)), ant colony optimization (Nia et al. (2014); Huang and Lin (2010)), simulated annealing algorithm (Maiti and Maiti (2005)) and other authors to find optimal decisions for inventory control problem. The flower pollination algorithm (FPA) is one of such popular metaheuristic algorithms. It was developed by Yang (2012) and successfully implemented to obtain the solution of several real-life complex optimization problem (Abdelaziz et al. (2016)). Recently, Nabil (2016) hybridized FPA and clonal selection algorithm (CSA) (De Castro and Timmis, 2002) and proved that the modified flower pollination algorithm (MFPA) can outperform popular metaheuristic algorithms such as bat algorithm, firefly algorithm, genetic algorithm, and simulated annealing. From the applicability point of view of this study, MFPA is used to solve the developed constraint optimization model. The Mantegna algorithm is used to generate the L$\acute{e}$vy Flight (Mantegna (1994)). To enhance performance of the FPA algorithm, Nabil (2016) proposed a MFPA by hybridizing FPA (Yang (2012)) and clonal selection algorithm (De Castro and Timmis, 2002) to improve the convergence rate. The MFPA algorithm selects best m solution from the population and clones proportionally to the fitness of the solution. The clonal property inspired from clonal selection algorithm ensures high explorability of the pollen, which increase the probability of finding the global optimum.

A metaheuristic optimization procedure was used by Horng and Lin (2017) for assemble-to-order (ATO) inventory system. They developed an ordinal optimization based metaheuristic (OOHM) Algorithm which covers three modules, namely meta-modelling, exploration, and exploitation. They found that the proposed algorithm gives better result than the three heuristics. Parsopoulos et al. (2015) investigated differential evolution (DE) algorithm for the dynamic lot sizing model. DE is a population-based algorithm and they compared the results with the existing state-of-art algorithms. DE established as an efficient algorithm for the specific dynamic type of problems. A metaheuristic optimization approach was utilized by Sarkar et al. (2019) within a closed-loop supply chain management for returnable transport items. Shaabani and Kamalabadi (2016) used simulated annealing for an inventory routing problem. The inventory was in perishable in nature. The use of the metaheuristics approach and its application in food processing industry was analyzed by (Wari and Zhu, 2016). They optimized both the system operation and the processing process. $\acute{a}\acute{o}$ (Cárdenas-Barrón et al., 2019) introduced a heuristic approach for reduction of waste for retail industry for a specific product. They proved that heuristic approach can save time and effort for any industry to obtain the optimum result. Even though there were several metaheuristic algorithms, but it was proved for the dynamical modelling the MFPA converges over the other algorithms. Thus, it is utilized for the proposed model.

The organization of the paper is as follows: Section 3 describes the assumptions and corresponding notation for this model. In Section 4, the mathematical formulation of the model is presented and Section 5 discusses the MFPA Algorithm. The numerical example is provided in Section 6 and the sensitivity analysis is given in the Section 7. Section 8 provides the managerial insights and finally Section 9 concludes by highlighting the major findings, limitations, and future research directions.

3. Notation and assumptions

The following notation and assumptions are used to formulate this model.

3.1. Notation

Index	
I	consecutive time period, $i = 1, 2, 3$
Decision variables	
$s_i(t)$	dynamic investment rate for promotion at $t \in [\mu_{i-1}, \mu_i]; \mu_0 = 0; \mu_3 = T$
p_i	selling price per unit time $t \in [\mu_{i-1}, \mu_i]; \mu_0 = 0; \mu_3 = T$
I_0	initial inventory level, $I_0 > 0$
u	preservation technology investment, $u \geq 0$
T	length of the replenishment cycle
Parameters	
$I(t)$	inventory holding position, $t \in (0, T]$
$S(t)$	impact of promotion on demand rate at time $t \in (0, T]$
K	replenishment cost per order ($/order)
C	purchasing cost per unit ($/unit)
c_d	disposal cost per unit ($/unit)
H	inventory holding cost per unit per unit time ($/unit/unit time)
θ_1	deterioration rate under the natural condition
θ_0	reduced feasible deterioration rate under preservation technology investment
Ξ	efficiency coefficient of preservation technology investment
β_i	price elasticity of products at any time $t \in [\mu_{i-1}, \mu_i]; i = 1-3; \mu_0 = 0; \mu_3 = T$
α_i	marginal cost for investment at time $t \in [\mu_{i-1}, \mu_i]; i = 1-3; \mu_0 = 0; \mu_3 = T$
Π	total profit per unit time ($/time)

3.2. Assumptions

The following assumptions are used to formulate the mathematical model.

1. The model considers a single-type of items with an attractive dynamic rate in promotion and a MFPA Algorithm is developed to solve the model by using Prontryagin's maximum principle.
2. The market demand pattern $D(t, p_1, p_2, p_3, S(t))$ is a price-promotion- and trapezoidal-type and its functional form is as follows:

$$D(t, p_1, p_2, S(t)) = \begin{cases} a_1 + b_1 t - \beta_1 p_1 + \gamma_1 S(t) & \text{if } 0 \leq t \leq \mu_1 \\ D_0 - \beta_2 p_2 + \gamma_2 S(t) & \text{if } \mu_1 \leq t \leq \mu_2 \\ a_2 e^{-b_2 t} - \beta_3 p_3 + \gamma_3 S(t) & \text{if } \mu_2 \leq tT \end{cases} \quad (1)$$

The demand pattern is closely related with Wu et al. (2016). The demand increases directly with respect to time through the initial and growth stage, i.e. $[0, \mu_1]$. In the maturity stage $[\mu_1, \mu_2]$, it becomes steady with respect to time and ultimately starts declining in the final stage $[\mu_2, T]$. In all three stages, the additional fluctuation occurs due to prices and investment in promotion. The promotion is used to accomplish objectives like, building product awareness, attract new customers, providing information about the store location, stimulating demand, increase sales in off-seasons. Therefore, the impact of the promotion cannot be ignored if the retailer would like to determine tangible the optimal replenishment policy and investment decision. Moreover, it is commonly observed that the retailer sets different prices during different stages of the product lifespan, and effective pricing strategy always helps the retailer to maximize profits on sales. These motivate the proposed study to integrate the effect of promotion and prices under the trapezoidal-type demand.

3. It is assumed the rate of investment in promotion is not uniform through different stages of the product lifespan. The retailer can adjust investment rate and price in each stage. Moreover, it is natural that the impact of the promotion diminishes as time progress. The following differential equation is for time evolution of promotional investment and its impact is considered as follows:

$$\dot{S}(t) = \begin{cases} s_1(t) - \delta_1 S(t) & \text{if } 0 \leq t < \mu_1 \\ s_2(t) - \delta_2 S(t) & \text{if } \mu_1 \leq t \leq \mu_2 \\ s_3(t) - \delta_3 S(t) & \text{if } \mu_2 \leq t, \end{cases} \quad (2)$$

where δ_i; $i = 1, 2, 3$, represents the decay rate of the promotional effect in three consecutive periods, where $\delta_1 \leq \delta_2 \leq \delta_3$. Further, the retailer's investment in promotion involves quadratic instantaneous cost functions at each stage measured by the following form:

$$C(s_i(t)) = \frac{\alpha_i s_i^2(t)}{2} \quad (3)$$

4. The rate of deterioration is

$$\theta \equiv \theta(u) = \theta_0 + (\theta_1 - \theta_0) e^{-\xi u} \quad (4)$$

under the influence of preservation technology investment u. The existing literature gave the concept that the deterioration rate tends to zero if the retailer puts immense investment. It is commonly observed that the product like milk, vegetable, cake deteriorates naturally and it is impossible to eliminate deterioration rate fully. Therefore, a threshold value θ_0 is considered for representing the permissible rate of deterioration under the influence of feasible investment in preservation technology. Note that, for $\theta_0 = 0$, it is similar to existing literature. It is another important note that $\theta \to \theta_1$ if $u \to 0$, i.e. the rate of deterioration remains unchanged if the retailer does not invest in preservation technology. Similarly, $\theta \to \theta_0$ if $u \to \infty$, i.e. the deterioration rate reaches its threshold value under the large investment of the retailer.

5. The replenishment rate is instantaneous. Due to the deteriorating nature of items, inventory level becomes zero at the end of a replenishment cycle. Thus, the shortages are not allowed. Deteriorating items should not be repaired or reworked.

4. Model formulation

Based on the assumptions, the mathematical formulation of the proposed model is developed in this section. The demand and deterioration are the causes of depletion of inventory and the governing differential equations represent the inventory level as follows:

$$\dot{I}(t) = \begin{cases} -a_1 - b_1 t + \beta_1 p_1 - \gamma_1 S(t) - \theta I(t) & \text{if } 0 \leq t \leq \mu_1 \\ -D_0 + \beta_2 p_2 - \gamma_2 S(t) - \theta I(t) & \text{if } \mu_1 \leq t \leq \mu_2 \\ -a_2 e^{-b_2 t} + \beta_2 p_2 - \gamma_3 S(t) - \theta I(t) & \text{if } \mu \leq t \leq T, \end{cases} \quad (5)$$

where $I(0) = I_0$ and $I(T) = 0$. Using the rate of change of inventory during several time intervals, it is easy to calculate the revenues and other system costs, which are needed to decide the dynamic investment in promotion. By that, the management of the industry would decide how much investment they can allow for the promotion.

a. Sells revenue (SR) in the cycle [0, T]

Within any supply chain, the customer is the source of revenues for the retailer. Therefore, the revenue plays very important role for any decision making process. The basic revenue can be found by the multiplication of demand and selling price. However, nowadays, due to complexity of the business market, it is not possible to fixed demand always. It may be possible different in several cases. The demand and selling price are different in different interval. Thus, the sells revenue (SR) in the cycle [0, T] can be calculated as follows:

$$SR=\frac{1}{T}\left[p_1\int_0^{\mu_1}\left[a_1+b_1t-\beta_1p_1+\gamma_1S(t)\right]dt+p_2\int_{\mu_1}^{\mu_2}[D_0-\beta_2p_2+\gamma_2S(t)]dt\right.$$
$$\left.+p_3\int_{\mu_2}^{T}\left[a_2e^{-b_2t}-\beta_3p_3+\gamma_3S(t)\right]dt\right]$$

b. Ordering cost (OC)

Generally the ordering cost is used onetime for ordering products by the retailer or many times but is considered as constant. For this model, the fixed ordering cost is as follows:

$$OC=\frac{K}{T}.$$

c. Inventory holding cost(HC) in the cycle [0, T]

The average inventory is different in different time interval. Thus this can be obtained as

$$HC=\frac{h}{T}\left[\int_0^{\mu_1}I(t)dt+\int_{\mu_1}^{\mu_2}I(t)dt+\int_{\mu_2}^{T}I(t)dt\right].$$

d. Purchasing cost (PC)

To obtain more profit it is most important that the purchasing cost should be low. purchase basic items, the industry has to pay the purchasing cost, by which they can hold the inventory for future. The cost can be found as

$$PC=\frac{cI_0}{T}.$$

e. Disposal cost (DC) in the cycle [0, T]

Due to deterioration, the managers have to invest some funds for disposing items. The management never prefers to use this fund, but it is very difficult to change it to zero level. This is the reason that the preservation technology costs are used to reduce this disposal cost. However, the disposal cost can be calculated as

$$DC=\frac{\theta c_d}{T}\left[\int_0^{\mu_1}I(t)dt+\int_{\mu_1}^{\mu_2}I(t)dt+\int_{\mu_2}^{T}I(t)dt\right].$$

f. Investment in preservation technology (IPT)

To reduce the disposal cost of the whole system, management always invests on preservation technology. Thus, the cost can be calculated as

$$IPT=\frac{u}{T}.$$

g. Investment in promotion (IP)

To motivate more sell of products, the common strategies by the management system is investment more in promotion, but it increases the total system cost. The optimum way is to use the dynamic investment in promotion purpose. The investment can be calculated as follows:

$$IP=\frac{1}{T}\left[\int_0^{\mu_1}\frac{\alpha_1s_1^2(t)}{2}dt+\int_{\mu_1}^{\mu_2}\frac{\alpha_2s_2^2(t)}{2}dt+\int_{\mu_2}^{T}\frac{\alpha_3s_3^2(t)}{2}dt\right].$$

Hence, the total profit per unit time, Π is given by

$$\Pi=\frac{1}{T}\left[p_1\left((a_1-\beta_1p_1)\mu_1+b_1\frac{\mu_1^2}{2}\right)+p_3\left(\frac{a_2(e^{-b_2\mu_2}-e^{-b_2T})}{b_2}-\beta_3p_3(T-\mu_2)\right)\right.$$
$$+p_2(\mu_2-\mu_1)(D_0-\beta_2p_2)-(K+cI_0+u)$$
$$+\int_0^{\mu_1}\left[\gamma_1p_1S(t)-(h+\theta c_d)I(t)-\frac{\alpha_1s_1^2(t)}{2}\right]dt$$
$$\left.+\int_{\mu_1}^{\mu_2}\left[\gamma_2p_2S(t)-(h+\theta c_d)I(t)-\frac{\alpha_2s_2^2(t)}{2}\right]dt+\int_{\mu_2}^{T}\left[\gamma_3p_3S(t)-(h+\theta c_d)I(t)-\frac{\alpha_3s_3^2(t)}{2}\right]dt\right] \tag{6}$$

Therefore, one can obtain the solution of the above optimization problem to get optimal dynamic investment rates, selling prices, replenishment time, and preservation technology investment which maximize the retailer's total profit. The simplified form of the discussed problem is

$$\begin{aligned}
&\text{Max } \Pi(p_1,p_2,p_3,u,s_1(t),s_2(t),s_3(t),I_0,T)\\
&\text{subject to}\\
&\dot{S}(t)=\begin{cases} s_1(t)-\delta_1S(t) & \text{if } 0\le t<\mu_1\\ s_2(t)-\delta_2S(t) & \text{if } \mu_1\le t\le\mu_2\\ s_3(t)-\delta_3S(t) & \text{if } \mu_2\le t\end{cases}\\
&\text{and}\\
&\dot{I}(t)=\begin{cases} -a_1-b_1t+\beta_1p_1-\gamma_1S(t)-\theta I(t) & \text{if } 0\le t\le\mu_1\\ -D_0+\beta_2p_2-\gamma_2S(t)-\theta I(t) & \text{if } \mu_1\le t\le\mu_2\\ -a_2e^{-b_2t}+\beta_2p_2-\gamma_3S(t)-\theta I(t) & \text{if } \mu_2\le t\le T\end{cases}\\
&I(T)=0,\ S(0)=S_0,\ T\ge\mu_2, \text{ and } p_i>c\ ,
\end{aligned} \tag{p1}$$

where $s_i(t)$ represents the control variables, $I(t)$ and $S(t)$ represent the state variables, and p_1, p_2, p_3, u, and T represent static variables. Generally, the initial replenishment quantity I_0 is considered as a decision variable in the above optimization problem. This study considers the replenishment quantity I_0 as a dependent decision variable. Once the decision variables $p_1, p_2, p_3, u, s_1(t), s_2(t), s_3(t), T$ are obtained, the replenishment quantity I_0 can be determined. For analytical tractability, several authors assumed that purchase cost is zero under a dynamic environment (Xue et al. (2016)), this restriction is relaxed. Now, Pontryagin's maximum principle is employed to obtain control variables for the optimal dynamic investment rates. Then, MFPA is applied to find ultimate decisions.

5. Solution methodology

The optimization has been done through two important theorems in the following section.

5.1. Dynamic investment strategy

This subsection assumes that p_1, p_2, p_3, u, and T are given and reformulate the optimization problem (p1) as given below:

$$\Pi_1 = \int_0^{\mu_1}\left[\gamma_1 p_1 S(t) - (h+\theta c_d)I(t) - \frac{\alpha_1 s_1^2(t)}{2}\right]dt + \int_{\mu_1}^{\mu_2}\left[\gamma_2 p_2 S(t) - (h+\theta c_d)I(t) - \frac{\alpha_2 s_2^2(t)}{2}\right]dt + \int_{\mu_2}^{T}\left[\gamma_3 p_3 S(t) - (h+\theta c_d)I(t) - \frac{\alpha_3 s_3^2(t)}{2}\right]dt \tag{p2}$$

Subject to the same set of constraints. The following Theorem 1 gives the nature of the investment.

Theorem 1. *The Hamiltonian function of the dynamic problem (p2) is concave with respect to the dynamical investment $s_i(t)$; whereas the dynamical investments depend on the adjoint variables of the Hamiltonian function.*

Proof. *It is assumed that all the functional forms in the problem (p2) are non-negative, continuous, and differentiable on $[0, T]$. The Hamiltonian function H for the above optimization problem (p2) is formulated as follows*

$$H = \begin{cases} \gamma_1 p_1 S(t) - (h+\theta c_d)I(t) - \dfrac{\alpha_1 s_1^2(t)}{2} + \lambda_1[s_1(t) - \delta_1 S(t)] & \text{if } 0 \le t \le \mu_1 \\ \quad + \lambda_2[-a_1 - b_1 t + \beta_1 p_1 - \gamma_1 S(t) - \theta I(t)] & \\ \gamma_2 p_2 S(t) - (h+\theta c_d)I(t) - \dfrac{\alpha_2 s_2^2(t)}{2} + \lambda_1[s_2(t) - \delta_2 S(t)] & \text{if } \mu_1 \le t \le \mu_2 \\ \quad + \lambda_2[-D_0 + \beta_2 p_2 - \gamma_2 S(t) - \theta I(t)] & \\ \gamma_3 p_3 S(t) - (h+\theta c_d)I(t) - \dfrac{\alpha_3 s_3^2(t)}{2} + \lambda_1[s_3(t) - \delta_3 S(t)] & \text{if } \mu_2 \le t \le T \\ \quad + \lambda_2\left[-a e^{-bt} + \beta_2 p_2 - \gamma_3 S(t) - \theta I(t)\right], & \end{cases} \tag{7}$$

where λ_1 and λ_2 are adjoint variables associated with states equations $\dot{I}(t)$ and $\dot{S}(t)$, respectively. It can be found from Eq. (7) *that each component of the Hamiltonian function is composed of two parts: the first part is the integrant of objective functional, and the second part consists the right hand side of the state equations which denotes the indirect contribution to the objective functional from the value of the changes $S(t)$ and $I(t)$. Note that, the initial condition $I(0)$ and the terminal conditions of $S(T)$ remain free, which introduce the following transversality conditions as $\lambda_1(T) = 0$ and $\lambda_2(0) = 0$ and the adjoint variables λ_1 and λ_2, must satisfy the following differential equations*

$$\dot{\lambda_1} = -\frac{\partial H}{\partial S} = \begin{cases} \delta_1\lambda_1 + \gamma_1\lambda_2 - p_1\gamma_1 & \text{if } 0 \le t \le \mu_1 \\ \delta_2\lambda_1 + \gamma_2\lambda_2 - p_2\gamma_2 & \text{if } \mu_1 \le t \le \mu_2 \\ \delta_3\lambda_1 + \gamma_3\lambda_2 - p_3\gamma_3 & \text{if } \mu_1 \le t \le \mu_2 \end{cases} \tag{8}$$

$$\dot{\lambda_2} = -\frac{\partial H}{\partial I} = \begin{cases} (h+\theta c_d) + \lambda_2\theta & \text{if } 0 \le t \le \mu_1 \\ (h+\theta c_d) + \lambda_2\theta & \text{if } \mu_1 \le t \le \mu_2 \\ (h+\theta c_d) + \lambda_2\theta & \text{if } \mu_2 \le t \le T. \end{cases} \tag{9}$$

Solving the differential Eq. (9), *one yields*

$$\lambda_2(t) = \frac{h+\theta c_d}{\theta}(e^{\theta t} - 1), \; 0 \le t \le T. \tag{10}$$

Substituting Eq. (10) *into* Eq. (8) *and solving, one can obtain*

$$\lambda_1(t) = \begin{cases} \gamma_1\left(p_1 + \dfrac{h+\theta c_d}{\theta}\right)\dfrac{1 - e^{-\delta_1(\mu_1 - t)}}{\delta_1} + \dfrac{\gamma_1(h+\theta c_d)e^{\theta t}}{\theta(\theta - \delta_1)}(1 - e^{(\theta-\delta_1)(\mu_1 - t)}) & \text{, if } 0 \le t < \mu_1 \\ \quad + \lambda_1(\mu_1)e^{-\delta_1(\mu_1 - t)} & \\ \gamma_2\left(p_2 + \dfrac{h+\theta c_d}{\theta}\right)\dfrac{1 - e^{-\delta_2(\mu_2 - t)}}{\delta_2} + \dfrac{\gamma_2(h+\theta c_d)e^{\theta t}}{\theta(\theta - \delta_2)}(1 - e^{(\theta-\delta_2)(\mu_2 - t)}), & \text{if } \mu_1 \le t < \mu_2 \\ \quad + \lambda_1(\mu_2)e^{-\delta_2(\mu_2 - t)} & \\ \gamma_3\left(p_3 + \dfrac{h+\theta c_d}{\theta}\right)\dfrac{1 - e^{-\delta_3(T - t)}}{\delta_3} + \dfrac{\gamma_3(h+\theta c_d)e^{\theta t}}{\theta(\theta - \delta_3)}\left(1 - e^{(\theta-\delta_3)(T - t)}\right), & \text{if } \mu_2 \le t < T. \end{cases} \tag{11}$$

It is to be noted that $\frac{\partial\lambda_1(t)}{\partial t} = -\gamma_3(p_3 + c_d + h/\theta)e^{-\delta_3(T-t)} + \frac{\gamma_3(h+\theta c_d)}{\theta(\theta-\delta_3)}(e^{\theta t}\theta - \delta_3 e^{(\theta-\delta_3)T+\delta_3 t}) < 0, \forall t \in [\mu_2, T]$, *i.e.* $\lambda_1(t)$ *decreases in final stage of the product lifespan. To maximize the Hamiltonian H with respect to* $s_i(t)$, *the first order condition is obtained by solving* $\frac{\partial H}{\partial s_i(t)} = 0$. *On simplification,*

$$s_1(t) = \frac{\lambda_1(t)}{2\alpha_1},\ 0 \le t \le \mu_1 \tag{12}$$

$$s_2(t) = \frac{\lambda_1(t)}{2\alpha_2},\ \mu_1 \le t \le \mu_2 \tag{13}$$

$$s_3(t) = \frac{\lambda_1(t)}{2\alpha_3},\ \mu_2 \le t \le T. \tag{14}$$

decay rates are identical at each stage.

Proof. *At the beginning of the declining stage,* $\lambda_1(\mu_2) = \frac{\gamma_3}{\delta_3}(p_3 + c_d + h/\theta)(1 - e^{-\delta_3(T+t)}) + \frac{\gamma_3(h+\theta c_d)}{\theta(\theta-\delta_3)}(e^{\theta t} - e^{(th-\delta_3)T+\delta_3 t})$, *thus the solution exists if*

$$p_3 > \frac{(h + c_d\theta)(e^{\delta_3\mu_2}(\theta - \delta_3(1 - e^{T\theta})) - e^{\delta_3 T}(\theta - \delta_3(1 - e^{\theta\mu_2})))}{(e^{\delta_3 T} - e^{\delta_3\mu_2})\theta(\theta - \delta_3)}(=\Gamma_1, say). \tag{15}$$

Substituting $p_1 = p_2 = p_3$, $\gamma_1 = \gamma_2 = \gamma_3$, *and* $\delta_1 = \delta_2 = \delta_3$, *one can obtain*

$$\frac{\partial\lambda_1(t)}{\partial t_i} = -\gamma_3(p_3 + c_d + h/\theta)e^{-\delta_3(T-t_i)} + \frac{\gamma_3(h+\theta c_d)}{\theta(\theta-\delta_3)}\left(e^{\theta t_i}\theta - \delta_3 e^{(\theta-\delta_3)T+\delta_3 t_i}\right) < 0,$$

where $t_1 \in [0, \mu_1]$, $t_2 \in [\mu_1, \mu_2]$, *and* $t_3 \in [\mu_2, T]$. *Therefore, the investment rate of the retailer decreases continuously, if the retailer considers always uniform price, promotion sensitivity, and decay rates are identical at each stage.*

Substituting Eq. (12) ~ (14) *in* Eq. (2), *one can find that the optimal path representing the promotional level as follows*:

$$S(t) = \begin{cases} \dfrac{\gamma_1(p_1+\omega_1)\left(\dfrac{1-e^{-\delta_1 t}}{\delta_1} - e^{-\delta_1\mu_1}\omega_2\right)}{2\alpha_1\delta_1} + \dfrac{\gamma_1\omega_1\left(\dfrac{e^{\theta t} - e^{-\delta_1 t}}{\theta+\delta_1} - e^{(\theta-\delta_1)\mu_1}\omega_2\right)}{2\alpha_1(\theta-\delta_1)} + \dfrac{\lambda_1(\mu_1)e^{-\delta_1\mu_1}\omega_2}{2\alpha_1} + S_0 e^{-\delta_1 t} \\ \text{if } 0 \le t \le \mu \\ \dfrac{\gamma_2(p_2+\omega_1)\left(\dfrac{1-e^{\delta_2(\mu_1-t)}}{\delta_2} - e^{-\delta_2\mu_2}\omega_9\right)}{2\alpha_2\delta_2} + \dfrac{\gamma_2\omega_1\left(\dfrac{e^{\theta t} - e^{(\theta+\delta_2)\mu_1}e^{-\delta_2 t}}{\theta+\delta_2} - e^{(\theta-\delta_2)\mu_2}\omega_9\right)}{2\alpha_2(\theta-\delta_2)} + \dfrac{\lambda_1(\mu_2)e^{-\delta_2\mu_2}\omega_9}{2\alpha_2} \\ +S(\mu_1)e^{-\delta_2(t-\mu_1)} \text{if } \mu_1 \le t \le \mu_2 \\ \dfrac{\gamma_3\left(p_3+\omega_1\right)\left(\dfrac{1-e^{\delta_3(\mu_2-t)}}{\delta_3} - e^{-\delta_3 T}\omega_{10}\right)}{2\alpha_3\delta_3} + \dfrac{\gamma_3\omega_1\left(\dfrac{e^{\theta t} - e^{(\theta+\delta_3)\mu_2}e^{-\delta_3 t}}{\theta+\delta_3} - e^{(\theta-\delta_3)T}\omega_{10}\right)}{2\alpha_3(\theta-\delta_3)} + S\left(\mu_2\right)e^{-\delta_3(t-\mu_2)} \\ \text{if } \mu_2 \le t < T. \end{cases} \tag{16}$$

Moreover, $\frac{\partial^2 H}{\partial s_i(t)^2} = -2\alpha_i < 0$ *and* $\frac{\partial^2 H}{\partial s_i(t)\partial s_j(t)} = 0\ \forall\ i \ne j$, *therefore H is concave with respect to* $s_i(t)$ *and the optimal path of* $s_i(t)$ *depends on* $\lambda_1(t)$.

Finally, Theorem 2 *can be proved for the optimum strategy for the retailer.*

Theorem 2. *The dynamical investment rate of the retailer decreases continuously if always the retailer's uniform-price, production-sensitive, and*

[*See all values in* Appendix A]. *Finally, substituting* Eq. (16) *in* Eq. (5), *the optimal path representing the inventory level in the entire replenishment cycle is obtained by the following equations.*

$$I(t)=\begin{cases} I\left(0\right)e^{-\theta t}-\dfrac{(a_1-\beta_1 p_1)(1-e^{-\theta t})}{\theta}-b_1\left(\dfrac{t}{\theta}-\dfrac{1-e^{-\theta t}}{\theta^2}\right)+\gamma_1 S\left(0\right)\omega_3 \\ -\dfrac{\gamma_1}{2\alpha_1}\left[\dfrac{\lambda_1(\mu_1)e^{-\delta_1\mu}\omega_{11}}{2\delta_1}+\dfrac{\gamma_1}{\delta_1}(p_1+\omega_1)\left(\dfrac{1}{\delta_1}\left(\dfrac{1-e^{-\theta t}}{\theta}-\omega_3\right)-\dfrac{e^{-\delta_1\mu_1}\omega_{11}}{2\delta_1}\right)\right. \\ \left.+\dfrac{\gamma_1\omega_1}{(\theta-\delta_1)}\left(\dfrac{1}{\theta+\delta_1}\left(\dfrac{e^{\theta t}-e^{-\theta t}}{2\theta}-\omega_3\right)-\dfrac{e^{(\theta-\delta_1)\mu_1}\omega_{11}}{2\delta_1}\right)\right] \text{ if } 0\le t\le \mu_1 \\ I\left(\mu_1\right)e^{-\theta(t-\mu_1)}-\dfrac{(D_0-\beta_2 p_2)(1-e^{-\theta(t-\mu_1)})}{\theta}-\gamma_2 S\left(\mu_1\right)e^{\delta_2\mu_1}\omega_5 \\ -\dfrac{\gamma_2}{2\alpha_2}\left[\dfrac{\gamma_2}{\delta_2}(p_2+\omega_1)\left(\dfrac{1}{\delta_2}\left(\dfrac{1-e^{\theta(\mu_1-t)}}{\theta}-e^{\delta_2\mu_1}\omega_5\right)-\dfrac{e^{-\delta_2\mu_2}\omega_{12}}{2\delta_2}\right)\right. \\ \left.+\dfrac{\gamma_2\omega_1}{(\theta-\delta_2)}\left(\dfrac{e^{\theta t}-e^{2\theta\mu_1}e^{-\theta t}}{2\theta(\theta+\delta_2)}-\dfrac{e^{(\theta+\delta_2)\mu_1}\omega_5}{\theta+\delta_2}-\dfrac{e^{(\theta-\delta_2)\mu_2}\omega_{12}}{2\delta_2}\right)+\dfrac{\lambda_1\mu_2 e^{-\delta_2\mu_2}\omega_{12}}{2\delta_2}\right] \text{if } \ \mu_1\le t\le\mu_2 \\ I\left(\mu_2\right)e^{-\theta(t-\mu_2)}+\dfrac{\beta_3 p_3(1-e^{-\theta(t-\mu_2)})}{\theta}-\dfrac{a_3\left(e^{-b_3 t}-e^{(\theta-b_3)\mu_2}e^{-\theta t}\right)}{\theta-b_3}-\gamma_3 S\left(\mu_2\right)e^{\delta_3\mu_2}\omega_7 \\ -\dfrac{\gamma_3}{2\alpha_3}\left[\dfrac{\gamma_3(p_3+\omega_1)}{\delta_3}\left(\dfrac{1-e^{\theta(\mu_2-t)}}{\theta\delta_3}-\dfrac{e^{\delta_3\mu_2}\omega_7}{\delta_3}-\dfrac{e^{-\delta_3 T}\omega_{13}}{2\delta_3}\right)\right. \\ \left.+\dfrac{\gamma_3\omega_1}{(\theta-\delta_3)}\left(\dfrac{e^{\theta t}-e^{2\theta\mu_2}e^{-\theta t}}{2\theta(\theta+\delta_3)}-\dfrac{e^{(\theta+\delta_3)\mu_2}\omega_7}{(\theta+\delta_3)}-\dfrac{e^{(\theta-\delta_3)T}\omega_{13}}{2\delta_3}\right)+\dfrac{\lambda_1(\mu_3)e^{-\delta_3\mu_3}\omega_{13}}{2\delta_3}\right] \text{ if } \mu_2\le t\le T. \end{cases} \tag{17}$$

Using Eq. (17), *the replenishment quantity I_0 can be obtain by using the condition $I(T)=0$. Therefore, one needs to find the following optimization problem to get optimal decision*

$$\text{Max } \Pi(p_1,p_2,p_3,u,s_1(t),s_2(t),s_3(t),I_0,T) \tag{p3}$$

subject to

$$\begin{cases} a_1-\beta_1 p_1+\gamma_1 S(0)\ge 0 \\ D_0-\beta_2 p_2+\gamma_2 S(\mu_1)\ge 0 \\ a_2 e^{-b_2\mu_2}-\beta_2 p_2+\gamma_3 S(\mu_2)\ge 0 \\ I(T)=0, S(0)=S_0, p_1>c, p_2>c, p_3>\text{Max}\{c,\Gamma_1\}, T\ge\mu_2, \end{cases} . \tag{c1}$$

where the constraints set $c(1)$ ensure non-negative demand throughout the product lifespan. Due to the complexity and nonlinear characteristics of both the objective function and constraints, the analytical solution is difficult to find. Therefore, the MFPA Algorithm for solving the above optimization problem (Nabil (2016)) *is given in the following section.*

5.2. Modified flower pollination algorithm (MFPA)

Flower pollination is a biological nature-based process, where pollen is needed and transferred from one to another flower for the sake of fertilization. The process of flower pollination happens in two ways, cross-pollination or self-pollination. The cross-pollination occurs if pollens are transferred among different plant species. The self-pollination may occur, if pollens are transferred within the same flower or from different flower of same plant species. The Algorithm is based on four rules inspired from pollination process of nature's flowering plants Yang (2012).

Rule 1 The biotic is treated as global pollination and that of cross-pollination, where the pollinators follow the Le′vy distribution (See for reference, Mantegna (1994)).
Rule 2 The abiotic can be recognized as local pollination and that of self-pollination.
Rule 3 Insects act like pollinators as they can develop the constancy of flower. This flower constancy and reproduction probability are equivalent. Similarity of two flowers is proportionate to the reproduction probability.

Rule 4 Use of the switch probability can solve the switching of problem between local and global pollination, with slight biased toward local pollination.

Global pollination is usually biotic, that means pollination is done by global pollinators like insects and birds. Over long distance, their movements can be mimicked by using the L e′vy Flight. Rule 1 and Rule 3 can be represented mathematically as

$$x_i(t+1)=x_i(t)+\gamma_1 L(g^*-x_i(t)).$$

Here $x_i(t)$ is the i*th* pollen or solution vector and g^* is the current best solution at iteration t, where γ_1 is a scaling factor or shape parameter introduced for controlling the step size. The step size L is used to mimic biotic pollination that follows Le′vy distribution. In this study, Mantegna Algorithm (Mantegna, 1994) is used to generate L. Similarly, the local pollination can be represented by Rule 2 and Rule 3 and its mathematical form is given as follows:

$$x_i(t+1)=x_i(t)+\delta\gamma_2\left(x_j(t)-x_k(t)\right).$$

Here $x_j(t)$ and $x_k(t)$ are pollen from different flowers of same plant species. This mimics flower constancy in a small neighborhood. Mathematically, this becomes a local random walk, if $x_j(t)$ and $x_k(t)$ comes from same flower species, where δ follows a uniform distribution defined over the interval $(0,1)$. Although, the global and local pollination can happen in any ratio, but in reality, local pollination tends to happen more than the global pollination. To simulate this phenomenon, a switch probability p is always used to switching two types of pollination by using Rule 4. The MFPA Algorithm to solve the developed optimization problem is described below:

Algorithm. MFPA

In this study, the penalty method is employed to find the solution of a nonlinear optimization problem with the inequality constraints. For the optimization problem

$$\text{Min } f(x),\ x=(x_1,x_2,\ldots,x_n)^T\in R^n$$

$$\text{subject to } g_i(x)\le 0,\ (i=1,2,\ldots,N).$$

The idea is to define a penalty function such that an unconstrained

Define the objective function $f(x)$, $\mathbf{x}=(x_1, x_2, x_3, x_4, x_5)$

Create an random initial population **pop** of size **n** and identify the best solution (g^*)

Identify $p \in [0,1]$ which is a switch probability between global and local pollination

While(gen < Max Generations Number)

  Define a uniformly distributed random number $c_1 \in (0,1)$

  If $c_1 > p$

    For each x_i in **pop**

      Draw a (5-dimensional) step vector **L** by using **Mantegna Algorithm**

        Compute $\sigma^2 = \left[\frac{\Gamma(1+\lambda)}{\lambda\Gamma(1+\lambda/2)} \frac{sin(\pi\lambda/2)}{2^{(\lambda-1)/2}}\right]^{1/\lambda}$

        Generate $U \sim (0, \sigma^2)$ and $V \sim (0,1)$. Compute $L = \frac{U}{|V|^{1/\lambda}}$

      Global pollination via $x_i(t+1) = x_i(t) + \gamma_1 L(\mathbf{g}^* - x_i(t))$

    End for

  Else

    Select best **m** solutions from **pop** to form **Clonespop** population

    Clone solutions in **Clonespop** proportional to objective function

    For each solution in **Clonespop**

      Delta follows uniform distribution in the interval [0, 1]

      Choose j and k from **pop**

      perform local pollination via $x_i(t+1) = x_i(t) + \gamma_2\delta(x_j(t) + x_k(t))$

    End For

  End if

  Select best m solutions from **pop** and **Clonespop** to form **NewPop** population

  replace **pop** by **NewPop**

  Find the current best solution g*

    After successive r iterations, keep g* and replace **pop** by a newly generated one

    gen=gen+1

  End while

  print $\mathbf{g}^*$

Table 2
Input data of numerical example.

$a_1 = 30$	$a_2 = 100$	$b_1 = 5$	$b_2 = log2/8$	$D_0 = 50$
$\beta_1 = 0.5$	$\beta_2 = 0.5$	$\beta_3 = 0.5$	$\gamma_1 = 0.4$	$\gamma_2 = 0.4$
$\gamma_3 = 0.3$	$\delta_1 = 0.25$	$\delta_2 = 0.3$	$\delta_3 = 0.3$	$\mu_1 = 4$
$\mu_2 = 8$	$\theta_1 = 0.4$	$\theta_0 = 0.2$	$\xi = 0.05$	$S_0 = 50$
$h = 0.5$($/unit/ unit time)	$c = 20$($/unit)	$c_d = 5$($/unit)	$\alpha_1 = \alpha_2 = \alpha_3 = 1$	$K = 500$($/order)

optimization problem can be constructed from a constrained optimization problem. A pseudo-objective function yielded using the penalty method is as follows.

$$\text{Min } F(x) = f(x) + \sum_{i=1}^{N} \psi_i \text{Max}(0, g_i)^2(x),$$

where ψ_i is a non-negative large number corresponding to the i-th constraint, known as the penalty number (Yeniay (2005)).

6. Numerical experiment

This section explores characteristics of the proposed model and the solution procedure with two numerical examples. The sensitivity analysis performs a crucial role to find out the changes of key parameters over the optimal solution and convergence rate of the Algorithm. The following parametric values are considered to illustrate the model as given in Table 2.

Some of them are chosen based on previous studies (Tsao and Sheen, 2008). The parameters related to the Algorithm based on some preliminary parametric studies are as follows: $n = 30$, $m = 10$, $p = 0.7$, $\gamma_1 = 1$, $\gamma_2 = 1.5$, $\lambda = 1.5$, and cloning array $= [7, 6, 5, 4, 3, 2, 1, 1, 1, 1]$.

By applying hybridized flower pollination Algorithm, the optimal decision of the proposed model is obtained as $p_1 = \$81.11$/unit, $p_2 = \$96.32$/unit, $p_3 = \$81.78$/unit, $T = 8.85$ time unit, $u = \$87.21$/unit, $I_0 = 448.52$ units, and $\Pi = \$1348.82$/cycle. The optimal investment rate in promotion and the corresponding promotional level are governed by following relations:

$$\begin{aligned} s_1(t) &= 70.8628 - 31.4823e^{0.202554t} + 14.2673e^{0.25t} && \text{if } 0 \le t \le \mu_1 \\ s_2(t) &= 69.1923 - 15.3285e^{0.202554t} + 1.24483e^{0.3t} && \text{if } \mu_1 \le t \le \mu_2 \\ s_3(t) &= 44.6242 - 11.4964e^{0.202554x} + 1.71617e^{0.3t} && \text{if } \mu_2 \le t \le T \end{aligned}$$

$$S(t) = \begin{cases} 283.451 - 192.42e^{-0.25t} - 69.5652e^{0.202554t} + 28.5342e^{0.25t} & \text{if } 0 \le t \le \mu_1 \\ 230.641 - 116.646e^{-0.3t} - 30.5011e^{0.202554t} + 2.07465e^{0.3t} & \text{if } \mu_1 \le t \le \mu_2 \\ 148.747 + 265.706e^{-0.3t} - 22.8758e^{0.202554t} + 2.86022e^{0.3t} & \text{if } \mu_2 \le t \le T. \end{cases}$$

The graphical representation of the investment rate in promotion

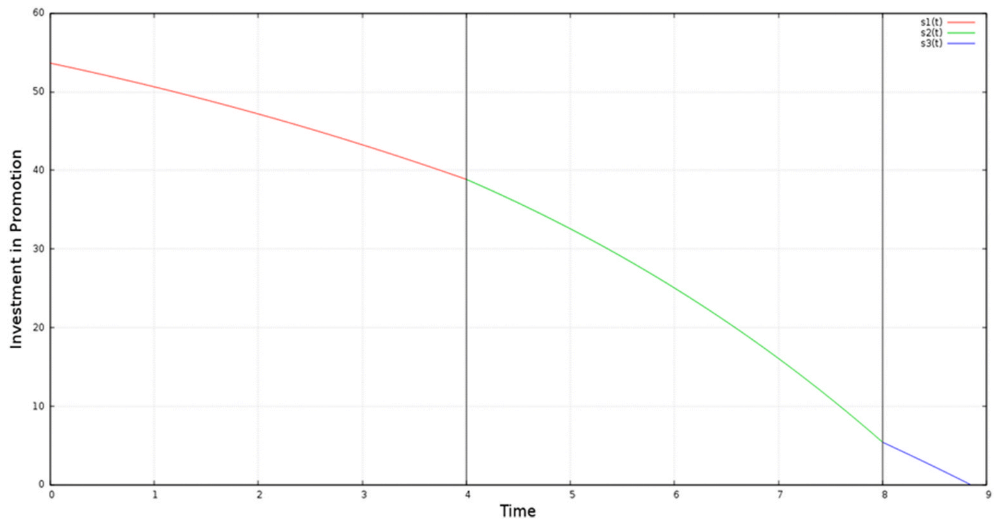

Fig. 1. Investment rate at different stages of the product lifespan.

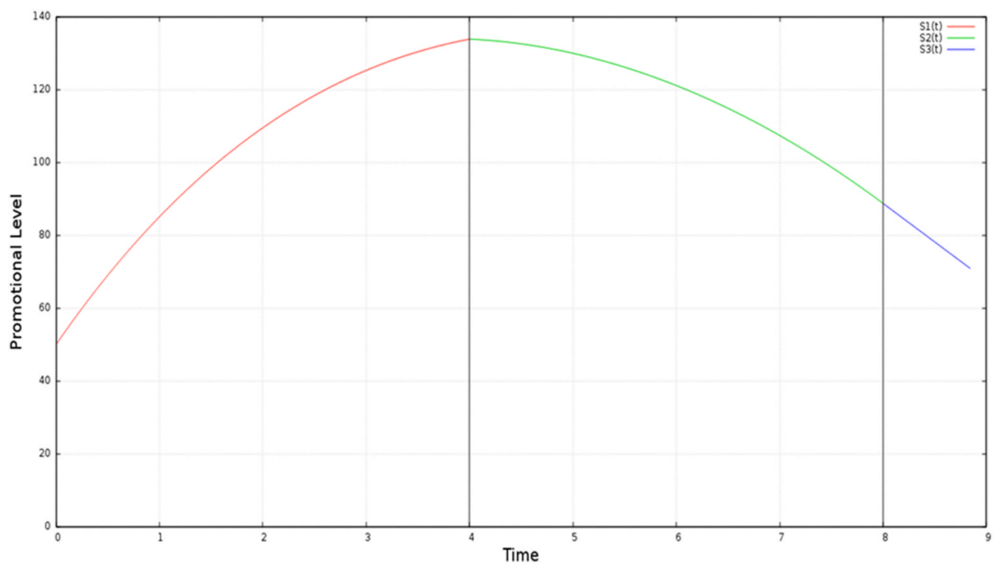

Fig. 2. Promotion level at different stages of the product lifespan.

and the corresponding promotional level are depicted in Figs. 1 and 2.

From Fig. 1, it is observed that the investment rate in promotion decreases gradually until zero at the end of the product lifespan and the rate of decrements are different. Whenever the replenishment cycle starts, the on-hand inventory level is relatively high, price is not be as sensitive as in later stages, and the retailer wants to expand market, therefore the rate of decrement of investment is less compared to the final stage. On the contrary, at the final stage, the competition is fierce and the retailer needs to set lower price to liquidate inventory and retain the market share, the rate of decrement of investment in promotion is highest compared to previous stages. From Fig. 2, it is observed that the impact of promotion in demand is increases initially due to high investment, but it decreases gradually at the end of the cycle. It happens due to less investment and decay effect. Note that, if $\theta_0 = 0.35$ the optimal solution is obtained as $p_1 = \$69.04, p_2 = \$75.62, p_3 = \$70.76$, $T = 8.18$ time unit, $u = 54.86, I_0 = 308.28$ units, $INV = \$3153.85$, and $\Pi = \$1,216.06$. Therefore, the investment in preservation technology is not profitable for the retailer in such scenario.

Fig. 3 indicates only the objective function values (Π) with respect to the number of iteration obtained for the different values of switch probability **p**. Similarly, theFig. 4 provides only the objective function values (Π) with respect to number of iteration obtained for the different values of λ. Although the convergence rate depends on **p** and λ, but in every scenarios, one may obtain the global optimal solution.

To verify whether the price discrimination is profitable and justify the necessity of determination of optimal replenishment time, the following numerical examples are studied as in Table 3.

Example 1 represents the scenario, where the retailer can consider several prices in different stages, in Examples 2~4 the retailer sets same price for any two periods, in Example 5 the retailer sets uniform price throughout the product lifespan, and in Example 6~7 the effect of price discrimination is analyzed by considering the finite time horizon $T = 12$. From the Table 3, one may observe that the price discrimination throughout the product lifespan is always profitable for the retailer. Besides, it gives the opportunity to investment in both preservation technology and promotion. It is observed that under the promotion-price- and trapezoidal-type demand, the price of products in maturity stage is greater to compare to the initial and decline stage and the market penetration pricing policy is suitable for the retailer in all above scenarios to gain the maximum profit. The numerical illustrations justify that the retailer should determine the optimal replenishment time to earn maximum profit instead of using predetermined one.

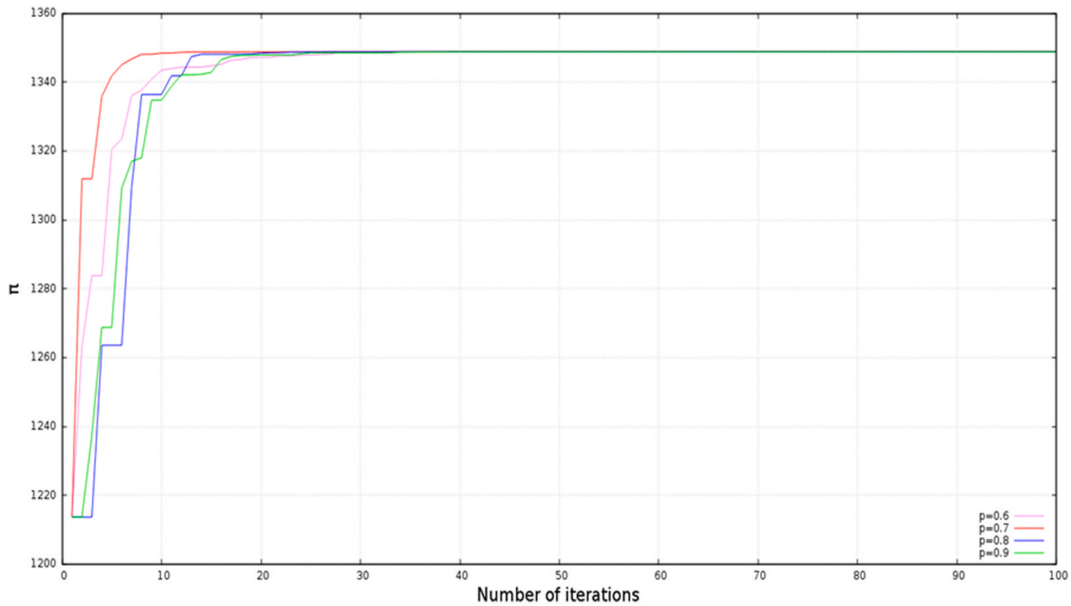

Fig. 3. Convergence indicator in improving solution for different values of p.

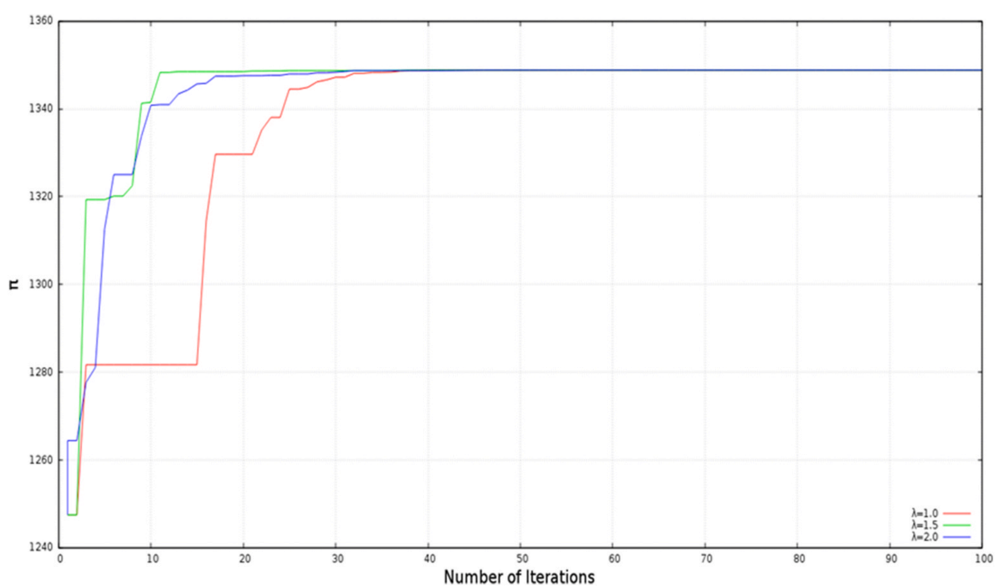

Fig. 4. Convergence indicator in improving solution for different values of λ

Table 3
Impact of price discrimination over the finite or infinite planning horizon.

Serial number	p_1 ($/unit)	p_2 ($/unit)	p_3 ($/unit)	T (cycle)	u ($/cycle)	I_0 (units)	IP ($/cycle)	Π ($/cycle)
1	81.11	96.32	81.78	8.85	87.21	448.52	11,552.82	1348.82
2	86.32	86.32	86.32	8.86	83.65	425.85	10,391.42	1325.48
3	87.18	87.18	80.03	8.73	84.32	431.25	10,598.35	1327.25
4	81.24	95.31	81.24	8.85	87.23	445.62	11,456.43	1345.80
5	80.42	93.01	93.01	8.63	85.40	431.47	10,870.93	1341.96
6	83.76	103.29	83.01	12	102.32	578.04	14,587.12	1201.24
7	87.65	87.65	87.65	12	98.02	542.54	12,710.31	1164.98

7. Sensitivity analysis

The sensitivity analysis is carried out on the optimal policy with respect to the price sensitive parameters β_1, β_2, and β_3. The results are given in Table 4. When the value of one parameter varies, all others remain unchanged.

The price sensitivity of demand and the total profit are closely interrelated. It is intuitive to obtained that, as price sensitivity of

Table 4
Sensitivity analysis with respect to the price sensitive parameters.

	% change	p_1($/unit)	p_2($/unit)	p_3($/unit)	T(cycle)	u($/unit)	I_0units	Π($/cycle)
β_1	− 30%	128.69	104.18	83.38	8.00	97.70	488.26	2002.23
	− 10%	92.28	98.21	82.76	8.62	90.31	457.67	1495.59
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	72.48	94.80	81.03	9.03	84.70	441.09	1238.57
	+ 30%	60.02	92.53	79.96	9.31	80.66	429.48	1084.43
β_2	− 30%	99.68	191.79	103.81	8.92	111.03	690.99	2452.33
	− 10%	84.64	114.60	86.03	8.81	94.36	492.61	1563.53
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	78.60	83.38	78.79	8.90	80.09	418.03	1197.69
	+ 30%	75.12	66.06	74.80	9.02	63.72	376.83	1000.19
β_3	− 30%	86.16	109.87	122.60	10.86	102.68	615.38	1543.22
	− 10%	82.28	99.44	91.03	9.41	90.57	487.23	1379.48
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	78.60	93.86	74.70	8.34	85.15	417.61	1335.07
	+ 30%	75.12	92.31	60.12	8.00	84.29	397.90	1332.43

Table 5
Sensitivity analysis with respect to the promotion sensitive parameters.

	% change	p_1($/unit)	p_2($/unit)	p_3 ($/unit)	T(cycle)	u($/unit)	I_0units	Π($/cycle)
γ_1	− 30%	66.46	90.84	79.33	8.99	63.30	351.72	1180.98
	− 10%	75.50	94.18	80.81	8.90	80.31	411.15	1283.17
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	87.80	98.99	82.99	8.79	93.56	493.52	1428.68
	+ 30%	106.47	106.92	86.61	8.68	105.53	619.28	1656.72
γ_2	− 30%	72.49	72.81	73.08	8.47	54.45	310.51	1099.05
	− 10%	77.52	85.88	78.02	8.69	77.81	388.02	1240.12
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	86.23	111.96	87.22	9.05	96.78	537.62	1506.41
	+ 30%	110.00	188.40	111.90	9.79	119.35	964.29	2226.95
γ_3	− 30%	79.74	92.68	75.14	8.11	84.46	403.24	1332.75
	− 10%	80.52	94.76	79.15	8.58	85.88	429.97	1340.40
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	81.87	98.37	85.08	9.15	89.16	472.48	1361.44
	+ 30%	84.35	105.03	95.50	9.96	96.22	550.30	1407.44
S_0	− 30%	75.95	93.81	80.56	9.11	86.32	441.46	1185.36
	− 10%	79.40	95.50	81.37	8.93	86.92	446.33	1292.74
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	82.21	97.12	82.18	8.76	87.50	450.55	1406.52
	+ 30%	86.19	98.65	82.98	8.58	88.06	454.12	1526.88

products increases, profit decreases, and the investment opportunity of the retailer either reduces the deterioration or in promotion. The results in Table 4 justify the common intuition. But, the characteristics of the optimal solution changes significantly. While β_1 and β_2 increase, the replenishment cycle time of the retailer increases, but contrary happens for β_3. Therefore, the retailer should reduces the length of the replenishment cycle to gain the maximum profit, if the price sensitivity is high in the final stage. Notably, the optimal investment decisions and profit are influenced by the price sensitivity of products during maturity stage, and price sensitivity in the final stage is more influential then the initial stage for determining the optimal preservation technology investment. From Table 4, one can observe whether to apply market skimming or penetration or mix pricing strategy should be decided based on the price sensitivity at different stages. Therefore, the retailer should monitor the price sensitivity and accordingly decides the pricing strategy to obtain the optimal profit. The promotional investment usually used to make consumer less price-focused, and more concerned with product benefits. As consumer becomes more sensitive in promotion, the investment becomes more profitable. From Table 5, one may find that as γ_i, $i = 1, 2, 3$ increases, the investment in promotion also increases. Due to the higher demand, the retailer can charge more and it gives opportunity to invest not only in the promotion, but in the preservation technology. Implicitly, the profit of the retailer and the replenishment cycle time are largely influenced by the promotion sensitivity in the maturity state. Besides, as S_0 increases, the profit of the retailer increases. Therefore, the reputation of the retailer makes a large impact on the profitability. Therefore, the investment in promotion not only enhance the profit of the present cycle, but it creates the opportunity to earn the higher profit in future.

From Table 6, one may observe that as the decay rate increases the profit of the retailer. In other words, if the investment in promotion is not effective to induce consumers, then it may become catastrophe for the profitability of the retailer. Besides, the profit of the retailer is highly sensitive with the decay rate in the initial and growth stage compared to decline stage. If the decay rate is high, then the retailer should reduce the length of replenishment cycle as well as the price of the product to enhance the impact on promotion as much as possible. Moreover, the higher decay rate reduces the opportunity of invest in the preservation

Table 6
Sensitivity analysis with respect to the parameters related to decay rate.

	% change	p_1($/unit)	p_2($unit)	p_3 ($/unit)	T(cycle)	u($/unit)	I_0units	Π($/time)
δ_1	− 30%	96.63	115.68	90.12	8.99	100.92	588.32	1676.28
	− 10%	85.18	101.31	83.95	8.88	91.64	484.87	1434.80
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	77.73	92.25	79.99	8.82	82.80	418.76	1278.09
	+ 30%	72.45	86.02	77.29	8.80	73.59	372.95	1169.09
δ_2	− 30%	93.91	133.23	103.78	9.98	106.80	701.48	1696.01
	− 10%	83.92	104.33	86.67	9.16	92.78	503.89	1425.00
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	79.08	90.62	78.27	8.59	82.51	408.92	1293.97
	+ 30%	76.13	82.52	73.29	8.17	74.40	352.56	1214.27
δ_3	− 30%	81.66	97.79	83.48	9.12	88.62	467.21	1353.83
	− 10%	81.25	96.70	82.21	8.92	87.55	453.29	1350.16
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	80.99	96.02	81.43	8.79	86.94	444.73	1347.73
	+ 30%	80.81	95.52	80.87	8.69	86.52	438.54	1345.89

Table 7
Sensitivity analysis with respect to the parameters related to deterioration.

	% change	p_1	p_2	p_3	T	u	I_0	Π
		($/unit)	($/unit)	($/unit)	(cycle)	($/unit)	units	($/cycle)
θ_0	− 30%	84.86	102.22	84.21	8.90	91.09	453.66	1399.46
	− 10%	82.40	98.37	82.65	8.87	88.70	451.99	1366.11
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	79.77	94.14	80.81	8.81	85.45	442.45	1331.16
	+ 30%	76.88	89.33	78.55	8.68	80.83	419.92	1294.87
θ_1	− 30%	81.10	96.29	81.77	8.84	68.86	448.21	1350.89
	− 10%	81.10	96.31	81.77	8.84	82.74	448.44	1349.33
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	81.11	96.32	81.78	8.85	90.86	448.58	1348.41
	+ 30%	81.11	96.33	81.78	8.85	96.62	448.68	1347.76
ξ	− 30%	81.05	96.24	81.74	8.85	114.46	448.73	1344.78
	− 10%	81.09	96.30	81.77	8.85	94.57	448.58	1347.74
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	81.12	96.33	81.78	8.84	81.00	448.47	1349.73
	+ 30%	81.14	96.36	81.80	8.84	71.10	448.38	1351.16

technology.

Compare to the existing literature on preservation technology, in this study it is assumed that the retailer can reduce the rate of deterioration up to a threshold limit (see Table 7). For example, milk or vegetables deteriorates with respect to the time, and the retailer needs to apply different measures to preserve the product which requires adjustable investment. The proposed results implies that the threshold value, i.e., how much the deterioration can be reduced, is important for providing preservation technology investment. The results shows that the profit of the retailer is highly sensitive on that threshold value. The retailer needs less amount of investment in the preservation technology for the product with smaller amount of deterioration rate. Products's price are sensitive to the threshold value. It is intuitive to find as ξ increases profit of the retailer increases and investment in preservation technology decreases. The sensitivity analysis of the model justify the fact. Therefore, the retailer should observe the nature of the product, and accordingly the management takes the preservation technology investment decision.

From Table 8, one may obtain that the impact of holding cost and disposal cost are not enormous like the unit cost of the product on the profitability of the retailer. It is quite natural that as c, c_d, and h increases, the profit of the retailer decreases. The above results justify it. But the rate of the decrement is highly sensitive with c, and as c increases the rate of investment in promotion and preservation technology decreases. Moreover, the price at the maturity stage is highly sensitive to the unit cost of the product. The replenishment time decreases with respect to all cost parameters due to the higher deterioration rate. It is clear that as the promotion investment cost coefficients α_i $(i = 1, 2, 3)$ increases, profit, replenishment cycle time, and price of products decreases. If the promotion cost coefficients are relatively large, then the higher promotional cost insists the retailer to invest less in preservation technology. To capture more profit from expanding demand, the retailer becomes motivated to set low price. Subsequently, the inefficient promotion activity always harms the retailer's profitability. Notably, under the promotion-price-and trapezoidal-type demand, the effectiveness of the retailer in promotion during initial and the growth stage is a key factor to achieve the higher profit.

8. Managerial insights

A constant investment for a deteriorated products might not be beneficial on the perspective of the economical balance. It is already established that the deterioration rate is not constant always. The controllable deterioration is more beneficial for the industry. The manager can control the deterioration rate such that the lifespan of the product will increase rather than the normal lifespan. This gives profit to the industry economically that the product can serve more iifespan

Table 8
Sensitivity analysis with respect to the cost parameters.

	% change	p_1	p_2	p_3	T	u	I_0	Π
		($/unit)	($/unit)	($/unit)	(cycle)	($/unit)	units	($/cycle)
c	− 30%	83.01	101.07	82.31	8.94	89.24	461.88	1655.89
	− 10%	81.74	97.90	81.96	8.88	88.02	452.92	1450.55
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	80.47	94.74	81.58	8.81	86.24	444.11	1247.72
	+ 30%	79.20	91.60	81.17	8.74	83.76	435.21	1047.39
c_d	− 30%	81.75	97.18	81.75	8.85	89.13	465.99	1357.17
	− 10%	81.33	96.61	81.77	8.85	87.78	454.50	1351.78
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	80.88	96.01	81.78	8.84	86.72	442.40	1345.69
	+ 30%	80.43	95.37	81.77	8.81	85.98	429.82	1338.90
h	− 30%	81.43	96.76	81.77	8.85	88.51	457.35	1353.14
	− 10%	81.22	96.47	81.78	8.85	87.64	451.50	1350.30
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	81.00	96.16	81.78	8.84	86.78	445.51	1347.30
	+ 30%	80.77	95.85	81.77	8.83	85.93	439.38	1344.12
α_1	− 30%	105.43	127.37	94.69	9.29	107.81	693.34	1725.42
	− 10%	85.84	102.41	84.38	8.95	93.14	497.96	1421.98
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	77.75	91.97	79.89	8.76	81.68	412.71	1297.48
α_2	+ 30%	73.30	86.13	77.31	8.64	70.90	363.78	1230.58
	− 30%	84.87	112.36	91.77	9.55	100.00	679.60	1446.74
	− 10%	81.91	99.68	83.88	9.02	90.46	476.61	1368.66
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	80.51	93.88	80.26	8.71	84.57	427.92	1334.75
	+ 30%	79.70	90.61	78.23	8.53	80.53	399.87	1316.31
α_3	− 30%	81.18	96.50	82.07	8.89	87.40	451.05	1349.36
	− 10%	81.12	96.36	81.85	8.86	87.25	449.14	1348.95
	0%	81.11	96.32	81.78	8.85	87.21	448.52	1348.82
	+ 10%	81.09	96.28	81.72	8.84	87.17	448.04	1348.71
	+ 30%	81.07	96.23	81.64	8.83	87.12	447.32	1348.56

within a system. The preservation gives the way to the industry manager for increasing the lifespan of the deteriorated products. But, the interesting relation between the deterioration and the preservation is that after a threshold value of the deterioration, the preservation technology cannot protect that product from the full deterioration. Whenever the deterioration rate of a product is inversely proportional to the time, the constant investment is not worthy. The dynamical investment is appropriate and realistic rather than the constant investment. This study finds the threshold value of the deterioration. Now, the worthy dynamic investment for the deterioration is that the investment can be adjusted based on the time and the situation of the product. If the deterioration level reaches to the threshold value, the investment will no longer beneficial for that product. Thus, the manager can save the investment money. The industry manager can investment money based on the product's quality and the time. The promotion of a product is established as fruitful for the deteriorated, products for fast selling. Thus, the industry manager will earn more revenue from those products whose lifespan are controlled by some dynamic investments of the preservation technology. The manager can decide the investment amount based on the quality of the product at that specified time.

9. Concluding remarks

This paper analyzed a dynamic promotion rate based on several price range and studied an optimum investment to reduce the rate of deterioration by preservation technology considering their lifespan. Those products were used, whose demand pattern follows the trapezoidal-type. Due to the dynamic investment rate and trapezoidal demand, the profit was maximized with Pontryagin's maximum principle and MFPA Algorithm together, to calculate simultaneously selling price, investment in promotion, investment in preservation technology, and optimal order quantity. Using the MFPA algorithm, the mathematical model obtained the optimum profit of the maximization problem. As the time-dependent promotional effort was used within the variable demand, thus, the control theory was utilized to solve the proposed study. Utilizing the time-dependent investment for retailer's promotional effort, the study obtained the optimum dynamic investment based on time. Numerical study gave the benefit of the dynamic investment. Through the sensitivity analysis, the effectiveness of the key parameters are discussed. The present study contributed to the literature in the following ways. First, the dynamics of investment in promotion and its impacts were studied analytically under the price and promotion dependent trapezoidal-type demand. The investment rate in promotion was decreases as time progress and the retailer can earn more profit by monitoring investment strategy. Second, under the trapezoidal-type demand, the retailer should apply the price discrimination at each stage of the product lifespan and determined the optimal replenishment time to get the optimal profit. Moreover, the retailer should decide pricing strategy, i.e., penetration or skimming or their combination according to price sensitivity of the product at different stages. Third, invest in the preservation technology is not always profitable for the retailer and the retailer should take decision on this issue based on nature of the product. Finally, the retailer may determined the optimal replenishment time to gain the maximum profit under the promotion-price-and trapezoidal-type demand.

Possible future extensions from this can be done in several ways. For instance, one may analyze the impact of dynamic investment in preservation technology on optimal replenishment decision. It is assumed that the retailer has opportunity to set uniform price throughout each stage of the demand function, therefore it will be an interesting idea to form the model by assuming the impact of the price differentiation at each stage. In addition, researchers may extend the model by considering partial backlogging, time-dependent deterioration item, boundary points of each stage as fuzzy number. Lead time plays an important role for the deteriorated products. This study can be extended by using stochastic lead time (Malik and Sarkar, 2020) within the inventory model. The advanced payment policy for the can be used in the extended study (Mashud et al. (2020)). Combination with dynamical investment and advanced payment strategy may make more strongest study than the present scenario. The carbon emission scenario (Mishra et al. (2020)) can be an extension of the model in the virtue of the environmental perspective to save the nature along with the sustainable movement of the inventory in different places. The sustainability is one of the main concerns of the industry (Sarkar and Sarkar, 2020). This study considered only single type of products. If multiple products are considered and those products are considered as substitutable/complementary to each other then a big competitive pricing strategy can be considered. That will be a big research area for the retailer's point of view (Sarkar et al. (2017)).

Appendix A. List of additional notation used to simplify expression

$$\frac{h+\theta c_d}{\theta} = \omega_1;\ \frac{(e^{\delta_1 t}-e^{-\delta_1 t})}{2\delta_1} = \omega_2;\ \frac{(e^{-\delta_1 t}-e^{-\theta t})}{\theta-\delta_1} = \omega_3;\ \frac{e^{\delta_1 t}-e^{-\theta t}}{\theta+\delta_1} = \omega_4;\ \frac{e^{-\delta_2 t}-e^{(\theta-\delta_2)\mu_1}e^{-\theta t}}{\theta-\delta_2} = \omega_5;$$
$$\frac{e^{\delta_2 t}-e^{(\theta+\delta_2)\mu_1}e^{-\theta t}}{\theta+\delta_2} = \omega_6;\ \frac{e^{-\delta_3 t}-e^{(\theta-\delta_3)\mu_2}e^{-\theta t}}{\theta-\delta_3} = \omega_7;\ \frac{e^{\delta_3 t}-e^{(\theta+\delta_3)\mu_2}e^{-\theta t}}{\theta+\delta_3} = \omega_8;\ \frac{(e^{\delta_2 t}-e^{2\delta_2\mu_1}e^{-\delta_2 t})}{2\delta_2} = \omega_9;$$
$$\frac{(e^{\delta_3 t}-e^{2\delta_3\mu_2}e^{-\delta_3 t})}{2\delta_3} = \omega_{10};\ \omega_4-\omega_3 = \omega_{11};(\omega_6 - e^{2\delta_2\mu_1}\omega_5) = \omega_{12}\ (\omega_8 - e^{2\delta_3\mu_2}\omega_7) = \omega_{13}.$$

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